UNISA

A global perspective of mathematics teaching: Implications for South Africa

Inaugural lecture

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Abstract

Mathematics is a fundamental requirement to pursue a career in an economic sector that the 2010 report on South African Millennium Development Goals (MDG) considers essential for economic development and growth. The report further cites poor quality education provision as an impediment to acquisition of knowledge and skills that are essential for participation in the key economy sectors. At the centre of the concern raised in the report is poor learner performance in mathematics which is attributed to poor mathematics teaching. Hence, the focus of the inaugural lecture is on mathematics teaching.

The lecture is premised on the notion that effective mathematics teaching is critical for successful learning. For teachers to be effective in their teaching it is paramount for them, among others, to deeply understand the learning process and their role in learning so that they can prepare and plan lessons in such a way that learning can be more conceptually inclined. Furthermore, the lecture attempts to unravel and profile the state of mathematics teaching in some of the countries that have achieved better than South Africa in the international mathematics tests such as TIMSS, PISA and SACMEQ. In particular, this is done by comparing the teaching of mathematics in South Africa with the way it is done in Netherlands, Finland, Japan, United States of America and Zimbabwe. The comparison is largely intended to provide insight into ways mathematics is taught in these countries and factors behind their success in mathematics learning that can be of benefit to South Africa. Through this understanding, in particular it is hoped that South Africa could review and improve on its mathematics teaching approach and strategies. Largely drawing on the findings of research done, either on my own or jointly, issues such as gender-stereotypes; inadequate teacher content knowledge; the quality of teacher education; the status of teaching profession and school leadership tend to hinder effective mathematics teaching. Attempts are also made in this lecture to illustrate how the quality of teacher education and status accorded to the teaching profession in South African compare with those in Finland and Japan where teaching enjoys high social status; is meant for the best achievers; and pays competitive salaries.

It is therefore concluded that quality mathematics teaching in South Africa will continue to be a phantom unless

- there is a quality teacher education that refreshes teachers' competencies;
- teachers make efforts to understand how their learners think and learn, and recognise the learning experiences of their learners; and
- teachers are given the necessary support by the authorities.

Introduction

It's really a distinct honour and special privilege that I have been granted by the University of South Africa to deliver this inaugural lecture. I feel very elated for the opportunity to present the second inaugural lecture in the Institute for Science and Technology Education (ISTE) and the first in the College of Graduate Studies. From its humble beginnings in 2006 the Institute has been on a growth trajectory as evidenced in the swelling numbers of students enrolled and graduating from its degree programmes; of publications finding their way into peer-reviewed journals and conference proceedings; and of academic forums (i.e. seminars, symposia, workshops and conferences) it continues to organise on regular basis. I am pleased to be associated with the achievements of ISTE such as having the proceedings of its annual international conference that it started in 2010 being accredited by the Department of Higher Education; yearly graduating an ever increasing number of doctoral and master's students; and for developing and offering a unique honours programme in mathematics, science and technology education. The achievements of ISTE are largely ascribed to its visionary and enterprising leadership of the outgoing head, Professor HI Atagana and the supportive and diligent staff to whom I pay tribute.

Inaugural lectures are meant to showcase and celebrate the University's newly appointed professors, who have to illustrate what they stand for in their respective disciplines. I have noted that in some universities, there is a time lag that even stretches into years between when one is appointed as a professor and when the professor gives the inaugural lecture. It is a thing that is not practised at UNISA, where a professor gives an inaugural lecture within a year of appointment.

That said, I think it may now be opportune to declare my credentials as a mathematics educationist and profess the discipline of mathematics education in the Institute for Science and Technology Education. As a mathematics educationist I consider myself a mathematician with an inclination to the teaching and learning of mathematics. My research objective has essentially been to seek ways to improve learner performance in mathematics and to teach mathematics in a familiar context as required by the curriculum. Hence I conceptualised my research around the notion that mathematics learning can be better and fun if it happens in a familiar context with the teachers' role being to facilitate and mediate learning. To this end, teachers have

- to be aware that learners bring knowledge and experiences to class that should be recognised when teaching;
- to formulate strategies that enable learners to discover and develop knowledge; and
- to create an atmosphere in class that promotes learning.

Hence, part of my research has been in ethno-mathematics, where I identified the mathematical aspects embedded in the socio-cultural activities and artefacts. The intention was to debunk the perception

- that mathematics has a Eurocentric origin, is a body of knowledge that is abstract and has no connection or relevance to our everyday lives;
- that mathematics is only a subject taught and learned in formal classroom settings; and
- that socio-cultural activities and artefacts have no potential use in the teaching and learning of mathematics.

Teaching mathematics in a familiar context entails adopting an ethno-mathematical approach in its teaching and learning. Obviously, teachers have to be accustomed to this approach through an appropriate teacher development programme where they are familiarised with the concept of ethno-mathematics and trained on how to use the ethno-mathematical approach. An ethno-mathematical approach is

> a learner-centred activity-oriented teaching approach that focuses on the mastery of mathematics content and induces affinity for mathematics through relevant real-life activities that are familiar to learners (Mogari, 2014: 3).

The study project on the ethno-mathematical approach somehow stimulated my interest on teacher development particularly on ways to enhance teacher proficiency in teaching. It is for this reason I became involved in the UniveMalaShi project that was underpinned by a systemic reform model to optimise the implementation of a new curriculum. The project was conceptualised around the view that a new curriculum can be successfully implemented if teachers are adequately prepared and empowered with necessary knowledge and skills and enjoy the support of the district officials, school management, parents and the community at large. We therefore designed a teacher in-service programme (i.e. the UniveMalaShi project) to influence teachers' attitudes and beliefs about the new curriculum with a view to effecting the necessary teacher change as well as improving and aligning, accordingly, their content knowledge and classroom practice (Onwu & Mogari, 2004). Research activities in the project were intended to provide a mirror with which the project could see itself by evaluating and constantly reviewing the implementation process of the systemic reform model.

Furthermore, the continued poor performance in grade 12 science and mathematics examinations prompted us, at the Institute for Science and Technology, to come up with an initiative that can help redress and improve the results. It is on this basis that a Winter School Programme for Mathematics, Science and Technology teachers was conceptualised. This is an annual programme that seeks to improve the teachers' subject matter knowledge and classroom practice. At the beginning of each year we attend the Gauteng Education Department's Roadshows. These are sessions that take place in each district in Gauteng where the respective subject specialists and grade 12 teachers meet and discuss the examiner's report on how learners handled various questions in the preceding end-of-the-year examination. This provides us with inkling on learners' difficulties and deficiencies. The data are then corroborated with that we glean from teachers and learners on topics in the syllabi they have difficulties with. Based on this comprehensive data we then organise the Winter School programme for the year. Research is then conducted to do the process and impact evaluation of the programme (Atagana, Mogari, Kriek, Ochonogor, Ogbonnaya & Makwakwa, 2009; Atagana, Mogari, Kriek, Ochonogor, Ogbonnaya, Dhlamini & Makwakwa, 2010 & 2011; Atagana, Mogari, Kriek, Ochonogor & Makwakwa, 2012 & 2013).

I have also looked into teachers' competencies and readiness to deliver quality teaching (Makwakwa & Mogari, 2011, 2012 & 2013), teachers' misconceptions (Mogari, 1997), subject matter knowledge (Ogbonnaya & Mogari, 2014; Makwakwa & Mogari, 2011 & 2013) and beliefs (Mogari, 2014; Onwu & Mogari, 2004). I have lately started working on problem solving in mathematics (Dhlamini & Mogari, 2012 &

2013; Mogari & Lupahla, 2013) because it is the backbone of mathematics. For a learner to perform well in mathematics, sound and well-established problem solving skills are essential. Thus, I have mapped problem solving skills of learners and looked into how high-achieving learners go about solving non-routine problems (Mogari & Lupahla, 2013); and investigated possible ways to improve learners' problem solving skills (Dhlamini & Mogari, 2012). I consider this area of research important because it has potential to provide further insights into the deficiencies and difficulties learners have in solving mathematical problems.

Background

The 2010 report on South African Millennium Development Goals (MDG) indicates

- That South Africa has a sophisticated infrastructure, well-developed private sector and stable macro-economy, but suffers inequality in education and this has compromised quality education provision in most areas.
- That poor quality education increases school dropout rate and impacts adversely on the process of development of knowledge and skills, thus compromising creativity.
- That a largely poorly educated community tends to have high levels of unemployment, poverty and poor standard of living while, on the other hand, people endowed with the appropriate knowledge and skills as a result of quality education are better positioned to obtain meaningful and decent formal employment and can also easily adjust to a dynamic and fast-moving knowledge-based economy (MDG, 2010).

It is common knowledge that South Africa has prioritised economic growth in order to provide highly needed and sustainable jobs particularly in key economic sectors that are listed in the 2010 MDG report, which are mining, agriculture, tourism, transport, energy and manufacturing. I also consider construction, health, education and finance as important economic sectors for economic development. For one to participate adequately in most of these sectors, he has to acquire the necessary knowledge and skills through a formal university programme that requires a pass in matriculation mathematics. For example, a pass in matriculation mathematics is an essential requirement if one is to read engineering, accounting, medicine, building science, actuarial science, agricultural science and applied science. According to the National Council of Teachers of Mathematics (NCTM) (2000) ‘mathematical competence opens doors to productive futures’ (p1).

For South Africa to achieve its goals of development it certainly has to emphasize the teaching of mathematics, given that teaching plays a critical role in learning. I note that the link between teaching and learning is well documented (see, for example, Hiebert & Grouws, 2007; Anthony & Walshaw, 2009; Kersting, Givvin, Sotelo & Stigler, 2010). Furthermore, in its report on 2013 Grade 12 Mathematics Examination performance sent to the Department of Basic Education by the Association of Mathematics Education of South Africa (AMESA) wrote:

Paper 1

> The paper could be classified as a very reasonable paper. If learners were taught well, then a level 4 (minimum 50%) would be within reach (p2).

Paper 2

> It would appear from learners’ views after the paper that they found the questions quite challenging, possibly because of the higher level 4 proportion of the paper. At the same time if learners were taught properly and all the work was covered in class, then there were sufficient questions in the paper for learners to pass (p8).
>
> AMESA (2013)

In essence, AMESA bemoans the quality of mathematics teaching that, seemingly, disadvantaged learners in the development of the requisite knowledge and skills to solve the given problems in the examination. In addition, the examiner expressed concern that teachers did not meaningfully explain concepts in context; learners were not attuned to move from word problems to equations; and learners could not solve questions requiring high-order thinking skills hence learners did not generally achieve satisfactorily in 2013 Grade 12 Mathematics Examination (Chauke, 2014). In sum, the respective reports by AMESA and the examiner attribute learners’ inadequacy to poor quality teaching. Modisaotsile (2012) adds that on average teachers in the township schools teach for 3½ hours a day as compared to 6½ hours in the former model C schools. It is against this background that the focus of my lecture is on mathematics teaching. In this lecture, I will explore the role and impact of my research on the teaching and learning of mathematics; compare the teaching of mathematics in South Africa with countries that have achieved better in the international tests on mathematics in order to get insight into their ways of mathematics teaching and factors behind their success in mathematics; discuss some factors that hinder effective mathematics teaching; and lastly I will make recommendations that can contribute to the improvement of mathematics teaching in South Africa and spell out my research plans for the future.

Mathematics learning

From a point of view of cognitive science, Piaget's theory indicates that a learner learns on his own without any intervention by perceiving either an object or a phenomenon in his environment. His mind then processes what he has perceived. If what he has perceived is compatible with his cognitive structure it gets processed with ease and this is assimilation. Where there is no match between what a learner has perceived and his cognitive structure, the cognitive structure modifies itself to enable the processing of what has been perceived and this is accommodation. To this end, learning may be perceived as the process whereby knowledge is created through transformation of experience (Kolb & Kolb, 2005). What Piaget's theory suggests is that learning is a continuous process that happens independently of any external facilitation. Snoek and Wielenga (2001) concur with Piaget and define learning as a continuous process where a learner makes efforts to connect new information with prior experiences, individual ideas, insights and conceptual notions. To illustrate this notion of learning by perceiving, let me share with you the story of Carl Friedrich Gauss (1777 – 1855), whose father was a foreman in a masonry firm. Gauss knew how to reckon before he could even talk. He used to watch his father prepare wages of his workforce. When he was three, one day he corrected an error in his father's calculations of wages. May be it is because his schema for reckoning were well established, hence at the age of 10 his teachers revealed that there is nothing more they could teach him because he seemed to be knowing everything they tried to teach him (Burton, 1985). I, in turn, tried to investigate the genesis of the geometrical knowledge the miniature wire car makers used. I wanted to establish how the car makers develop the geometrical knowledge they used in the activity of making cars with wire. My findings were that the boys developed their knowledge by first observing others and then engaging in 'hands-on' activities, following the procedures of the more skilled peers (Mogari, 2004). Therefore, it is my view that when a teacher plans his teaching, he should consider learners' prior knowledge, thinking and perceptions.

On the other hand, Vygotsky theory considers social interaction key in the learning process. According to the theory a learner learns by actively participating in a sociocultural context and learning happens in the zone of proximal development (ZPD) which is defined as

> the distance between the actual development level as determined by independent problem solving and level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers (Vygotsky, 1978: 33).

In terms of Vygotsky's theory, it is the teacher's responsibility to ensure that learners function above their actual level of development. The teacher has to mediate learning by monitoring the learners' group-work and ensure that the class groups are diversified as far as possible in terms of ability so that the more knowledgeable learners can mediate their peers' learning. In my study on the origin of the geometrical knowledge of the wire car makers, I also found that the more skilled boys coached and guided their novice peers and this enabled them to develop knowledge or perfect what they already knew (Mogari, 2004). We also investigated the effects of group-work approach on the teaching of some mathematical aspects to grade 10 learners in the township schools and our study found that the group-work approach has potential to facilitate learning (Dhlamini & Mogari, 2013). Nevertheless, my research has revealed that gender-stereotypes tend to manifest themselves during group work where boys subdued girls consequently disadvantaging them in their learning (Mogari, 2001) [Later, I will revisit the issue of gender-stereotypes].

From the preceding discussion on the two theories, the notion that a learner's mind is a tabula rasa that has to be filled up with knowledge is dispelled. I also gather that learning does not happen passively through transmission, it is instead a continuous process of knowledge construction that happens better through active participation, either individually or collaboratively, on a given task. Secondly, interacting with surroundings or those around you is also essential in facilitating knowledge construction and, lastly, successful knowledge construction depends on existing knowledge or what the learner already knows. Thus, from the lamentation by AMESA, it is important that teachers should teach 'properly' and 'cover all the work' to avoid disadvantaging learners (AMESA, 2013).

Another issue that plays a significant role in learning are ways and means to facilitate the process and make it effective and meaningful so that learners can use what they have learnt in their everyday lives. My work on ethno-mathematics attempts to do this (see Mogari, 2002; 2007 & 2014). I used a socio-cultural activity of making wire cars in the learning of some aspects of Euclidean geometry (see Mogari 2002 & 2007). Using such an activity entails designing the necessary teaching-learning support material and providing wires and other required materials to make cars. Learners develop mathematical knowledge by following the instructions set out in the teaching-learning support material as they make the cars (see Mogari 2002 & 2007). Similarly, as we shall see later, the Dutch use the Realistic Mathematics Education (RME) approach and the Japanese put emphasis on solving real world problems to facilitate mathematics learning.

I therefore consider the insight into learning fundamentally important to teachers, especially when they prepare and plan lessons. Good teachers know their learners; can identify their learners' learning difficulties or factors that have potential to impede learning; understand how learners think and what they are thinking; anticipate possible learners' misconceptions; organise content in a way learners will find it easier to learn; come up with appropriate teaching strategies to facilitate learning and make it exciting; and organise necessary 'resources' to help learners learn content better and meaningfully. If mathematics learning is to be improved, it is paramount to understand the learning process (Stigler & Hiebert, 1999) and for teachers to understand their role in learning.

Mathematics teaching

Learners in some countries have been performing consistently well on international mathematics tests such as the Trends in International Mathematics and Science

Studies (TIMSS) and Programme for International Student Assessment (PISA). Therefore, one wonders: what is it that mathematics teachers in these countries have and do differently in their teaching that tends to produce desirable results. Perhaps, it might be of interest to examine briefly how mathematics is taught in these countries.

Netherlands: The idea of infusing real life and contextual problems into mathematics teaching is encouraged in Holland through the realistic mathematics education (RME) approach. In RME,

> Mathematics must be connected to reality, stay close to children and be relevant to society, in order to be of human value. Instead of seeing mathematics as subject matter that has to be transmitted, I see the idea of mathematics as a 'human activity'. Education should give students the "guided" opportunity to "re-invent" mathematics by doing it. This means that in mathematics education, the focal point should not be on mathematics as a closed system but on the activity, on the process of mathematization, going from the world of life into the world of symbols.
> Freudenthal (1977)

Freudenthal emphasises learning mathematics, through interactions with peers or the teacher, from real life and contextual problems and from what has been invented or discovered. This made Treffers (1991) to come up with two forms of mathematization, namely, horizontal mathematization (i.e. using mathematics aspect to organise and solve a problem embedded in real setting) and vertical mathematization (i.e. reorganising mathematics aspects to discover connections between concepts and strategies and then using what has been discovered). Education is thought to go beyond just acquiring knowledge and skills, instead its primary goal should be to have capacity to connect knowledge and information from various sources, to check it and restructure it until it is relevant and valuable for one's everyday life (Snoek & Wielenga, 2001). RME thus rejects the mechanistic and procedure-focused way of teaching and learning mathematics instead it encourages learners to learn mathematics by developing and applying mathematical concepts and tools in daily-life problem situations that make sense to them.

(Van den Heuvel-Panhuizen, 2003: 9)

Japan: As in South Africa, the Japanese government decides on the curriculum and determines the duration of mathematics lesson and number of mathematics lessons per week (Judson, 1999). Mastrull (2002) indicates that mathematics lessons are generally scheduled during first periods of the day. Hiebert and Stigler (1999) point out that mathematics teaching in Japan follows a structured problem solving approach where teachers play a less active role and learners are afforded time to invent their own solution procedures, analyse and proving. Hiebert and Stigler further indicated that a Japanese mathematics lesson focuses strictly on a particular concept with the teacher connecting different parts of the lesson to ensure coherence. A typical lesson starts with a review of the previous lesson; then the problem of the day is presented; learners would then work through the problem either individually or in groups; learners' solution strategies and procedures will then be discussed by the teacher and class; and the teacher will conclude the lesson by highlighting and summarizing the major points as well as giving learners not more than four problems as homework (Hiebert & Stigler, 1999). According to Mastrull (2002), there is nothing extraordinary about the intelligence of the Japanese learners, they are undoubtedly characterised by hard work where they are in school for $5\frac{1}{2}$ days in a week and about 80% of them receive supplementary school because they know that good performance determines the type of high school, college (i.e. university), graduate school and eventually career they will follow.

Finland: It is reported that teaching in Finland is scheduled for 45 minutes with a 15-minute break in between lessons, which are meant to refresh so that teachers can be more effective in their teaching and learners can stay attentive and sharp (Savola, 2010). Problem solving has also been given much emphasis in Finland since 1980s (Pehkonen, 2008). In class, teachers use the traditional teacher-centred whole-class interaction because they believe in building strong relationships with their learners, but teachers still make a point to voluntarily interact with individual learners during seatwork and insist on the use of proper terminology by learners to explain and justify their thinking (Savola, 2010). Learners are given homework daily and teachers hold learners accountable for completion of the homework (Savola, 2010). The benefits of all these initiatives are evident in the learners' achievement in TIMSS & Pisa.

United States of America: In the United States of America, there is no national curriculum. Each State or education district (within a state) develops its own curriculum, assessments policies and procedures as well as professional development initiative are aligned to the national academic standards that stipulate what a learner should know and be able to do and a standardized math tests is used to determine these competencies (Ferrini-Mundy, 2000). Mathematics teaching is geared towards preparing learners for the workplace, further education and for citizenship. The National Council for Teaching Mathematics (NCTM) developed Principles and Standards that insist on mathematics being taught as an activity and guide the instructional efforts to shape the learners' abilities to explore, conjecture, reason logically, formulate and solve problems, communicate mathematically and develops their confidence in doing mathematics (NCTM, 1992: vi); and for teachers to be effective in mathematics teaching they should understand what learners know and need to learn, and then challenging and supporting them to learn it well (NCTM, 2000: 2). It is however noted that the teaching of mathematics is done differently from district to district or state to state across the country. For example, the state of Connecticut developed 'Math Connections' that seeks to show learners the interrelationship among topics within mathematics plus between mathematics and other subjects and also introduces learners to technology through computers and graphic calculators (Hopkins, 2008). According to Hopkins 'Math Connections' has been acclaimed; because

i. In post-high school education, learners who used the programme tend to outperform those who did not.
ii. It helps learners apply their knowledge, critical-thinking and problem-solving skills.
iii. It integrates different topics in mathematics and as a result learners no longer have a piece-meal perception of mathematics (i.e. as a set of disjoint aspects).

iv. It provides real world situations and context to make sense of various mathematical aspects, and it thus enables learners use mathematics in their everyday lives.
v. It has made mathematics exciting and fun to learn.

I note that somehow there are similarities between ‘Math Connections’ and the ethno-mathematical approach I used in my research (see Mogari, 2002, 2007 & 2014). The mathematics teaching approach in Connecticut is different from that of other parts of the United States of America. Dossey, Halvorseen and McCrone (2012) noted heavy reliance on textbooks that contain theoretical tasks intended to practice routine problem solving procedures. This means that no connection is made between real life context and mathematics aspects in class. Calculator use has become popular over the years (Dossey et al, 2012) and in contrast calculators are forbidden in Japan (Mastrull, 2002). In our study on the effects of calculator use on the arithmetic proficiency of undergraduate mathematics 1 students, we found that those who started to use calculators early in their schooling and those who habitually used calculators were disadvantaged in mental computations (Mogari & Faleye, 2012). Dossey et al (2002) indicate that the mathematics curriculum is topic-based and learners tend to select topics to study. There have however been attempts to blend topics such as algebra, geometry, functions and data analysis in highly connected and integrated body of knowledge where emphasis is on modelling and applications but, regrettably, the number of learners enrolled in such mathematics courses has remained small (Hirsch, 2007).

Zimbabwe: The school curriculum in Zimbabwe is underpinned by Ralph Tyler’s four considerations of purpose, experience, organisation and evaluation (Bilbao, 2008). According to Ncube (2013) effective mathematics teaching is more prevalent in private schools than in public schools where teachers display low morale due to low salaries and poor working conditions. Nevertheless there are still traces of hard-work and commitment among some teachers, who unlike their disheartened colleagues make clear the topics they intend on teaching, the objectives to be achieved and they also link content taught with the one already taught (Ncube, 2013). In a typical lesson, a teacher first introduces the content of the day then provides illustrative example(s) and thereafter learners are given tasks to collaboratively work on

(Mtwetwa, 2013). Mtwetwa also reports that problem-solving activities are virtually non-existent in lessons, and despite group-work approach being generally used, responding in chorus to questions the teacher poses is predominant.

South Africa: In the South Africa's Curriculum and Assessment Policy Statement (CAPS) document, mathematics teaching in grades 10 – 12 has been allocated $4\frac{1}{2}$ hours per week and is taught in order 'to develop mental processes that enhance logical and critical thinking, accuracy and problem solving' (Department of Basic Education, 2011: 8). The CAPS document also encourages the incorporation of real life and contextual problems in mathematics teaching (Department of Basic Education, 2011: 8), because teaching mathematics in the learners' familiar context tends to facilitate learning and make it fun (Mogari, 2002). My research has showed that incorporating real life activities in mathematics teaching can facilitate learning (Mogari, 2002 & 2007) and so does our research on the use of context-based problem solving instruction (see Dhlamini & Mogari, 2012). Nevertheless, mathematics teaching tends to be examination driven where rote learning and memorisation are the norm. My research confirms this observation because I found that teachers who participated in an in-service programme on an ethno-mathematical approach, when they returned to their classes resorted to structured direct teaching because they were eager to adequately prepare learners for the examination that was drawing close (Mogari, 2014). Nevertheless they only used the ethno-mathematical approach upon request and used the approach perfectly for that matter (Mogari, 2014). Indeed, one would have expected the teachers to have enthusiastically adopted the ethno-mathematics approach in their teaching particularly because this is what the curriculum document stipulates and encourages.

Thus, I do believe the above cases of mathematics teaching provide insights into the processes of teaching that tend to either facilitate or impede learning. For example, the Japanese, Dutch, Finnish and Connecticut scenarios illustrate ways to improve learning as well as to develop learners' high order thinking and reasoning skills, and sophisticated problem solving skills, while the general American and South African cases show ways of mathematics teaching that lead more towards procedural and superficial learning.

Some Impediments to Mathematics Teaching

Drawing on our research and literature there are factors that hinder effective mathematics teaching.

1. *Gender-stereotypes*

Gender-stereotypes in the teaching and learning of mathematics have a considerable history. In countries such as the United States of America, England and Australia girls were not allowed to learn mathematics or any of its sub-disciplines for the better part of the 19th century because they were thought to be lacking the necessary capacity to do well, and it was only in the last century that a large number of girls started enrolling in mathematics (Leder, 1992). Nevertheless, fewer girls were still participating in mathematics in early 1980s (Fennema, Wolleat, Pedro & Becker, 1981) and this is still the case in the contemporary era (see Watt, 2007; Mandina, Mashingaidze & Mafuta, 2013).

There is evidence that gender-stereotypes have influenced gender differences in mathematics achievement (see Nosek et al, 2009). After reviewing a number of studies, Cai (2003) noted that despite it being evident that the gender-related performance difference in mathematics was decreasing, girls continued to show lower educational aspirations in mathematics; less confidence about their mathematical ability and lower perception about the usefulness of mathematics to them.

Research shows that girls showing interest in studying mathematics are made to change their minds by being inappropriately treated by teachers (see, for example, Jung & Chung, 2005; Scantlebury, 2008; Spielman, 2008) or boys (Jung & Chung, 2005; Scantlebury, 2008; Park, Behrman & Choi, 2011). Gender-stereotypes were also evident in my study on the use of an activity of making a miniature wire car in the teaching of some mathematical aspects (Mogari, 2002). I have argued that social factors in some instances have forced girls to opt for careers not related to mathematics, and this is regardless of their noted potential in mathematics (Mogari, 2010). Based on the findings of my study I concluded that gender-stereotypes disadvantage girls, hamper their learning and instil a sense of inferiority in them (Mogari, 2002 & 2010).

Evidently gender-stereotypes rear their heads in terms of teachers' gendered selection that favours boys for participation in various lesson activities and detestable lesson management that made boys dominate lessons, thus quality of mathematics teaching is compromised.

2. *Inadequate Teacher Content Knowledge*

Evidence showing inadequate level of teacher content knowledge abounds. For example, my research showed misconceptions a group of practicing mathematics teachers had about infinity (∞) (Mogari, 1997). We found that Grade 11 teachers battled to teach probability because of their poor content knowledge (Makwakwa & Mogari, 2011). Our research at ISTE shows that there are high school teachers with a limited knowledge of some topics in the syllabus (see Atagana, Mogari, Kriek, Ochonogor, Ogbonnaya & Makwakwa, 2009; Atagana, Mogari, Kriek, Ochonogor, Ogbonnaya, Dlamini, & Makwakwa, 2010 & 2011). Deficient teacher content knowledge is also prevalent in studies by Bansilal, Brijlall and Mkhwanazi (2014) and Taylor (2011). Teachers with limited content knowledge will not be a position to simplify aspects of content so that learners can learn them with ease and also make the content interesting and relevant by relating it to real life context.

Furthermore, judging by the achievement of South African learners in TIMMS & PISA and supported by the findings by Onwu and Mogari (2004); Mogari et al (2009); Atagana et al (2009); Atagana et al (2010 & 2011); Taylor (2011) and Bansilal et al (2014), I am of the view that the South African teacher education programmes are less rigorous in terms of the extent to which they prepare teachers. Obviously one cannot be effective in teaching with some inadequacies. Put differently, one cannot simply teach well what she/he does not know well. Moreover, that our research showed that teachers with better quality qualifications tend to be more effective in their teaching (Mogari, Kriek, Stols & Ogbonnaya, 2009). I refer to such qualifications as those that are rich on subject content knowledge, pedagogical content knowledge and pedagogical knowledge and focus evenly on all these forms of knowledge.

3. Quality of Teacher Education

Our research has showed that a well-designed and effectively taught teacher development programme has capacity to improve and develop teachers (Mogari et al, 2009) who are adequately prepared to teach more effectively (Onwu & Mogari, 2004). Shulman (1986) showed that successful mathematics teaching is a function of teacher content knowledge, pedagogical knowledge and pedagogical content knowledge. Thus, it is my view that quality mathematics teaching is a function of teacher's performance which in turn depends on his ability and competency as well as the quality of his teacher education programme. Suffice it to say at this point that the quality of a teacher qualification depends on the quality of the teacher development programme. Also noting that we found a strong significant positive relationship between teacher qualification and learner achievement (Mogari et al, 2009), I now compare the teacher preparation process in Finland and Japan with that of South Africa.

Finland: According to Sahlberg (2010) until the mid-1970s primary school teachers were trained in teacher colleges while secondary school teachers were prepared in universities, and this arrangement changed towards the end of the 1970s so that all pre-service education was university based. During teacher preparation in Finland, a balanced development of personal and professional competency is a priority where secondary school mathematics teacher trainees are taught discipline mathematics as well as mathematics didactics with emphasise on pedagogical content knowledge (Sahlberg, 2010). Westbury et al (2005) indicated that during pre-service teacher training there is a strong focus on building pedagogical thinking skills that enable teachers to manage the teaching process according to educational norms and practice. The Finnish tend to favour a systematic approach to introduce and implement change in the education system. For example, systematic efforts were used to promote and implement problem solving in mathematics teaching by building it into pre-service and in-service education, seminars were then organised for teacher educators to introduce and demonstrate to teachers the use of problem solving approach (Pehkonen, 2008). A typical Finish teacher is a subject-focused and content expert, and one needs a minimum qualification of a master's degree to teach either in primary or secondary school while a bachelor's degree is an entry requirement for a teaching position in preschool and kindergarten (Sahlberg, 2010).

Teachers work independently and are trusted to make decisions they deem necessary to empower learners (Hancock, 2011).

Japan: A master's degree is also a minimum entry qualification into teaching in Japan (National Institute for Educational Research (NIER), 2013). Pre-service education covers mathematics content knowledge, pedagogical content knowledge and pedagogy, with most weighting given to content knowledge followed by pedagogy, and upon completion of their pre-service teachers have to take a competency examination before they can be appointed (Erbilgin & Boz, 2013). Shimizu (1999) reports that teachers are taught how to pose problems and anticipate learners' responses; to consider the diversity of experience and knowledge learners bring to class; and to intensely write a lesson's plan well. Shimizu further indicates that the novice teachers (i.e. newly appointed teachers) participate in induction programmes for a year. The induction programme entails workshops and mentoring, where novice teachers first observe their mentors' teaching, the mentors will then supervise and monitor novice teachers teaching (Shimizu, 1999). There is also an ongoing lesson study programme which is an in-service teacher development programme that deepens content knowledge, enhances understanding of pedagogy and develops ability to observe and understand learning (Mastrull, 2002). In this programme teachers stay behind after school and observe a colleague teaching followed by a critical discussion of the observed lesson.

South Africa: Pre-service teacher training was largely offered in the teacher training colleges until early in the 2000s. The teacher training colleges offered a two-year programme (i.e. Junior Secondary Teachers' Certificate), with a minimum admission requirement of a school leaving certificate, to students aspiring to be secondary school teachers, while prospective primary school teachers were enrolled in a two-year programme (Primary Teachers' Certificate) that required someone with only a junior secondary certificate (i.e. standard 8). It was only in the late 1970s that a three-year teacher training programme (i.e. Senior Secondary Teachers' Certificate) was introduced for those intending to teach matriculation. It was only in the early 1980s that three-year diploma programmes, with a minimum admission requirement of school leaving certificate, were introduced to replace all teachers' certificate programmes. The curricula of the diplomas still gave methodologies more attention than discipline content knowledge and subject matter knowledge. From the age profile of active teachers, it is evident that a considerable number of them obtained their pre-service from the training colleges. The demise of teacher training colleges in early 2000s led to universities taking over pre-service training and offered it through degree programmes that, on average, offer more discipline content knowledge and methodologies. In-service teacher education is in a form of formal university programme (e.g. Advanced Certificate in Education (ACE)) or informal programme largely offered by non-governmental organisations (NGOs) or tertiary institutions as part of their research projects.

4. Status of Teaching Profession

Finland: The sudden improvement in achievement by Finish learners on international tests since late 1990s is largely attributed to the quality of teachers (Westbury et al, 2005; Sahlberg, 2010). Teaching in Finland is the most admired profession; enjoys high status; and attracts a good salary and becoming a teacher is a very competitive process where only the best and the brightest normally fulfil the selection requirements which are mainly the highest scores and excellent interpersonal skills (Sahlberg, 2010). For example, in 2008 an average annual salary of a teacher was about $38 500 (OECD, 2008).

Japan: The status of teaching has been constantly improving since the end of World War II (NIER, 2013). Rapid industrialisation and economic growth increased demand for more skilled competent work force. This led to an intensified demand among learners for quality education that would prepare and enhance their chances for good employment. The status of the institution one studied in became a critical factor in securing employment. Thus, there was a stiff competition for places in best schools and tertiary institutions and a sudden high need for competent and dedicated teachers. Government enacted a law to improve teachers' salaries, and as a result there was sudden influx into teaching (NIER, 2013). Savvylearners (2000) reports that in 1985 a 40 year old teacher earned between US $33100 and $36200. Teaching has since become a prestigious career and the Japanese teachers are held in high esteem, command the general respect of the public largely and enjoy high social status (NIER, 2013). According Savvylearners, teachers are expected to infuse cultural values throughout school activities and to be concerned about students' lives both in and out of school. Their efforts and influence often extend into the home and the community.

South Africa: A typical learner who obtained remarkable results in mathematics and physical science in grade 12 would prefer to pursue a career in engineering, built sciences, computer studies, medicine, actuarial science, and so on. People in these careers are generally held in high esteem and also are perceived to enjoy better salaries. During the days of teacher training colleges, learners generally went into teaching after failing to gain admission into their intended career studies. Pre-service teacher training is now offered by universities that normally require a matriculation exemption for admission into degree programmes, and thus limits the number of learners eligible for admission into teaching qualifications. For example, in 2013 only 30.6% of learners passed with a matriculation exemption and for mathematics, of all the learners who sat for grade 12 examinations only 42.96% wrote mathematics and only 40.5% of them scored over 40% (Department of Basic Education, 2014b). Despite few learners being drawn into teaching, the teachers' salaries remain less competitive to attract a big number of recruits. The annual salary range for teachers with a relative education qualification value (REQV) of 14 and above is R185 184 - R419 085 and significant amount of experience would be required for a teacher to be on the maximum level (Moneyweb, 2014).

From the three cases, I note that competitive salaries and high social status afforded to teachers tend to encourage more high achievers to consider teaching as a career. This augurs well for the profession as it attracts people with deep mathematics knowledge and research shows that the level of teachers' content knowledge impacts on the quality of teaching (Shulman, 1986).

5. *School Leadership*

Quality leadership is a critical factor behind the success of an organization or institution. It actually sets the tone and cultivates a culture of commitment and performance in the personnel. Our research has showed that a well performing and high achieving school is related to strong leadership and good support by the principal and district officials (Onwu & Mogari, 2004). Furthermore, Modisaotsile

(2012: 5) points out that schools that are poorly resourced achieve good results because of strong leadership given by the principal. For example, in his study that sought to determine reasons for consistent good performance by poorly resourced schools with large classes, Mathebula (2004) found that the schools had policies that were enforced and adhered to; well-functioning monitoring and evaluation systems; motivated, committed and hardworking teachers who fulfilled their teaching responsibilities as expected; and strong element of collegiality and co-operation among staff. Mathebula found that the principals provided leadership that inspired teachers, made them to teach with enthusiasm and diligence, and this, in turn, stimulated learners and made them to work harder hence the good performance they consistently showed.

It is my deduction therefore that quality school leadership stimulates and enhances teacher and learner performance; and creates school conditions that are conducive for effective teaching and learning, while bad leadership demoralizes; compromises performance; and creates a dysfunctional school. Thus, I claim that sound leadership in a school engenders successful mathematics teaching.

Conclusion

In this lecture,

i. I have demonstrated that indeed mathematics teaching in South Africa is different from other countries. It is examination driven and hinges largely on transmission and rote learning. Mathematics is not portrayed as an activity, a resource or a social construct, instead it is generally taught as an abstract body of knowledge where learners have to prove and memorise theorems, and solve problems by simplifying and finding the value of the unknown.

ii. My research involving ethno-mathematics has highlighted the social dimensions and utility of mathematics; and how ethno-mathematics can be used as an instructional approach that has potential to facilitate and enhance learning as well as making it fun.

iii. Our research has provided evidence that in-service teacher education tends to be effective when it enjoys the support of school and education authorities (i.e. from circuit office upwards).

iv. I have noted that attracting the best achievers into teaching, as is the case with the Japanese and Finnish, is highly beneficial.

v. I have discussed the effects of effective school leadership, gender-stereotypes, elevating the status of teaching profession and quality teacher development efforts on mathematics teaching.

vi. I delved into the learning process with a view to highlighting the critical role played by teachers in learning.

It is therefore my conclusion that quality mathematics teaching in South Africa will continue to be a phantom unless

- there is a quality teacher education that refreshes teachers' competencies;
- teachers make efforts to understand how their learners think and learn, and recognise the learning experiences of their learners; and
- teachers are given the necessary support by the authorities.

Recommendations

Further to it being customary to make recommendations at occasions of this nature, I think it is paramount to exhibit how the preceding discussion can benefit mathematics, mathematics education, and mathematics teacher education. The following are my recommendations:

1. We have noted from the discussion that countries that recruit the best into teaching tend to perform well in international tests. Perhaps South Africa should replicate this practice and strive to identify and attract best achievers to the teaching profession. Obviously this can't just happen unless the status of the teaching profession is improved considerably by revising remuneration scales so that teachers enjoy the same benefits coupled with being afforded the same respect as engineers, chartered accounts, actuaries, and so on.
2. Admission requirements into pre-service teacher education has to be stringent to ensure that people being recruited and trained as teachers are of fitting qualities, i.e. have the necessary aptitude to help learners learn and 'soft skills' needed to guide and mould learners' characters into, at least, accountable, responsible and sincere adults.

3. I think there is a need to restructure pre-service teacher education programme to ensure that there is a strong balanced emphasis put on content knowledge, pedagogical knowledge and pedagogical content knowledge; and to reshape the teachers' beliefs about mathematics and its teaching.
4. There should be ongoing in-service teacher education that continuously retool and refresh the knowledge and skills of teachers and reshape their beliefs and thinking about mathematics teaching so that teachers can be more effective and enterprising in their teaching.
5. There should be mandatory induction and mentorship programmes for novice teachers that, among others, help them fit and settle well into the system as well as guide and coach them on the nitty-gritties of teaching.
6. The link between quality leadership and good performance is well documented. I think it is incumbent upon a school manager to create conducive atmosphere in the school for effective teaching and successful learning. It is common knowledge that complacency often creeps in when there is a constant repetition of same tasks; one starts to perform those tasks without thought; and one develops a false sense of security. I therefore reckon it would be appropriate to make appointments in to school management positions (i.e. principal, vice-principal and head of departments) fixed-term rather than permanent.
7. There should be more recognition given to research and its findings in mathematics education. In fact research should inform mathematics teaching and mathematics teacher development initiatives.
8. Mathematics teaching should not happen in isolation from other subjects across the curriculum. Mathematics should be taught in way that demonstrates its importance and usage across the curriculum. Such an initiative has potential to make learners realize the relevance and value of mathematics and hopefully might change their attitude and stimulate their interest to learn more mathematics.
9. Teachers are expected to produce results, and this stands out as critical measure of their performance during appraisal. Hence their teaching is examination driven and promotes rote learning and encourages memorization. Perhaps, expectations placed on teachers should rather be on the extent to which learners use mathematics efficiently in their daily lives and

this should be determined by learners’ capacity to solve real context mathematically-oriented problems rather than the obtained scores.

Future Research Plans

Learner participation and performance in mathematics in the grades 10 - 12 remains minimal. In 2013, for example, well under 50% grade 12 learners wrote mathematics examination and around 40.5% of them scored 40% or more (Department of Basic Education, 2014). Of concern was learners’ capacity to solve problems that required high-order thinking skills and to represent word problems with appropriate equations (Chauke, 2014). This is an indictment of the quality mathematics teaching that seems to fall short to encourage and stimulate conceptual learning. There is therefore a need to come up with a teacher development initiative that would expose teachers to teaching strategies that facilitate meaningful and conceptual learning so that learners can have capacity to solve problems requiring high order thinking skills and non-routine real context word problems. In sum, the envisaged teacher development initiative will seek to reshape the teachers’ beliefs and perceptions about mathematics and its teaching, hone and enhance their problem solving skills followed by training them to teach mathematics in a real life context as well as via problem solving where emphasise, among others, will be on ensuring that real life context or problems really serve as a resource to scaffold learners to conceptual learning. Research will focus on process and impact evaluation of the teacher development programme.

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