# The Role of Mathematics in First Year Students' Understanding of Electricity Problems in Physics 

by

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## Declaration

I declare that The Role of Mathematics in First Year Students' Understanding of Electricity Problems in Physics is my own work and that all the sources that I have used or quoted have been indicated and acknowledged by means of complete references.

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#### Abstract

Mathematics plays a pertinent role in physics. Students' understanding of this role has significant implications in their understanding of physics. Studies have shown that some students prefer the use of mathematics in learning physics. Other studies show mathematics as a barrier in students' learning of physics. In this study the role of mathematics in students' understanding of electricity problems was examined. The study undertakes a qualitative approach, and is based on an intepretivist research paradigm.

A survey administered to students was used to establish students' expectations on the use of mathematics in physics. Focus group interviews were conducted with the students to further corroborate their views on the use of mathematics in physics. Copies of students' test scripts were made for analysis on students' actual work, applying mathematics as they were solving electricity problems.

Analysis of the survey and interview data showed students' views being categorised into what they think it takes to learn physics, and what they think about the use of mathematics in physics. An emergent response was that students think that, problem solving in physics means finding the right equation to use. Students indicated that they sometimes get mathematical answers whose meaning they do not understand, while others maintained that they think that mathematics and physics are inseparable.

Application of a tailor-made conceptual framework (MATHRICITY) on students work as they were solving electricity problems, showed activation of all the original four mathematical resources (intuitive knowledge, reasoning primitives, symbolic forms and interpretive devices). Two new mathematical resources were identified as retrieval cues and sense of instructional correctness. In general, students were found to be more inclined to activate formal mathematical rules, even when the use of basic or everyday day mathematics that require activation of intuitive knowledge elements and reasoning primitives, would be more efficient.

Students' awareness of the domains of knowledge, which was a measure of their understanding, was done through the Extended Semantic Model. Students' awareness of the four domains (concrete, model, abstract, and symbolic) was evident as they were solving the electricity questions. The symbolic domain, which indicated students' awareness of the use of symbols to represent a problem, was the most prevalent.


Key terms; first year physics students; mathematics in physics; mathematical resources; intuitive mathematics; reasoning primitives; extended semantic model; electricity problems; students understanding

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Philosophers reminisce about the meaning of life. When one has been blessed with a family like mine; a loving wife, two beautiful kids (a girl and a boy) then luckily you don't have to think about what the purpose of life is. You just experience it. With regard to my studying, I experienced my family's full support in so many different ways. When I'm seated and toiling on the laptop my kids would know that "Daddy is doing homework". Sega - true to character - just kept a mindful distance...To this immediate experience in the meaning of my life (Sega, Lebo and Larona) I say... thank you very much..

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## Contents

Declaration ..... i
Abstract ..... ii
Acknowledgements ..... iii
List of Tables ..... ix
List of Figures .....
List of Abbreviations ..... xi
Language Editing Certificate ..... xii
Turn it In Report ..... xiii
Turn it in Receipt ..... xiv
Chapter 1 Introduction and Background ..... 1
1.1 Introduction ..... 1
1.2 Study context ..... 3
1.3 Problem Statement ..... 5
1.4 Rationale for the Study ..... 6
1.5 Objectives and Research Questions ..... 7
1.6 Operational definition of key terms ..... 8
1.7 Dissertation overview ..... 9
Chapter 2 Literature Review ..... 10
2.1 Introduction ..... 10
2.2 Contrasting mathematics and physics ..... 10
2.3 Problem solving in physics ..... 15
2.3.1 Use of Algorithms and Heuristics in Problem Solving ..... 16
2.3.2 Multi - Step Strategy ..... 16
2.3.3 Group Work ..... 18
2.4 The dichotomy in students' use of mathematics in physics ..... 18
2.4.1 Mathematics as indispensable in students' learning of physics ..... 18
2.4.2 Mathematics as a barrier in students' learning of physics ..... 20
2.5 Students' learning outcomes on electricity as a topic in physics ..... 22
2.5.1 Students use of mathematics in the topic of electricity ..... 22
2.5.2 Students conceptual understanding of electricity ..... 23
2.5.3 Students' misconceptions of electricity ..... 24
2.6 MPERG and related studies on students' use of mathematics in physics ..... 26
2.6.1 Students' interpretation of constants and variables ..... 26
2.6.2 Semantic analysis ..... 27
2.6.3 Mathematics - Physics entanglement ..... 29
2.7 Mathematical thinking in physics. ..... 30
2.7.1 Mathematical Resources ..... 30
2.7.2 Epistemic Games and Frames ..... 30
2.8 Summary of the observations ..... 31
Chapter 3 Conceptual Framework. ..... 34
3.1 Introduction ..... 34
3.2 General Systems Theory (GST ..... 34
3.3 Extended Semantic Model (ESM) ..... 36
3.4 Some relevant approaches for students' use of mathematics in physics ..... 39
3.4.1 Integration approach ..... 39
3.4.2 Modeling approach ..... 39
3.5 Design of the conceptual framework ..... 45
3.5.1 Electricity layer ..... 45
3.5.2 Mathematical Resources layer ..... 46
3.5.3 MATHRICITY ..... 48
3.6 Application of MATHRICITY through analysis of a typical first year ..... 50
electricity question ..... 50
3.7 Chapter summary ..... 51
Chapter 4 Research Method ..... 53
4.1 Introduction ..... 53
4.2 Research Design ..... 53
4.3 Instruments ..... 55
4.3.1 Expectation survey ..... 55
4.3.2 Test scripts ..... 56
4.3.3 Focus group semi - structured interviews ..... 57
4.4 Validity and Reliability of the Instruments ..... 57
4.4.1 Validity and Trustworthiness of SERMP ..... 57
4.4.2 Validity and Trustworthiness of the focus group interview ..... 59
4.4.3 Validity and Trustworthiness of the test Scripts ..... 59
4.4.4 Pilot study ..... 60
4.5 Participants ..... 60
4.6 Analytical Framework ..... 61
4.6.1 Survey and interview analysis ..... 61
4.6.2 Scripts analysis ..... 62
4.6.3 Integrating all the analyses ..... 63
4.7 Ethics ..... 63
4.8 Summary ..... 64
Chapter 5 Students' Expectations on the Use of Mathematics in Physics ..... 65
5.1 Introduction ..... 65
5.2 Students' response to the SERMP ..... 65
5.3. Emergent responses from the SERMP ..... 67
5.3.1 Students agree ..... 67
5.3.2 Students are neutral ..... 68
5.3.3 Students disagree ..... 68
5.3.4 Summary of emergent responses ..... 68
5.4. Students' Epistemological Frames ..... 68
5.5. Epistemological frame: What students think it takes to learn physics ..... 70
5.5.1 Use of equations in learning physics ..... 72
5.5.2 Memorization in learning physics ..... 74
5.5.3 Conceptualization in learning physics ..... 75
5.6. Epistemological frame: What Students think about the Use of Mathematics in Physics ..... 76
5.6.1. The meaning of mathematical answers ..... 78
5.6.2. Relationship between mathematics and physics ..... 79
5.7 Summary ..... 83
Chapter 6 Students' test scripts ..... 85
6.1. Introduction ..... 85
6.2 Analysis of students' work on Electric Force ..... 87
6.2.1 Analysis of Student V1's work on Q1A ..... 88
6.2.2 Analysis of Student M3 on Q1A ..... 92
6.2.3 Analysis of Student M5'swork on Q1 A ..... 94
6.2.4 Summary of the three students' work on $\mathrm{Q}_{1} \mathrm{~A}_{1}$ ..... 97
6.3 Analysis of students work on Electric Field question ..... 98
6.3.1 Analysis of student V1's work on Q1B ${ }_{2 a}$ ..... 99
6.3.2 Analysis of student M4 on Q1B 2 a ..... 104
6.3.3 Analysis of student M5's work on Q1B ..... 108
6.3.4 Summary of the three students' (V1, M4, M5) work on Q1B ${ }_{2 \mathrm{a}}$ ..... 111
6.4 Analysis of students' work on Electric Circuit question ..... 112
6.4.1 Analysis of Student M6 on Q2B 2 ..... 112
6.4.2 Analysis of Student H1's work on Q2B 2 ..... 114
6.4.3 Analysis of student H2's work on Q2B 2 ..... 116
6.4.4 Summary of the three students' (M6, H1, H2) work on Q2B ${ }_{2}$ ..... 117
6.5 Chapter Summary ..... 118
Chapter 7 Summary of Study and Findings ..... 121
7.1. Study summary ..... 121
7.2 Discussion of findings ..... 123
7.2.1 What they think it takes to learn physics ..... 124
7.2.2 What students think about the use of mathematics in physics ..... 125
7.2.3 How students used mathematics in the physics topic of electricity ..... 126
7.2.4 Updated MATHRICITY ..... 133
7.3 Conclusion ..... 134
7.3.1 Students Expectations ..... 135
7.3.2 Mathematical Approaches ..... 136
7.3.3 Types of understanding ..... 137
7.4 Limitations of the study ..... 141
7. 5 Implications and further studies ..... 142
REFERENCES ..... 146
APPENDICES ..... 154
Appendix A: MPEX ..... 155
Appendix B: VASS ..... 158
Appendix C: EBAPS ..... 164
Appendix D: SERMP ..... 170
Appendix E: CONSENT FORM ..... 173
Appendix F: UNISA Ethics Clearance ..... 174
Appendix G: University of Botswana Research Permission letter ..... 176
Appendix H: Interviews ..... 177
Appendix I: Instructor Solutions to Test Questions ..... 192
Appendix J: Students Use of Units, Variables and Constants ..... 196
Appendix K: Students' Solutions to Questions ..... 206

## List of Tables

TABLE 3.1: LIST OF INTUITIVE RESOURCES ..... 42
TABLE 3.2: LIST OF ABSTRACT REASONING PRIMITIVES ..... 43
TABLE 3.3: TEMPLATE FOR USING THE DEVELOPED CONCEPTUAL FRAMEWORK (MATHRICITY) ..... 51
TABLE 4.1: STUDY DESIGN ..... 54
TABLE 5.1: STUDENTS' FREQUENCY RESPONSE TO SERMP QUESTIONNAIRE (N= 193). 66
TABLE 5.2: SERMP ITEMS RELATING TO WHAT STUDENTS THINK IT TAKES TO LEARNPHYSICS71
TABLE 5.3: SERMP ITEMS RELATING TO WHAT STUDENTS THINK ABOUT THE USE OF MATHEMATICS ..... 77
TABLE 6.1: A STEP-BY-STEP DESCRIPTION OF STUDENT V1' WORK ON Q1A ..... 89
TABLE 6.2: A STEP-BY-STEP DESCRIPTION OF STUDENT M3' WORK ON Q1A 1 ..... 92
TABLE 6.3: A STEP-BY-STEP DESCRIPTION OF STUDENT M5'S WORK ON Q1A ..... 95
TABLE 6.4: A STEP-BY-STEP DESCRIPTION OF STUDENT V1'S WORK ON Q1B ..... 100
TABLE 6.5: A STEP - BY - STEP DESCRIPTION OF STUDENT M4'S WORK ON Q1B ${ }_{2}$ ..... 105
TABLE 6.6: A STEP-BY-STEP DESCRIPTION OF STUDENT M5'S WORK ON Q1B ${ }_{2 A}$ ..... 109
TABLE 6.7: A STEP-BY-STEP DESCRIPTION OF STUDENT M6'S WORK ON Q2B 2 ..... 113
TABLE 6.8: A STEP-BY-STEP DESCRIPTION OF STUDENT H1' WORK ON Q2B 2 ..... 114
TABLE 6.9: A STEP-BY-STEP DESCRIPTION OF STUDENT H2'S WORK ON Q2B ..... 116
TABLE 6.10: SUMMARY OF APPLICATION OF MATHRICITY ON STUDENTS’ SOLUTIONS ..... 119

## List of Figures

FIGURE 1: GREENO'S EXTENDED SEMANTIC MODEL ..... 36
FIGURE 2: FOUNDATION OF CONCEPTUAL FRAMEWORK ..... 46
FIGURE 3: MATHEMATICAL RESOURCES ..... 47
FIGURE 4: MATHRICITY ..... 49
FIGURE 5: QUESTION 1A ${ }_{1}$ ..... 87
FIGURE 6: STUDENT V1'S SOLUTION TO Q1A ..... 88
FIGURE 7: STUDENT M3 SOLUTION TO Q1A ..... 92
FIGURE 8: STUDENT M5 SOLUTION TO Q1A ${ }_{1}$ ..... 94
FIGURE 9: QUESTION 1B ${ }_{2}$ ..... 98
FIGURE 10: STUDENT V1 SOLUTION ON Q1B ..... 99
FIGURE 11: STUDENT M4 SOLUTION TO Q1B ${ }_{2 A}$ ..... 104
FIGURE 13: STUDENT M5 SOLUTION TO Q1B $2_{2 A}$ ..... 108
FIGURE 14: QUESTION 2B ..... 112
FIGURE 15: STUDENT M6 SOLUTION TO Q2B 2 ..... 112
FIGURE 16: STUDENT H1 SOLUTION TO Q2B ${ }_{2}$ ..... 114
FIGURE 17: STUDENT H2 SOLUTION TO Q2B ..... 116
FIGURE 18: MATHRICITY (UPDATED VERSION) ..... 134

## List of Abbreviations

EBAPS: Epistemological Belief Assessment Physics Survey
EF: Epistemological Frame
ESM: Extended Semantic Model
FLAP: Flexible Learning Approach to Physics
GST: General Systems Theory
MPERG: Maryland Physics Education Research Group
MPEX: Maryland Physics Expectation
SERMP: Students Expectation of the Role of Mathematics in Physics
UB: University of Botswana
VASS: Views Assessment Student Survey

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To the best of my knowledge, all the proposed amendments have been effected and the work is free of spelling, grammar, structural, and stylistic errors. I can confirm that the standard of language use meets the stringent requirements for the award of a senior degree.

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## Chapter 1 Introduction and Background

When on the morning of 09 June, 2014, as I was right in the middle of my write-up of this thesis, a colleague from the administration department walked into my office, saw a periodic table on the wall and commented, "eish this one... this ...this ...this chemistry, physics I never really understood a thing at school. But you know I was good in mathematics but this one...this one ... I never really got $i t$ ".

I responded with a subtle but wry smile, because:

1. She just talked about what I was doing and have been studying for the past five years (and she did not know that).
2. It also showed how individuals (even those seemingly lay in the science context) have a positive expectation about one's ability in mathematics, relative to the physical sciences!

### 1.1 Introduction

Studies have shown that mathematics has a significant role to play in students' learning of physics. Others studies doubt whether we know exactly the type of role. Thompson, Christensen, Pollock, Bucy, and Mountcastle (2009) declare that specific mathematical concepts are required for a complete understanding and appreciation of physics. Uhden, Karam, Pietrocola and Pospiech (2012) on the other hand posit that knowledge about the supportive use of mathematics in physics is still fragmented. However, physics education research has shown that in understanding physics, conceptualization and problem solving are two key factors. Problem solving is described as the heart of the work of the physicist (Fuller, 1982). Hestenes (1987) says problem solving is a process that involves following appropriate reasoning paths to obtain knowledge about physical objects or processes. In a majority of cases, such problem solving in physics involves the use of mathematics (Redish, 2005).

While he acknowledges the use of mathematics in physics, Redish (2005) points out that physicists use mathematics differently than mathematicians. McDermott (1991) cautions that students' use of maths in physics problems and the success in solving equations does not necessarily imply that a corresponding level of conceptual understanding was reached. Koichu (2010) extends that further to say even advanced mathematical knowledge does not guarantee advanced problem-solving behaviors. While they concur, Tinkers, Lambourne and Windsor (1999) maintain that students' mathematical skills are a great concern to physicists.

Basson (2002) says that notwithstanding the pivotal role of mathematics in physics, students' difficulty in transferring knowledge or skills across subjects is also evident where mathematics and physics are concerned. Students' ability to transfer knowledge and skills across distinct disciplines is an implicit assumption of contemporary education systems. Roberts, Sharma, Britton and New (2007) observes that little is known with regards to transfer issues; and that "the ability to transfer mathematics skills into a chosen science discipline is of crucial importance in students' development as scientists and in their future careers" (p. 420).

Most studies in the mathematics-in-physics terrain have dwelt on students' use and understanding of mathematics in physics (Larkin, 1980; Sherin, 2001; Kuo, Hull, Gupta \& Elby, 2013). Their direction of focus has been mainly to investigate how students use mathematics in physics and how effective that use may be. Where studies have been on mathematics in physics, the focus has mostly been on physics in general (Feynman, 1992; Redish, 2005; Redish \& Gupta 2009; Quale, 2011; Kuo et al., 2013), or again in the topic of mechanics. One entity who has worked extensively on the students' use of mathematics in physics is the University of Maryland Physics Education Group (MPERG) directed by Redish (http://www.physics.umd.edu/perg). They continue to explore ways in which the use of mathematics in physics could be optimised for meaningful learning of physics.

In this study the primary object of study is how mathematics influences students' understanding of physics, as shown by the way they use mathematics in physics. Using the physics topic of electricity as context, first year students' understanding was studied. The choice of electricity was a deliberate shift from the focus of most physics education research in introductory physics, where the concentration is mostly on the topic of mechanics (Basson, 2002; Carrejo \& Marshall 2007; Hestenes, 1992). The choice was also influenced by Mulhall, McKittrick and Gunstone (2001) observation that "electricity in some form is seen as a central area of physics/science curricula at all levels of education; primary, secondary and tertiary" (p. 576). An additional criterion for choosing electricity was that most questions that students are expected to work on are quantitative and thus provide an explicit use of mathematics. This study is not about mathematical problem solving per se; rather, it is about the role that mathematics plays for students to understand (or misunderstand) the topic electricity in physics. An investigation that encompasses students' mathematical approaches when solving electricity problems will elucidate this role.

Mathematical approaches are the methods, strategies and tactics that students display when solving physics problems (Schoenfeld, 1992). These have a direct effect on the quality of learning taking place in introductory physics classes. Students' approaches to a particular learning task are usually informed by their expectations (Redish, Saul \& Steinberg, 1998; Marshall \& Linder, 2005). These expectations are what students think is required of them to do well in, or pass a course and they are partly influenced by pedagogy as well as students' prior learning (Redish et al., 1998). Expectations in turn inform students' epistemological stance. How students use mathematics in physics is based on their expectations of the role of mathematics in physics. Since students' mathematical approaches are linked to their expectations, it is therefore always prudent to base a study that covers students' approach to a learning task and the quality of learning taking place, to their expectations of the course.

Through research - based observations on students' use of mathematics in physics similar to the ones mentioned above, Tinkers et al. (1999), from what has come to be known as Flexible Learning Approach to Physics (FLAP) offer plausible explanations and interventions. They reason that the decrease in students' familiarity with mathematics in physics is compounded by the increase in the diversity of backgrounds for recent university introductory physics students. These observations have among others led to widespread calls for the integration of mathematics and physics in physics teaching (Basson, 2002; Tinkers et al., 1999).

### 1.2 Study context

The first year students at the University of Botswana (UB) where this study was conducted come to the physics class from widely varied high school science backgrounds. They may all be enrolled for physics in the first year but will proceed in the second year to various disciplines such as engineering, earth sciences, environmental health, health sciences or physics teaching depending on their performance and career interest.

The students are admitted from high school having met the university admission requirements in the science subjects at Form 5 (Grade 12). In their first year, they all do the same physics course, which is algebra - based. They concurrently register for one of the two types of mathematics courses; one or two other science subjects (chemistry or biology) depending on their career interests as well as their capabilities.

There are four lecture streams of students in the first year doing physics; namely stream A, B, C and D . The streams, which are based on the availability of space, are also meant to make the large numbers of the physics students $(\mathrm{N} \approx 1000)$ manageable during teaching. Four different lecturers teach the streams with each stream consisting of approximately two hundred and fifty (250) students. The lectures from the different streams run concurrently.

The first year physics course (PHY 122) is three credits, which means three lecture hours per week. It is assessed through two continuous assessment tests in a semester. The examinations are written in November for the first semester and April for the second semester. This course also has a laboratory component and tutorials. The tutorial mark contributes to the final mark for the theory component, while the laboratory mark is separate.

The 3 hour weekly laboratory sessions are attended by sixty (60) students from two (2) tutorial groups ( 30 students per tutorial). Students in a single tutorial group are selected using a simple chronological order of their surnames. Only in cases where a student's lessons clash on the timetable, is a student slotted in a separate group. Students work in pairs and seldom in threes depending on the availability of space and equipment. All the students have laboratory manuals and they know beforehand which experiment they will be doing that week. The purpose of the laboratory session is so that students can get to understand through experimentation, the concepts that they did in the theory classes. Students may however be doing an experiment whose theory they have not yet done, or one whose theory was done a while ago.

The tutorial sessions are fifty (50) minutes weekly. There is an average of thirty (30) students in every tutorial group. Students are given about three to four questions as homework. The questions are mostly problem solving types and seldom conceptual. The tutors subsequently solve the problems for students on the board during tutorials. Students have to write a tutorial test every fortnight. The tutorial test involves students doing one of the problems that they were given a week ago, under supervision by tutors. Students then submit work on the particular question which is marked and contributes to the students' continuous assessment. These tutors are graduate students; physics degree holders, current and former secondary school physics teachers employed on one year running contracts designated either Temporary, Part - time or Temporary Full - time respectively.

### 1.3 Problem Statement

Hewitt (1998) quotes a taxi driver who upon hearing from the conversation in the taxi that the professor was attending a physics conference comments, "Whew... physics, I couldn't stand that subject, I hated the mathematics" (p.194). The taxi driver, who it would be safe to assume that came in contact with physics up to about high school, represents an even broader populace in terms of what individuals perceive physics to be. Even college, university students and some teachers would immediately think "mathematics" once the word physics is mentioned.

While it is incontestable that mathematics plays a significant role in the teaching and learning of physics, the paradox is that it is the use of mathematics in physics that is still a major deterrent in students' learning of physics (Albe, Venturini \& Lascours, 2001; Mualem \& Eylon, 2010). While it is still argued that the use of mathematics in physics is to simplify complex physical relationships and principles, the actual learning of physics by students portrays a contradictory picture (Redish, 2005).

Therefore there is a need to investigate how mathematics influences students understanding of physics, as shown by the way they use mathematics in physics. While most previous investigations focused on students' use of mathematics in physics, the current study proposes to scrutinize the role that mathematics plays in first year physics students with regard to their understanding of the specific physics topic of electricity.

Part of the focus of the study will be on how students' mathematical approaches could be influenced by their expectations. Previous studies on expectations are normally conducted in isolation. They are not linked to what actually happens in practice. Likewise, studies on mathematical approaches have been conducted in isolation (Uhden et al., 2012). This study will look at students' expectations of the use of mathematics in physics and link it to the actual practice, which are their mathematical approaches. A more holistic picture of the role mathematics in students' understanding of physics is expected then when one relates expectations, mathematical approaches, and subsequent understanding.

### 1.4 Rationale for the Study

Physics education research, like most science education studies provides varying and sometimes contrasting indicators about the genesis of students' attitudes, behaviour, skills and competencies in physics. The demarcation between physics education research that is focused at tertiary science education and that which is focused at high school level exacerbates this scenario. At tertiary level, physics education research that is driven mostly by physics departments concentrates more on the physics content as well as students understanding of it (Tinkers et al., 1999; Redish, 2003). At a high school level, studies that are mostly driven by university science education departments focus more on teaching methodologies and on the teacher (Antimirova, Goldman, Lasry, Milner-Bolton \& Thompson, 2009; Fensham, 1992; Tinkers et al., 1999). The result is a gap in our understanding of the genesis of students' expectations, and their competencies such as mathematical aptitude in physics when they exit high school and on entry to university as first year students.

In physics education research at tertiary level, more focus has also been on conceptual understanding rather than mathematical manipulations (Gaigher, Rogan, \& Braun, 2007). This study focuses on how the use of mathematics in physics by first year students aids their conceptual understanding of physics. This approach is comparable to one adopted by Kuo et al. (2013) who were focused on how and when students blend intuitive and formal mathematical ideas, and made a case that equations can express a holistic conceptual meaning.

Investigating the role of mathematics in students' understanding of physics is a case of interrelationships between two branches of knowledge, namely mathematics and physics as well the epistemological energies involved. The use of epistemology, a field concerned with ways of knowledge acquisition and validation will help expose how and if transfer of knowledge between mathematics and physics does occur. Investigating knowledge transfer is important, especially when noting the distinct fields of mathematics and physics. It demonstrates students' ability to apply what they have learned in one context to a different context (Basson, 2002).

With the above understanding, how the use of mathematics in physics by first year students at UB influences their learning of physics will be investigated through this study. The
mathematics engrained in physics problem-solving could play a significant part in students' understanding of physics and consequently their teaching of it after graduating from university as some of them go on to become physics teachers in high school.

While a number of studies regarding students' use of mathematics in physics have been done elsewhere (Woolnough, 2002; Tuminaro 2004; Uhden et al., 2012), and physics instructors have also made numerous comments on the topic, no studies to my knowledge have investigated how mathematics influences students' understanding of physics, as shown by the way they use mathematics in physics, with specific focus on the physics topic of electricity, and in a setting rich in diversity as UB. A lot of university physics courses elsewhere are also now streamlined along engineering, health, general science and education, as early as first year. The group for this study is diverse in that students with all these diverse capabilities and interests are in one class, doing the same course.

### 1.5 Objectives and Research Questions

This study has three objectives.

The first objective was to:

- Determine first year students' expectations of the role of mathematics in physics.

This is so as to establish a baseline on which students' mathematical approaches could be analysed.

The second objective was to:

- Determine what mathematical approach first year students use when solving electricity problems.

This is so as to establish a mathematical trend from which understanding of the physics could be inferred.

The third objective was to:

- Determine types of understanding that emerge when students solve electricity problems

This is so as to discern the variation and extent of understanding as indicated by their different mathematical approaches.

With a better understanding of students' approximate cognition on the mathematics embedded in physics; the expectation is that task design and problem solving exercises that involve the use of mathematics in particular, could be structured to highlight mathematical approaches that enhance student understanding of physics. The idiosyncrasy, if any with regard to the use of mathematics in the topic of electricity should also surface.

This study intends to focus on how mathematics influences first year students' understanding of physics, as shown by the way they use mathematics in the physics topic of electricity and answer the following research questions:
a. What are students' expectations of the role of mathematics in physics?
b. What mathematical approaches do students use when solving electricity problems?
c. What types of understanding emerge when students use certain mathematical approaches to solve electricity problems?

### 1.6 Operational definition of key terms

For the purpose of this study, the following terms and phrases shall mean:
Integration: Effectively combined use of mathematics and physics; when mathematics is used to optimal benefit in students' learning of physics.
Baseline: Foundation upon which emerging patterns of use of mathematics in physics can be explained. This foundation will be used to explain the variation of students' mathematical approaches.

Mathematical Resources: Conceptual or mental models that are activated when students use mathematics in physics.

Extended Semantic Model: A framework that describes students' understanding of their application of mathematics in physics.
MATHRICITY: Tailor - made conceptual framework describing students' use of mathematics in the physics topic of electricity.

### 1.7 Dissertation overview

The first chapter offers a background on the use of mathematics in physics and how it could lead to students understanding of physics. The various approaches that have previously been used to investigate the field are outlined. The divergence that the current study carves from the previous ones is stated. The problem statement, rationale and context of the study are given. Objectives are stated and research questions are teased out to give direction to the study.

In chapter 2, a review of previous and relevant research on students' use of mathematics in physics is presented. In chapter 3 the review crystallizes into a conceptual framework that will be used in analyzing students' work. Chapter 4 lays down the research method that was used in collecting data as well as that was used in analyzing it. Validity, reliability as well as ethical considerations are given. In chapter 5 results from an expectation survey as well as focus group interviews are presented and analyzed to offer a baseline to the study.

Students' actual work on electricity problems is analyzed by means of the developed conceptual framework in chapter 6 . Chapter 7 discuss the results from both chapters 5 and 6 , contrasts among themselves and with previous similar studies. The findings are summarized and conclusions made in line with objectives/research questions. Instructional implications and future studies that may arise are also offered.

## Chapter 2 Literature Review

### 2.1 Introduction

In this chapter previous work in the mathematics - in - physics literature is reviewed, critiqued and inferences made with specific attention to the study objectives.

The review starts broadly, with a general overview on how mathematics contrasts with physics. The varying pedagogical approaches with regard to the use of mathematics in physics are outlined. Students' use of mathematics in physics is also discussed within the context of problem solving. From their engagement in problem solving, it emanates that some students view mathematics as invaluable in their learning of physics, and that others still think that mathematics is a barrier.

The literature review is then narrowed to students' learning outcomes in electricity. Various learning outcomes are discussed with a deliberate focus on students' use of mathematics in the topic of electricity. Studies from a physics education research group with extensive work on mathematics in physics (MPERG) are purposely presented. These studies led to the last and very important sub - section to be considered, mathematical thinking in physics.

### 2.2 Contrasting mathematics and physics

The relationship between mathematics and physics is an ongoing debate in physics education research. However, there are some settled aspects of this debate that students should be able to demonstrate understanding of. One of these pertains to the notion that physics and mathematics are two strongly interdependent and closely linked areas in the scientific terrain.

A common perspective on the relationship between physics and mathematics is to perceive physics as applied mathematics. This perspective is but, only partly valid. This is partly because historically, it was only until the Maxwell's equations towards the end of $19^{\text {th }}$ century that the use of mathematics in physics became profound. While the Ptolemy ( $2^{\text {nd }}$ Century); Copernicus ( $16^{\text {th }}$ century); and especially Galileo ( $17^{\text {th }}$ century) eras did use some mathematics in physics experiments and observations, it was to a relatively minimal extent. Most of the physics during that period was without mathematics. James Clerk Maxwell (1831-1879) could be credited with pioneering the pivotal role of mathematics in physics as he applied the mathematics of calculus with ingenuity to electromagnetic concepts. Maxwell
precisely elaborated not just his theory of electricity and magnetism, but an outline of what was then still a novel approach to the use of mathematics in physics, explaining how mathematics ought to be used in physics (Tweney, 2011).

Proponents of conceptual physics propose a non - mathematical way of learning physics. They argue that students should learn and appreciate the physics concepts well before they can be asked to apply mathematics in solving physics problems. Hewitt (2010) continues his more than three decades development of a conceptual approach for learning physics; where students are engaged with analogies and imagery from real-world situations. This approach is said to build a strong conceptual foundation (Hewitt, 1998; 2006; 2010) that is necessary for introductory physics students. The students are expected to then be able to use equations of physics and better understand the relationship amongst concepts and the everyday world, through application of mathematics later in their advanced years of learning.

Effective ways in which students approach physics and subsequently achieve meaningful learning from the different physics pedagogical structures is the essence of contemporary physics education research. Be it in lectures, laboratories or a tutorial, dealing with mathematics in physics is one of contemporary physics students' major engagements (Redish, 2005). Feynman (1992) accentuates mathematics as an integral part of physics; that all the laws of physics are mathematical; and that it is impossible to explain honestly the beauties of the laws of nature (physics) in a way that people can feel, without their having some deep understanding of mathematics. Feynman (1992) is cited by Reif (1995) proclaiming, "Ordinarily, I try to get the pictures clearer, but in the end the mathematics can take over and be more efficient in communicating the idea of the picture" (p. 22).

It is however important for students to first understand that although physics and mathematics are so intertwined that it is difficult to deal with one exclusively without the other. Key physical concepts must be learnt before the mathematical formulas. Tinkers et al. (1999) quote Einstein (1954) proclaiming that, "the view that qualitative thought must precede quantitative calculations is neither new nor an invention of educationalists, yet as physics teachers we often forget this basic point" (p. 223). McDermott (2001) concurs and points out that students' understanding of important physical concepts and the ability to do the reasoning necessary to apply them is of greater lasting value than even correctly memorized formulas which are likely to be forgotten after the course ends.

Mathematics is the language through which physicists communicate to show the relationship between physics concepts, establishing some as laws, as well as in explaining physics principles. Mathematical symbols, what they represent and their manipulation are a convenient abbreviation for physicists (Redish, 2005). While the relationship between some physics concepts and the explanation of principles can be done qualitatively without mathematics, Reif (1995) cites Einstein emphasizing the importance of mathematics in physics by proclaiming, "The physicist work demands the highest possible standard of rigorous precision in the description of relationships such that only the mathematical language can give" (p. 23). Mathematics stamps a "scientific" signature on physics. It logically corroborates the qualitative physics theories, principles and laws through accurate and precise validity.

Mathematics plays a similar role in physics as it does in all other science and non-science subjects. In physics however, its role and the extent of its contribution is more than in most subjects. Mathematics elevates the scientific accuracy of physics above that of other sciences where less mathematics is used. The dual purpose of mathematics in physics is described as that of language plus logic (Feynman, 1992). Maxwell's view of mathematics in physics is described as "enhancing the formal derivational and calculation role of mathematics" and opening "a cognitive means for the conduct of 'experiments in the mind' and for sophisticated representations of theory" (Tweney, 2011, p. 687).

Being premised on the understanding that physical sciences are mathematical in character, Uhden et al. (2012) puts forth both philosophical and historical examples to illustrate how intrinsically, physics contain mathematics. They are the predictive power of mathematics in the invention of new physical theories; the mathematical nature of basic physics concepts; as well as the observation that great scientists could be classified as either or both mathematicians and physicists.

Physics and mathematics may be said to be different types of knowledge (Friegej \& Lind, 2006). They categorize these types of knowledge as:

Situational knowledge (knowledge about typical problem situations); conceptual knowledge (facts, concepts, principles of a domain); procedural knowledge (knowledge about actions which are important for problem solving) (p. 440).

Pettersson and Scheja (2008) also agree with the classification of knowledge as either conceptual or procedural. They describe conceptual knowledge as being particularly rich in relationships and can be thought of in terms of a connected web of knowledge. Procedural knowledge on the other hand refers to knowledge of rules or procedures for solving mathematical problems. Conceptual knowledge is further explained as a type of understanding that involves knowing both what to do and why whereas procedural knowledge involves simply, knowing how to do something.

In addition to being categorised as different "types", knowledge may also be classified in terms of quality. Friegej and Lind cite Anderson (1987) and Krems (1994) who classify "qualities of knowledge" as:

Hierarchical (superficial vs. deeply embedded); inner structure (isolated knowledge elements vs. well structured, interlinked knowledge); level of automation (declarative vs. compiled); and level of abstraction (colloquial vs. formal) (p. 440).

Understating of both the knowledge type and qualities of knowledge classifications will help put the contrasting of mathematics and physics into perspective. Physics entails the use of mathematics for a number of reasons. These include quantification, abbreviation, denoting relationship between phenomena, succinct portrayal of physical relationships and symbolic representation of phenomena (Redish, 2005). However, literature is abound that shows how physics and mathematics differ. For example, physics is meant to explain the interactions amongst objects and processes in the natural world, and come up with rules and generalization that govern these interactions. Whereas mathematics is touted as being about rigor, precision, exactness and accuracy (Hestenes, 1992), physics is about the best approximation (Buffler, Allie, Lubben \& Campbell, 2002). To Hestenes (1992; 2010), mathematics is sometimes called the science of patterns; whereas to Basson (2002), mathematics is concerned with; quantity, shape, data, space, and structure.

While mathematics is abstract, physics - with the exception of theoretical physics for advanced courses - deals with physical objects and processes. Roberts et al. (2007) argue that pure mathematics tends to be abstract, and not tied to physical context. Physics is concerned with exploring natural systems while mathematics is a logically structured body of knowledge which existed as a separate reality transcending the physical universe (McGinnis, 2003). The area of measurements and its concomitant use of units is one distinct aspect
between mathematics and physics. Mathematics involves calculations while physics involves applying the calculations in a natural context. Numbers in mathematics can stand for anything real or imaginary; they do not have to have no units. Numbers in physics however, quantify a physical entity which is measurable and therefore have units. In physics, symbols stand for ideas rather than quantities (Redish, 2005). In most cases, physics theories are based on experiment or observation, while mathematical theories simply exemplify the extent of the ingenious, almost artistic imagination of man (Feynman, 1992).

On another debate contrasting mathematics and physics Hestenes (2010) quotes a mathematician, Arnold (1997):

Mathematics is a part of physics. Physics is an experimental science, a part of natural science. Mathematics is the part of physics where experiments are cheap....In the middle of the $21^{\text {st }}$ centuries it was attempted to divide mathematics and physics. The consequences turned out to be catastrophic... (p. 14).

Discussion has also centered on how differently mathematics and physics are done despite their seemingly Siamese relationship. There is a fundamental difference in the way mathematics is done and the way science (physics) is pursued (McGinnis, 2003). He argues that there's a difference in the process of validation, in that mathematics involves congruence of numbers, while physics is concerned with congruence of concepts. McGinnis quotes that some pre-service science teachers who participated in their study, felt that "mathematics is more than just its connections to science" (p.30). The statement implies that mathematics can still exist independent of science (physics). The reverse cannot be said about science (physics).

With the awareness of most of the stated types and qualities of knowledge, Mazur (2009) observes that the type of knowledge organization in the current teaching in lectures and tutorials promote rote learning rather than critical enquiry. Problem solving, a common instructional practice in physics lectures and tutorials, and one where the use mathematics is predominant, is discussed in the next subsection, so as to establish the type or quality of knowledge from which it is based.

### 2.3 Problem solving in physics

Problem solving is a critical dimension in the use of mathematics in physics. It has been identified as a generic skill that is espoused by institutions as desirable, and expected as a key competency in students when they finally graduate (Billing, 2007). Studies on problem solving have been done in many other disciplines other than physics. These disciplines include mathematics, computer programming, engineering and medical science. There are notable similarities in the way problems are solved in these diverse disciplines.

Polya, in his 1945 article; How to solve it, could be credited with pioneering academic work on problem solving in general and his 1957 article is broadly cited for what he termed the 4step problem solving strategy which was to be applied to problem solving in general. This involved the distinct steps of: Understanding the Problem, Devising a Plan, Carrying out the Plan, and Looking Back. Since this seminal work, a lot has happened.

A significant focus of early research in physics education particularly around the beginning of the 1980's was on problem solving. The focus then was mainly to compare how experts and novices solved problems differently, and on metacognition (Tuminaro, 2004). One of the prevalent perceptions then was that problem solving was seen as a means to an end, and not as a goal in itself (Schoenfeld, 1992). A student engaged in problem solving was set to get to a correct solution, and not so much about the ontology of problem solving itself. Problem solving was mainly seen and used as a heuristic.

Problem solving in physics is regarded as the heart, an organ without which an organism is dead, or a system nonexistent (Fuller, 1892). Maloney (1994) agrees with Van Heuvelen (1991) that problem solving as a component of physics instruction is performed to enhance conceptual understanding of students, and that to a great extent; it involves mathematical manipulation of physics formulae. Contradictorily, students view problem solving as merely to determine the value of one or more unknown quantities (Van Heuvelen, 1991; Redish, 2003).

The use of mathematics in physics is outlined as calculation, derivation and representation (Tweney, 2011). While acknowledging the role of calculation and derivation as important, Tweney emphasizes the role of a special kind of problem solving in which relationships are seen across physical domains. More advanced problem solving behaviors would be observed
when students work within a conceptually-embodied mathematical world than when the focus is on symbolic and formal-axiomatic worlds (Koichu, 2010).

According to Van Heuvelen (1991), an appropriate order of knowledge construction based on cognitive and epistemological frameworks is imperative for students' effective learning of physics through problem solving He indicated that in this knowledge structure, students should be able to see relationships and similarities in diverse pieces of information. On the other hand knowledge organization is described as hierarchical for experts but fragmented for novices (Reif \& Heller, 1982). This hierarchically organized knowledge is deemed effective for problem solving while fragmented knowledge is much less dependable. In similar early studies on problem solving approaches between experts and novices (Larkin, McDermott, Simon \& Simon, 1980a; Chi, Feltovich, \& Glaser, 1981); experts were observed to organize knowledge by categorizing problems in terms of underlying concepts and principles, while novices used surface features. In physics education research, it is noted that experts have a lot of tacit knowledge and that their knowledge can be used to make scientific inferences and also that they are able to select various principles that can be applied in problem solving (Abdullah, 2006).

With a general picture of what the purpose of problem solving in physics is, or should be; different types of approaches used in problem solving are discussed.

### 2.3.1 Use of Algorithms and Heuristics in Problem Solving

Methods for solving problems can often be characterised in terms of algorithms or heuristics (Pretz, Naples \& Sternberg, 2003; Ormrod, 2004). Algorithm here refers to step-by-step procedures which when followed correctly, will guarantee a correct solution every time. Heuristics on the other hand refers to general strategies or "rules of thumb" for solving problems (Ormrod, 2004). Ormrod says that basic-level mathematics and physics problems are often solved using the heuristic of combining algorithms, by making use of several algebraic procedures in succession. Some of the well-known heuristics mentioned in the study include; successive refinements, means ends analysis, and working backward.

### 2.3.2 Multi - Step Strategy

Multiple representations are said to have cognitive as well as affective roles in students' learning, and that they enhance the development of conceptual understanding (Adadan, 2013). Notable studies that developed problem solving strategies that are specific to physics include Reif (1995), Heller and Heller (2001), and Redish (2003).

The multi-representational problem-solving strategy requires the use of pictorial, diagrammatic, procedural, and mathematical physics skills (Van Heuvelen, 1991; Reif, 1995). A pictorial representation portrays the situation, step by step from the start to the end of a process. This helps in the construction of diagrams that are more physical depictions of the process. The diagram serves several purposes. It summarizes the prominent features of a process and multiple diagrams can be used to describe more complex processes. Diagrams assist with construction of the mathematical representation of the situation (Van Heuvelen, 1991). The practice of constructing and interpreting diagrams of various kinds is claimed to contribute to the development of physical intuition (Hestenes, 1987). In using a multi representational strategy, "the solution to a problem relies on a whole series of representations with the value of the unknown being only a small and final part of the solution" (Reif, 1995, p. 4).The Reif strategy consists of a three step approach which includes: Analyse the Problem, Construction of a Solution, and Checks; where the student has to evaluate if the goal has been achieved, and that the final answer is sensible and consistent.

Some other extensive work on problem solving in physics was also done by Heller and Hollabaugh (1992). They proffer a multi-step strategy as an effective approach in problem solving. The strategy has evolved over time to encompass other pedagogical developments. When solving a problem, the Heller and Heller strategy recommends that students must begin with a qualitative approach and then progress toward a quantitative approach. These two main approaches can be summarized in terms of five steps. These are: Focus the Problem, which includes sketching a picture; Describe the Physics, which includes drawing a diagram; Plan a solution, which includes identifying the target quantity and illuminating unknowns; Execute the Plan includes putting in numerical values, and finally Evaluate the Answer, which means evaluating meaning and sensibility.

The critical stages of problem solving in physics as proposed by Reif \& Heller (1982); Heller, Keith \& Anderson (1992); and Heller \& Heller (2000) have subtle variations to those proposed by other researchers. In their congruence is that problem solving involves a chronological sequence of visualizing the problem; planning a solution; executing the plan, and finally checking and evaluating the solution. The executing the plan stage indicated here, is mainly mathematical.

### 2.3.3 Group Work

Group work is one other general strategy that has been demonstrated to be effective for students working on problem solving exercises (Johnson, Johnson \& Holubec, 1992). While it does not specifically address students' use of mathematics in physics, it is a better and more effective learning of physics in general as it offers students opportunities to practice problem solving strategies until they become more natural (Heller, Keith, \& Anderson,1992). It is based on the premise that groups can solve more difficult problems than individuals. Effective use of mathematics in physics would emerge if students are engaged, discuss and try to make sense out of physics problems. In these sessions, students can get practice developing and using the language of physics, which is to a significant extent mathematical. It is believed that group work is an effective activity that helps students learn (Johnson, Johnson \& Holubec, 1992).

The various problem solving strategies used or recommended for use by students show an inclination towards procedure - how students should do certain tasks. Less effort is spent on explaining what or why they are doing it. The use of heuristics and algorithms, as well as the multi-step strategy is comparable to the touted recipe or cook-book approach used in introductory physics laboratories (Redish 2003). Students may be just following these steps with little or no conceptual understanding of what they are doing. As a result of their use of mathematics in the various problem-solving experiences, one thing that comes out is that students develop either a positive or negative attitude towards learning physics (Kessels, Rau \& Hannover, 2006).

### 2.4 The dichotomy in students' use of mathematics in physics

Some students find mathematics helpful while others find it detrimental to them in their learning of physics. Maloney (1994) observes that for most students, "there is a dichotomy between learning to solve physics problems and solving physics problems to learn" (p.351).

### 2.4.1 Mathematics as indispensable in students' learning of physics <br> Hewitt (2006) proclaims that;

When the ideas of science are expressed in mathematical terms, they are unambiguous.... when findings in nature are expressed mathematically, they are easier to verify or to disprove [and that]...the methods of mathematics and experimentation led to enormous success in physics (p. 8).

Albe, Venturini and Lascours (2001) quotes Henry (1996) saying, "In the physics class, the routine use of at least some mathematics, essentially arithmetic, algebra or analytical tools, would seem to be inevitable" (p.198).

When focusing on the significance of mathematics in thermal physics, Thompson et al. (2009) note that, as with most physics areas, specific mathematical concepts are required for a complete understanding and appreciation of physics. Such mathematical applications as equations, graphs and diagrams they posit, simplify the analysis of complex physics problems.

Traditionally physics is presented in the form of rules, laws and principles, mathematically elaborated by formulas and equations (Tseitlin \& Galili, 2005). The authors note that many physics educators consider the mastering of mathematics as an indisputable premise for successful learning and study of physics. The same study quotes Hecht (1996a, b) and Hewitt (1998) who noted that in many universities the type of physics course is determined according to the mathematics used: "calculus", "algebra" and "conceptual" (without mathematics). Calculus - based and algebra - based vary in that for calculus, it is continuously changing quantities that are being calculated, while for algebra, it is the relationship between variables that is of interest. Students start using algebra at elementary stages, but engage calculus much later and thus introductory university level students are more likely to be familiar with algebra than with calculus.

Physics teachers have a tacit understanding strongly shared by students that the important aspects of physics have to do with manipulation of mathematical symbols (Mulhall \& Gunstone, 2008 cite Barros \& Elias, 1998). Several references in the same study are quoted all glorifying the importance of mathematics in physics. Wertheim (1997) is quoted saying, "a major psychological force behind the evolution of physics has been the a priori belief that the structure of the natural world is determined by a set of transcendent laws" (p. 436). Davies (1991) agrees; "the belief that mathematical laws of some sort underpin the operation of the physical world is now a central tenet of the scientific faith" (p. 436). Ayene, Damtie and Kriek (2010) conclude in their study on the level of mathematics required to do physics that students had difficulty in solving problems because they lacked some fundamental mathematical techniques.

### 2.4.2 Mathematics as a barrier in students' learning of physics

Transferring knowledge flexibly across different contexts has been reported as one of the shortcomings of physics and mathematics teaching. Physics students have been observed to be very deficient in the application of mathematics in physics, even when they are doing well in their mathematics courses and this has surprised many physics instructors (Redish, 2005). This resonates well with Basson's (2002) claim that even when students are proficient in their application of a certain skill in mathematics, the same students still struggle or fumble when required to apply the same skill in physics. Woolnough (2000) though expressing it slightly differently, supports both observations entirely. This knowledge transfer that is required of students is a high order cognitive skill and is related to one's meta-cognitive abilities (Roberts et al., 2009).

Still with regards to knowledge transfer, the use of mathematics like any other language is said to be context dependent (Basson, 2002). Most students who perform well in mathematics and physics fail to make substantial links between these contexts largely because of conflicts between the different belief systems (Woolnough, 2002). Woolnough (2000) points out that since there is the real world, the physics world, and the mathematical world, each with different characteristics and belief systems; then mathematics and physics are different belief systems which are ontologically different. The different belief systems are described as:

The real world is where phenomena, such as motion, are complicated. Many factors and influences act simultaneously, making these difficult to analyse. Experiments to try to analyse what is happening rarely work. The physics world is full of specific rules which apply to specific and often artificial situations. Some of these are counterintuitive and contradict what happens in the real world (McCloskey, 1983). There are many problems to be solved, which generally use a variety of equations. Experiments can be done and they will work as long as you have sophisticated equipment. Graphs can be drawn, and these describe what happened in the experiment. The mathematical world is full of rules which relate to $x$ and $y$, and to coefficients such as $a, b$ and $c$. There are many graphs which are characterized by $x$ and $y$ axes with scales that are even and without units (p. 264).

Students have been found to experience difficulties in manipulating different physical concepts (magnetic interaction, force, speed, current, field) simultaneously and in choosing those that are suitable for explaining a given problem (Maarouf \& Benyamna, 1997). These
are said to be compounded by the use of a high level mathematical formalism. A subsequent study (Albe et al., 2001) also concluded that for the students observed, there seemed to be a lack of clarity between the verbal explanation of the physics phenomena and the interpretation of the mathematics formula; and cites Gill (1999) as noting the paradox that students who do cope with the mathematics courses are still unable to apply them in context.

Woolnough (2000) reports that students regrettably become so firmly rooted in the familiarity of the algebra that they do not venture into the more forbidding territory of the physics behind the equation; noting that failure to assign units in some calculations is an indication that students have not yet begun to develop a conceptual link between the mathematics and the physical world. This conceptual link is what Bing and Redish (2008) refer to as "physical mapping"; or the interplay between the physical system of interest and the mathematics used to model it, which they further say, demonstrates that mathematics in physics class is only valid in so far as it reflects the physical system under study. Woolnough (2000) reports year 11 students failing to assign units to the calculated slope and concluding that this is because mathematical knowledge is carefully compartmentalized and that what students are being asked to do in physics contravenes the belief system they apply to work in their mathematics compartment.

Results from Alibert (1988) who was cited by Albe et al. (2001) contrasting university students' performance in mathematics and physics showed that in both subjects, the students systematically prefer automatic, algorithmic procedures. The study noted that these preferences are overwhelming to the detriment of reflection on the role and status of procedures in mathematics and in physics.

In a separate study aimed at identifying the origin of the difficulty in understanding the mathematics of differentials among high school and undergraduate physics students, Martinez -Torregrosai, Lo’Pez - Gay, \& Gras - Marti (2006) claim to have shown that only a small fraction of physics students in pre-university courses, and in the first courses of scientific-technical degrees, use the mathematics of differential calculus in physics fully understanding what they are doing. They went further and pronounced calculus as a situation that may worsen students' attitudes towards physics and mathematics.

More controversy continues however. Koichu (2010) for example, argues that several of the existing explanations regarding students' difficulties in problem solving are incomplete. He
refutes explanations such as, "students miss solutions since they invest little effort in planning", "students perform poorly when solving non-routine problems as they have too narrow a repertoire of tentative solution starts" and "students fail in solving non-routine problems since they lack certain heuristics in their arsenals" (p. 270-271). Koichu questions why students at times miss mathematically simpler ideas in preference for more involving formal mathematical approaches in their problem solving endeavors.

All this preceding literature is important, but in so far as it relates to mathematics - inphysics, in general. Since the current study is focused on electricity as a topic in physics, the next section looks at studies on the topic specifically with regards to students' mathematical approaches and patterns of understanding.

### 2.5 Students' learning outcomes on electricity as a topic in physics

As one of the physics topics commonly taught in first year, the topic of electricity impacts in the overall epistemological development of first year students. It is commonly taught after mechanics, and just before modern physics. Research on student learning in higher education illustrates that different academic disciplines have their distinctive ways of learning and that their successful studying involves adjusting to the discipline - specific way of what it means to learn in the subject area (Entwistle, 2005). Though it is normally presented as Electricity and Magnetism, the electricity component is distinct enough and can be analyzed as a separate entity. As in other physics topics, students' use of mathematics in problem solving is still a major part of their learning in the topic of electricity.

### 2.5.1 Students use of mathematics in the topic of electricity

Literature on the significance of mathematics in the physics topic of electricity is scanty. However, one such study on student difficulties with mathematics in electricity and magnetism (EM) indicates that students' struggle with fundamental EM concepts and this could be closely related to their difficulties with mathematics (Pepper, Chasteen, Pollock, \& Perkins, 2012). The authors note that students have difficulty combining physics ideas with mathematical calculations and that at times students do not access appropriate mathematical tools. Pepper et al. (2012) note that the various mathematical skills that include: using an appropriate mathematical tool, envisioning the spatial situation and connecting it to the mathematics and translating between physics knowledge and math tools are difficult on their own, and that combining them in the EM course may be even more challenging.

In a study focusing on problem solving in electrostatics, McMillan and Swadener (1991) observe that though the majority of students were able to solve the problems quantitatively, none of them was able to use qualitative reasoning in support of their solutions. In a study that deals with the use of mathematics in investigating the physics of electromagnetic concepts, Albe et al., (2001) reported that teachers indicated that they felt that the mathematical representation of physical phenomena was a real barrier to students understanding. The same study quotes Greca and Moreira (1997) who observed that most students in the second year of engineering school demonstrate poor organization of knowledge. This they say is demonstrated by the fact that students' mental representation of the magnetic field is a propositional representation, "a definition or a formula" (p. 198). These representations are manipulated routinely to resolve the traditional problems of electromagnetism. Greca and Moreira also put forward that, the emphasis placed on the mathematical aspects of field lines impedes physical understanding of the magnetic field [and electric field]. This could be interpreted to mean that instruction inadvertently focuses students' attention on the mathematics than on the concepts, which consequently is an ineffective approach in learning physics (Thong \& Gunstone, 2008; Gunstone, Mulhall \& McKittrick, 2009; Jones, 2010).

### 2.5.2 Students conceptual understanding of electricity

In order to develop conceptual understanding, Van Heuvelen (1991) says that construction of a knowledge structure is a necessary condition. This knowledge construction may be achieved when there is transfer of conceptual understanding from one context to another. However most studies, even those on conceptual understanding focus on student misunderstanding or shortcomings with regard to scientific concepts and also "relatively little effort has been put into exploring the nature of the understanding experienced by students in the course of studying" ( Pettersson \& Scheja, 2008, p. 767).

In a study involving the study of electromagnetic concepts, Thong and Gunstone (2008) state that students' knowledge structure often does not include key relationships in any form, neither mathematical nor qualitative; and that many introductory students' knowledge of the topic of electricity relies mainly in their explanations on a form of DC circuit and its associated aspects that includes: battery, parallel/series arrangement, Kirchhoff's Law and Ohm's Law. Gunstone et al. (2009) points to instruction as perpetuating students' abilities to solve only algorithmic DC circuit problems at the expense of anything else.

Conceptual change researchers offered conflicting interpretations regarding what constitutes effective teaching of the topic of electricity (Mulhall et al., 2001). The study identified two interrelated issues which are noted to be of fundamental importance but observed to be missing from the teaching and learning of electricity, namely: the range of models/analogies/metaphors appropriate in the teaching/learning of electricity, as well the meaning of conceptual understanding in electricity and how this changes with the different levels of education. This study demonstrated instances where even high school teachers could not differentiate between the electricity concepts of voltage, potential difference and EMF.

Undergraduate physics programs reveal little or no attention to students' conceptual understanding of electrical parameters such as voltage, potential difference and emf (Gunstone et al., 2009). They note that the focus is on more complex mathematical representations. The authors argue that the fact that some high school examination questions are sometimes observed to be conceptually inadequate may actually be an indication that even university academic physicists who were part of the panel drawing questions in the study, have some forms of confusion and inconsistency in the conceptual understanding of electricity. They further argue that the inconsistencies demonstrated by high school teachers may also be a result of their undergraduate university teaching.

### 2.5.3 Students' misconceptions of electricity

Electricity is not only a basic area in physics but also an area very fertile for students' alternative conceptions (Afra, Osta \& Zoubeir, 2009). Specifically with regards to the topic of electricity, Kenneth (2012) notes that, most students start with misconceptions that have become embedded over many years and are difficult to change. Kenneth explains that the major reason is that electrical concepts are counter - intuitive and non-sensory or abstract in nature.

Afra et al. (2009) say that since the application of electricity encompasses many aspects of everyday life, students tend to develop views and imagery of electrical concepts that are very different from scientific ones. In addition, Afra et al. cites earlier researchers who noted students view of voltage either as an outcome of a mathematical relation, or as a property of current. With regards to resistance, Afra et al. claim that studies also reveal that many students fail to develop a conceptual understanding about its role in a circuit. Dega, Kriek and

Mogese (2012) note that students' view of Ohm's law as the most important concept in electricity and magnetism, encourages some of these misconceptions.

One other misconception regarding electric field was in a study by Thong and Gunstone (2008) where students indicated that they understood electric field as not being affected by any addition of charges. Leppavirta (2012) observes that with regards to the example of two electric point charges, students think that "when the net charge of the first point charge is increased; it exerts greater force only on the second point charge but does not affect the force on the charge itself" (p.756). Leppavirta says this explanation fails to consider the symmetry of the electric forces. Students are known to have persistent misconceptions when they have to demonstrate understanding that Newton's laws extend to electric and magnetic situations ( Maloney et al., 2001) and Planinic, 2006 cited by Leppavirta, 2012).

Dega, Kriek and Mogese (2012) list and explain the following as categories of alternative conceptions in the topic of electricity: naïve physics; lateral alternative conceptions; ontological alternative conceptions; Ohm's p-prims, mixed alternative conceptions and loose ideas. Predominant misconceptions related to electric currents are listed by Afra et al. (2009) as:

- The unipolar model - where students do not recognize the need for a closed circuit, and therefore treat electric components as electric sinks that transform the current sent by a battery into light and/or heat.
- The attenuation model - whereby the current leaving a battery from one end is fused-up by the elements in the circuit, and the unused portion returns back to the other terminal of the battery.
- The sharing model, where the current sent by a battery is split and shared among the different components in the circuit. (p. 104).

Some of the models of students' misconceptions from simple electric circuits identified by Kapartzianis Kriek (2014) include:

[^1](Shipstone, Jung \& Dupin, 1988; Engelhardt \& Beichner, 2004); the battery as current source (Heller \& Finley, 1992; Borges \& Gilbert, 1999); battery and resistive "Superposition principle"; term confusion and rule application error (Koumaras et al., 1990; Engelhardt \& Beichner, 2004) and topology (Engelhardt \& Beichner, 2004)".

These lists of misconceptions explain what exacerbates students understanding of the topic of electricity. Compounded with students inefficient use of mathematics, studies on students' effective learning of electricity becomes imperative. There are no studies specifically orientated to look at how mathematics influences first year students' understanding of the topic of electricity. However, in-depth studies on students use and understanding of mathematics in physics in general have been conducted by Redish and University of Maryland Physics Education Research Group (MPERG). Though their approach is towards physics in general, this entity provides an opportune place to anchor the current study.

### 2.6 MPERG and related studies on students' use of mathematics in physics

Redish, together with the Maryland Physics Education Research Group (MPERG) have worked extensively on the relationship between mathematics and physics and how it affects students learning of physics (Redish, Steinberg \& Saul, 1996; Tuminaro, 2004; Redish, 2005; Tuminaro \& Redish, 2007; Bing \& Redish, 2007; Redish \& Gupta, 2009; Redish \& Bing, 2010). While the di Sessa (1993) p-prims and the Sherin $(1996 ; 2001)$ symbolic forms studies "ploughed" the cognitive field on students' use of mathematics in physics (see sections 3.4.2.1 and 3.4.2.2), Redish and MPERG studies have "cultivated" it extensively. These broad but extensive approaches are illustrated henceforth.

### 2.6.1 Students' interpretation of constants and variables

Introductory physics students are expected to successfully interpret many different mathematical entities including: numbers ( $2, e, 5 / 7$ ); universal constants (c, G, h ,k) experimental parameters $m, R, T, k$ (spring), initial conditions ( $\mathrm{x}_{0}, \mathrm{v}_{0}$ ), independent variables ( $\mathrm{x}, \mathrm{y}, \mathrm{t})$ and dependent variables ( $\mathrm{x}, \mathrm{y}, \mathrm{t}$ ) (Redish et al.,1996). The fact these (numbers, constants, experimental parameters, conditions and variables) are many and different; and that the difference is not always apparent as in the preceding case of independent and dependent variables, compounds the problem of students understanding the proper use of mathematics in physics.

The use of constants is a common problem related to the enterprise of deriving physics formulae and problem solving by students (Dawkins, Dickerson, McKinney \& Butler, 2008). It is rarely explicitly explained to students where the constants come from, whether they are natural values resulting from the relationship of a specific select set of physical variables, or they are a retrospective value inserted so as to make the formulae valid. These authors argue further that all the students know is that a constant is that which never changes. The basic mathematics of direct proportionality, inverse proportionality or ratio relationships are given peripheral attention in physics problem solving and so, in general students do not understand relationships, mathematical models and what they mean conceptually (Dawkins et al., 2008).

If interpretation of the above mentioned entities (numbers, constants, experimental parameters, conditions and variables) is correct and meaningful, students should be able to follow a reverse process, and formulate physics problems out of real-world situations (Redish, 2003). However, derivation of formulae, which may unpack the relationship between variables, is given minimal attention in physics problem solving (Van Heuvelen, 1991). Students do not seem to understand that symbols in physics have a different purpose, that they represent meaning about physical systems rather than expressing abstract relationships.

Redish (2005) advocates for developing physics curricula with the understanding that mathematics is used differently in physics since, "physicists and mathematicians label constants and variables differently; loading meaning onto symbols leads to differences in how physicists and mathematicians use and interpret equations; and that blending physical meaning with mathematics changes the way physicists look at equations" (p. 2). Tuminaro (2004) points out that; the equal symbol, the variables and the relationship between the variables are three things one must successfully interpret in order to understand an algebraic equation. Interpretation is closed linked with the language used, and the broad area of study for this is semantic analysis.

### 2.6.2 Semantic analysis

Similar to a host of previously mentioned studies, and even everyday perceptions, Redish and Gupta (2009) looked at mathematics distinctively as a language. They then implored the field of cognitive semantics, " $a$ subfield of linguistics that is concerned with how ordinary language is imbued in meaning" (p. 1) to find out how students make meaning with
mathematics in physics. This was on the premises that, whatever form of challenges students have with the use of mathematics in physics; the critical issue appears to be "making meaning". The study ushered in additional levels of; structure, interpretation and tools when it comes to the use of mathematics in physics context. "Making meaning" is therefore directly linked to interpretation.

The Redish and Gupta study illustrates that novices in physics problem solving, just like mathematicians, often focus on the grammar of an equation rather than the physical meaning. A further outcome of the study was that expert physicists often use implicit, tacit, or unstated knowledge (see section 2.4.1) in their application of mathematics to physics and that this contributes to students missing the meaning of use of mathematics in physics.

Mathematics adopts a terse and minimalist view - consisting mainly of heuristics, and sometimes devoid of meaning - while physics provides a much richer context (Redish \& Gupta, 2009). Evans and Green (2006) observe that the difference in the semantic structure reflects a difference in conceptualization and conceptual structure. They argue that this can be the case even though the objective information or meaning provided by an active sentence is identical to that provided by a passive sentence. In the active sentence, the focus is on the actor, while the passive formulation draws greater focus to that which is undergoing the action. The difference in focus could imply a different organization of the knowledge network, which could in turn lead to differences in how a subsequent domain of knowledge is accessed.

Talmy (2000) suggests that the semantic representation of concepts takes place through the interaction of dual systems whose relationship is complex and indirect. The dual systems are conceptual structuring system and a conceptual content system. The former is schematic while the latter is rich and highly detailed. The two descriptions are comparable to Sherin's symbolic template and conceptual schema (see section 3.4.2.2.). The resounding point is that understanding physics equations includes making many connections to stores of knowledge about mathematical operations and how those operations connect to physical meaning beyond variable definitions. One other plausible connection between mathematics and physics is succinctly demonstrated below, by at least one other of the MPERG studies.

### 2.6.3 Mathematics - Physics entanglement

While it is common to approach physics instruction as either conceptual or mathematical (see sections 2.1; 2.2; $2.3 \& 2.4$ ), Uhden et al. (2012) however argues that the use of mathematics in physics can lead to conceptual understanding of physics. This is in agreement with Tuminaro (2004) in "Mapping Mathematics to Meaning" epistemic game. Mapping mathematics to meaning is explained as a pattern of activities where students working on a physics problem begin with a physics equation, and then develop a conceptual story in the process.

At advanced levels, mathematics penetrates the physics concept to a level where the "two" become inseparable, to the extent that "it does not make sense to speak of conceptual or mathematical" (Uhden et al., 2012, p. 276). The study suggests that there are different levels of mathematical reasoning in physics and points out that there is a deep interrelationship between mathematics and physics. Ultimately the study developed a model that showed "important aspects of the mathematical character of physics" (p. 499). This has also been observed by Redish and Gupta (2009) when presenting the four steps of modeling, processing, interpreting, and evaluating as critical skills in the use of math in physics. They like Uhden et al. also observed that, "the physics and the math get entangled" (p. 3).

Still with regards to mathematics - physics entanglement, the use of mathematics in physics can be delineated into what is called structural mathematics and technical mathematics (Uhden et al., 2012). Structural mathematics is explained as the conceptual understanding of physics through mathematics, premised on the understanding that mathematics is in - built in physics principles/concepts. Technical mathematics on the other hand exists independently and is associated with pure mathematics manipulations. Kuo et al. (2013) concurs with the above, and conclude in their study that a certain level of physics understanding may result from exclusive mathematical manipulation; that students should be able to explain the physical meaning of their mathematical calculations.

Uhden et al. focuses primarily on what could be considered the role of mathematics in physics mostly from a philosophical-based experts' view point, devoid of empirical students' learning experiences. The educational implications are only mentioned through extrapolation. What mathematics in physics is, and how that relationship influences students understanding of physics are separate topics of engagement which can be explored distinctively. While the former; what mathematics in physics is, is important to put the study in perspective, it is the
latter; how that relationship influences students understanding of physics that is more important for the current study. This would be succinctly explored as students' mathematical thinking in physics.

### 2.7 Mathematical thinking in physics

Students' mathematical thinking in physics is described in a framework by Tuminaro (2004). He developed three major theoretical constructs namely; mathematical resources, epistemic games and frames. The framework introduces mathematical resources as the relevant cognitive structures for describing and analyzing mathematical thinking and problem solving. Epistemic games and frames provide the activation and context for the resources.

### 2.7.1 Mathematical Resources

Mathematical resources are described as knowledge elements activated in mathematical thinking and problem solving. They are units of thought or reasoning about use of mathematics in physics. In this theoretical bid students' mathematical knowledge consists of loosely organized bits of knowledge referred to as resources. Four types of knowledge elements which constitute mathematical resources are intuitive mathematics knowledge, reasoning primitives, symbolic forms and interpretive devices. These different types of mathematical resources are described in detail in section 3.4.2.2. Their relevance to the current study is demonstrated by the extent to which they contribute to the development of the theoretical framework (see section 3.5). Mathematical resources will play a significant role in identifying the mathematical approaches that students undertake when solving electricity problems. This is in accordance with the research question, "What mathematical approaches do students use when solving electricity problems?

### 2.7.2 Epistemic Games and Frames

Tuminaro describes epistemic games as "patterns of activities that use particular kinds of knowledge to create new knowledge or solve a problem" (p. 60). While epistemic games were first developed by Collins and Ferguson (1993) as normative, it was altered to be descriptive elements which in addition, are specific to physics as opposed to science in general (Tuminaro, 2004; Tuminaro \& Redish, 2007) . The six different games that students play in the context of problem solving in physics were identified to be: Mapping Meaning to Mathematics, Mapping Mathematics to Meaning, Physical Mechanism Game, Pictorial Analysis, Recursive Plug-and-Chug, and Transliteration to Mathematics. Some of these
games have been used to describe students mathematical approaches in the topic of electricity in this study (see sections 6.2, 6.3, 6.4).

Frames, is introduced to describe students' expectations when solving problems (Tuminaro, 2004). This explication of frames follows the work of Goffman (1974) and Tannen (1993). They are structures of expectation that determine how individuals interpret situations or events. Frames help in "understanding how or why students "choose" to play a particular epistemic game in a particular context" (Tuminaro, 2004, p. 6). Three different frames are identified as; quantitative sense-making, qualitative sense-making and rote equation chasing. The prior listed six epistemic games are couched in these frames. These epistemic games are contrasted with the epistemological frames as developed and applied in chapter 5 (see section 5.4, 5.5 and 5.6).

Some of the mathematical difficulties that students experience may actually be a result of epistemological framing (Bing \& Redish, 2006). Three types of' mindsets, which are in fact frames, have been identified in students' use of mathematics in physics. The students' mindsets are "calculation", "chunking", and "physical mapping". Calculation is when students pay attention to computational details, and how the mathematics is procedurally correct. Chunking entails hiding some mathematical detail, but "packaging parts of an expression together and seeing how the various packages relate to each other" (p. 420). Physical mapping on the other hand explains how mathematics in physics class is only valid insofar as it reflects the physical system under study. It focuses especially on the physical meaning behind numbers and their operations.

### 2.8 Summary of the observations

In this chapter, similarities and differences between mathematics and physics were discussed Mathematics was presented as a purely abstract discipline while physics represented some physical reality. Mathematics and physics were presented as different types of knowledge. Physics appeared to be favored by the conceptual knowledge description while mathematics would be favorably described as procedural knowledge. The use of mathematics in solving physics problems was outlined. Through illustration of various approaches in problem solving, the implicit but outstanding purpose appeared to be to enhance conceptual understanding of students.

What also surfaced is that while mathematics appears indispensable in students' learning of physics, in some instances, it can be a barrier. Some studies present mathematics and physics as inseparable. Others still advocate for a conceptual approach as a prerequisite to extensive mathematical formalism. The latter studies allude to the fact that inappropriately timed use of mathematics actually contributes to students difficulties in the learning of physics.

The dearth of literature with respect to use of mathematics in the specific topic of electricity also came to the fore. The few studies focusing on this area were all focused on whether there was conceptual understanding or not. No study specifically put mathematics first - as a possible cause and tried to evaluate the nature of understanding that result.

MPERG studies were isolated to illustrate the extent to which they contributed to the mathematics - in- physics literature. The MPERG studies are extensive, but also disjointed. They populate a very diverse investigation field that includes: students' interpretation of constants and variables; semantic analysis; mathematics - physics entanglement; epistemic games, frames and mathematical resources. In their congruence is the cognitive approach in investigating students' use and understanding of mathematics in physics. For this last point, MPERG studies have lent themselves credence to guide the current study.

One of the MPERG studies Tuminaro (2004) developed a composite framework on mathematical thinking in physics after a holistic investigation which includes all previously disjointed frameworks in the spectrum of research on students' use of mathematics in physics. Tuminaro claims that in addition to synthesizing previous studies on students use of mathematics in physics, he has demonstrated that; "the actual path that students follow during problem solving in physics varies from problem to problem and student to student" (p. 88). He says this fact is largely overlooked in many cognitive models of student problem solving. Tuminaro (2004) has in resonance with di Sessa (1993) adapted the general knowledge structure; where there exist general cognitive constructs that includes but are not exclusive to the structure of mathematics [or physics] concepts. This underscores the importance of likening students understanding of mathematics in physics to other cognitive processes that students may engage in, when studying other subjects different from mathematics or physics. The study purports to have developed a vocabulary and grammar as useful tools for understanding the nature and origin of students' mathematical thinking in physics. Tuminaro posits that the study offers all the knowledge and reasoning that is involved in mathematical
thinking and problem solving. This is why the MPERG studies and the Tuminaro (2004) study in particular have been identified as the "cornerstone" for the current study.

Tuminaro and other MPERG studies, as well as all the previously discussed studies were focused on how students use and understand the use of mathematics in physics, in general. The current study intends to take a step back and look at how students' use of mathematics in physics may be influenced by their understanding of the role that mathematics plays in physics.

This study will narrow the focus and cultivate specifically, the fresher topic of use of mathematics in the topic of electricity. As noted earlier, while it has become common for studies to focus on students' conceptual understanding of physics content, this study is retrospectively oriented to interpolate how students manipulation of mathematics in physics problem solving, may be influenced by their understanding of the role that mathematics plays in physics. More so, this study intends not only to develop and use an alternative framework, but also a domain (electricity) specific one. In the next chapter a conceptual framework that will be useful in analyzing the role of mathematics in students' understanding of the physics topic of electricity is developed.

## Chapter 3 Conceptual Framework

### 3.1 Introduction

This chapter presents the development of a conceptual framework for analyzing the role of mathematics in students' understanding of electricity. A suitable framework could not be found, for example frameworks such as FLAP were either too general (see section 3.4.1), or was on its own inadequate such as phenomenological primitives (see section 3.4.2.1). For that reason, it was decided to develop a framework suitable for this study from two widely used frameworks namely the General Systems Theory (GST) and Extended Semantic Model (ESM).

### 3.2 General Systems Theory (GST)

The General Systems Theory is a widely used framework that prescribes and explains relationships between subjects, content and ideas in both the natural and social sciences.GST claims to be about defining interrelationships amongst systems (Sergei \& Heather, 2002). A principal objective for this study, "to determine patterns of understanding that emerge when $I^{\text {st }}$ year students solve electricity problems" is about "defining interrelationships".

GST also claims to be about specifying systems (Sergei \& Heather, 2002). This resonates well with the other two objectives of this study; "to establish students' expectations of the role of mathematics in physics" and "to determine what mathematical approach $1^{\text {st }}$ year students use when solving electricity problems". The intention is to establish a baseline as well as any systematic approach in students' mathematical endeavors. The baseline should be provided by whatever information students give to suggest how they understand use of mathematics in physics. Part of the systems would be the emerging mathematical approaches by students as they solve physics problems, and as illuminated by the developed framework.

GST acknowledges the hierarchically ordered, self-contained way in which knowledge is presented within tertiary science education disciplines; but notes that the theory states that the biggest challenge could be in integrating knowledge and material from different disciplines (Sergei \& Heather, 2002). For this study, the self - containment of knowledge could be a precarious situation to disentangle. That "mathematics - in - physics" cuts across two
disciplines, and furthermore, that parsing physics churns out mathematics as a significant component heralds a challenging matrix for the investigation.

Sergei and Heather (2002) argue that the flagships of science education in constructivism (Von Glasersfeld, 1992; Tobin \& Tippins, 1993; Yager, 1995) and conceptual change (Hewson \& Thorley, 1989; Scott, Asoko \& Driver, 1992) can only be attained if facilitated through science 'maps' or outlines that identify interrelationships, connections and generalities of scientific knowledge in a valid manner. The role of mathematics in students understanding of physics would be better illuminated if the "interrelationships, connections and generalities" between the role of mathematics, students' understanding of physics and the physics topic in question are outlined.

The utility of the GST is elaborated as being able to; identify the system of which the unit in focus is a part; explain the properties or behaviour of the system and finally; explain the properties or behaviour of the unit in focus as part or function of the system (Skyttner, 2010). In this study, the role of mathematics, students' understanding of physics and the physics topic of electricity are the three units in focus. The expectation is that a systematic integration of the three should uncover the breadth of the objectives of the study.

From as far back as Boulding (1956), Von Bertalanffy (1968) to Skyttner (2010) the GST has evolved through various forms and has been hailed as a useful tool in mapping scientific knowledge by depicting relationships, connections and generalities. Through this framework, knowledge that is fragmented across subjects is harmonized. Information acquired from one area of science must be seen to fit into science as a whole. The theory advocates for students to have long-term and integrated understanding of science content, and also be able to apply their knowledge.

A framework would be a logical display of patterns and how the patterns relate to each other. For this study, an effective framework should be one that is consistently applicable in analysing students' use of mathematics in the physics topic of electricity. GST is preferred as a guiding tool for the development of the framework and analysis of this study in general because, as Tuminaro (2004) suggested, a general knowledge framework would offer a whole range of cognitive constructs [that should include the role of mathematics in students' understanding of physics].

### 3.3 Extended Semantic Model (ESM)

The Extended Semantic Model is a model of scientific problem solving and reasoning focusing mainly towards conceptual understanding. It was developed by Greeno (1989) with the intention to make sense of students' step - by - step progress when solving problems. It is able to show the extent of students' conceptual approach to problem-solving.

The ESM advocates for idealized problem solving that incorporates what it characterizes as the four domains of knowledge. These domains of knowledge are distinct areas of focus when solving physics problems. According to the ESM (fig.1) the domains of knowledge identifiable in problem solving are; concrete, model, abstract and symbolic. These areas of focus are purported to be cardinal for students' effective use of mathematics in solving physics problems (Gaigher et al., 2007). Effective use here implies there is concomitant understanding by the solver. The concrete domain includes physical objects and events. The model domain is about models of reality and abstractions. The abstract domain on the other hand includes concepts, laws and principles. Finally, the symbolic domain is concerned with language and algebra.


Figure 1: Greeno's Extended Semantic Model
Copied from Gaigher et al., 2007, p. 1093.

Greeno (1989) states that:
The concrete aspect is concerned with that which is physically sensible. Students must develop intuition that helps them make physical meaning from the physics problem they are solving. The model domain involves portraying models of reality and abstractions. In the abstract domain; concepts, laws and principles explain the physical or concrete aspect. Finally, the symbolic domain is concerned with symbolic ways of representing a problem, be it metaphorically in words, or through the mathematics of algebra, or both (p. 1093).

Scientific problem-solving and reasoning skills which lead to conceptual understanding are exemplified by correspondences between these domains (Greeno, 1989). Approaches to problem solving should show connection with other domains. Thus, a student solving a physics question that involves the use of mathematics is expected to indicate deliberate awareness of all or translation between the four domains as different areas of focus, to show there is conceptual understanding.

Greeno (1989) further describes the four domains as made up of two layers. The layers are denoted layer $a$ and layer $b$. Layer $a$ contains distinct items which are independent whereas layer $b$ consists of meaningful combinations of items from the respective domains in layer $a$.

To illustrate how items may be identified in layers $a$ and $b$ across all the four domains an example is presented by taking a familiar problem encountered in most introductory physics electricity questions. The example of two electric point charges

The $a$ layer will constitute, independent of one another the following: two electrons (concrete); two dots and two arrows (model); electric force, charge and distance (abstract); the symbols $F_{e}, q_{1}, q_{2}, r^{2}$ and constant $\mathrm{k}_{\mathrm{e}}$ (symbolic).

The $b$ layer will then show meaningful combination of items in the $a$ layer: repulsion between two electrons (concrete); two dots joined by two arrows pointing in opposite direction (model); relationship between electric force, charge and distance Coulomb's law (abstract); the mathematical relation $F_{e}=k_{e} q_{1} q_{l} / r^{2}($ symbolic $)$.

The ESM thus prescribes that awareness by the student of the different layers as well as the different domains is an indication of effective use of mathematics in physics. Students' difficulty with mathematics in physics by applying the ESM was researched by Gaigher et al. (2007). A step by step analysis of students working on problems indicated translation across the different domains. They observed discordance between students' successful algebraic solution and their conceptual understanding. They noted that experts on the other hand demonstrated use and translation between all knowledge domains as illustrated through the ESM. They further state that the successive representations, qualitative analysis, and use of general physics principles demonstrated by experts indicate translations between all four knowledge domains. Gaigher et al. proposed that students will develop conceptual understanding from making translations across the four knowledge domains. They state that, "the resulting network of links that develop between concrete situations; physics concepts; models and symbols amounts to a broad conceptual understanding of physics" (p. 1107). Gaigher et al. quotes Chekuri and Markle (2004) who argue that "although problem-solving in physics usually involves algebraic operations in the symbolic domain, the algebra should always be connected to the concrete, model, and abstract domains" (p. 1094).

While the Gaigher et al. study looked exclusively at the translation between the different ESM knowledge domains, taking one step before that should indicate how mathematics influences students to work and translate between the respective domains. This is the reason why in this study; the role of mathematics in students' understanding of physics, and not simply the familiar characterization of students' use of mathematics in physics, will be analyzed.

Koichu (2010) cites Schoenfeld (1992) who observed that the interplay between mathematical knowledge and students' strategic behaviors is not well understood by the research community. This necessitates future studies to clearly establish not only students’ mathematical approaches and patterns of understanding but more importantly the connection between the two. While the MPREG studies and their derivatives (see section 2.6) display a disjointed approach to the analysis of students' use of mathematics in physics, there is however some general agreement with regards to the use of mathematics in physics instruction. While the GST will be used to guide with the development of the framework structure, what follows are two of the overarching approaches that usher in the background and justification for the content.

### 3.4 Some relevant approaches for students' use of mathematics in physics

For students' effective use of mathematics in physics, integration and modeling constitute the spectrum of suggested alternatives.

### 3.4.1 Integration approach

Integration implies developing learning programs that involve knowledge transfer so that students recognize the connectedness and organization of different mathematics and science concepts (Basson, 2002). An extensive project called FLAP (Tinkers et al., 1999) that integrated mathematics and physics was developed in the UK for university teaching. The efforts FLAP made were addressing less mathematically prepared student populations because of the underlying assumption that students who perform poorly in their use of the mathematics in physics problem solving do so because they do not have requisite mathematical aptitude. Student under-preparedness is a valid claim and this study will incorporate FLAP's ideas. The framework that will be developed and used in this study will additionally look at other factors that affect students' use of mathematics in physics other than mathematical under-preparedness. The other factors could be using mathematics to model physics theories, laws, principles and how students interpret that.

### 3.4.2 Modeling approach

An alternative way of presenting the relationship between mathematics and physics is through representation or modeling. According to Hestenes (1992, 2010), modeling is the construction, validation and application of models; and for Tweney (2011), science rests on the construction and use of appropriate mental models. Mathematics is here touted as the science of patterns, conceding that mathematics is essential for students understanding of physics. This implies that pattern recognition skills are essential to understanding physics. Hestenes, however, cautions that a limitation of the modeling theory lies in students being able to distinguish the mathematical world from the physics world. Students need to be able to recognize the use and limitation of models and the modeling process. In applying the modeling theory, Hestenes (1992) says, students should understand that solutions are governed by physics concepts rather than mathematical operations. Hestenes (1996) is cited by Cabot (2008) as saying:

The great game of science is modeling the real world, and each scientific theory lays down the rules for playing the game. The object of the game is to construct valid models of real objects and processes. Such models comprise the content core of scientific knowledge. To understand science is to know how scientific models are constructed and validated. The main objective of science instruction should therefore be to teach the modeling game (p. 7).

Modeling is a powerful approach that has been widely applied to assist students' use of mathematics in physics meaningfully. It is broad in that it covers the nature of mathematics as applied to physics (and other STEM subjects) in general.

According to Megowan (2007) modeling in physics is different to modeling in mathematics. She explains that mathematical models help organize complex ideas by focusing on patterns and relationships. This she posits can be "fully described by the algebraic structure" (p. 12). Physics models on the other hand, she observes, constitute both geometric and algebraic structure since they represent "real spatial and temporal phenomena" (p. 12). Dawkins et al. (2008) noted that in general, however, students do not understand mathematical models and what they mean conceptually.

Koichu (2010) implores future studies to focus on the development of models that gives insight into the entire problem solving process. Other studies extending from the modeling theory (Sherin, 1996, 2001; Tuminaro, 2004) have investigated the nature of mathematics-inphysics at a much finer detail, and in the process uncovering practically more powerful understanding. The following subsection (3.4.2.1) presents phenomenological primitives as a cognitive approach towards modeling students' understanding of physics. The next subsection (3.4.2.2) details "mathematical resources" - a derivative of phenomenological primitives, as modeling mathematical thinking.

### 3.4.2.1 Phenomenological Primitives - modeling students understanding

In order to model students' understanding of physics, di Sessa (1993) came up with what he termed phenomenological primitives (p-prims). According to di Sessa p-prims are hypothetical knowledge structures that could be categorized according to source, size, function, and tendencies. P-prims are mental models. Facts, ideas, relations between concepts and habits constitute the spectrum of p-prims. Though theoretical by design, p-prims are
described as pragmatic mental models that could be used to describe students' preconceptions, thinking states as well as their problem solving potential. The p-prims are said to be activated in various and opportune circumstances which were comprehensively explained (di Sessa, 1993).

Though based largely on the McCloskey (1983) intuitive physics study, the di Sessa study uniqueness was in that it agitated for " a computational theory of common sense and intuitive knowledge and its evolution in scientific understanding" ( p. 174). The phenomenological primitives framework was developed in a physics context and had implications for instruction, students learning difficulties and also offered pedagogical resources for instructional design.

Di Sessa noted that [any] endeavor on physics understanding and physics learning is purely knowledge based. Congruent to that line of reasoning, di Sessa's p-prims and the breadth of the "towards an epistemology of physics" monograph formed a basis for substantial later studies on students' understanding of physics - especially those with a cognitive orientation, most of which are discussed here. The p-prim thesis has evolved as a backbone to subsequent epistemological studies in physics education, and notably in students' use of mathematics in physics (Sherin, 1996, 2001; Tuminaro, 2004; Jones, 2010). While they may not be immediately evident, p-prims will be the "seed" of the theoretical framework to be developed in this chapter, to illuminate how mathematics influences students' understanding of physics, as shown by the way they use mathematics in physics.

P-prims, just like mathematical resources (see section 2.7.1) are theoretical cognitive elements carving the manifold or knowledge - in - pieces framework. They differ in that pprims are broader and cover a much larger range of physics, even that which may not involve the use of mathematics. Mathematical resources on the other hand are, and convenient for this study, concerned with mathematics in physics only. Few studies focus on the strategic use of all these different types of mathematical resources together [to give a more holistic view of their application] (Wilkerson-Jerde \& Wilensky, 2011).

### 3.4.2.2 Mathematical Resources - modeling mathematical thinking

Students' mathematical thinking in physics is described in a framework by Tuminaro (2004). He developed three major theoretical constructs namely; mathematical resources, epistemic games and frames (see section 2.7). However, in the section, mathematical resources he
identified four types of mathematical resources being; intuitive mathematics knowledge, reasoning primitives, symbolic forms and interpretive devices. These mentioned types of mathematical resources were used in the development of the theoretical framework and will be discussed in the following paragraphs.

## Intuitive Mathematics knowledge

Intuitive mathematics knowledge is described as basic knowledge of mathematics like counting and "subitizing" that is learned at a very early age. Subitizing is explained as "the ability that humans have to immediately differentiate sets of one, two and three objects from each other" (Tuminaro 2004, p.40). The use of this type of mathematics knowledge has been seen to be activated by students even in more sophisticated and formal mathematics used in advanced physics courses. (Tuminaro, 2004, p. 45) gives examples of intuitive mathematics knowledge resources and their descriptions as in table 3.1 below:

Table 3.1: List of Intuitive Resources

| Intuitive Mathematics Knowledge |  |
| :--- | :--- |
| Subitizing | The ability to distinguish between sets of one, two, and three objects. |
| Counting | The ability to enumerate a series of objects. |
| Pairing | The ability to group two objects for collective consideration. |
| Ordering | The ability to rank relative magnitudes of mathematical objects. |

The intuitive mathematical knowledge resource is supported by Tweney (2011). Tweney explains that James Maxwell claimed that his work was founded upon that of Michael Faraday, and that the approach taken by Faraday, "while not mathematical in the usual sense (there are no formal equations in any of Faraday's works), was nonetheless 'intuitively mathematical" (p. 688). Koichu (2010) has a description for what is similar to intuitive mathematical knowledge and calls it "relatively basic thinking". He says this type of mathematical resource normally co-exists with advanced mathematical knowledge about how to solve problems.

## Reasoning Primitives

The reasoning primitive type of mathematical resource is a derivative of di Sessa' p-prims (1993) (see section 3.4.2.1). They are abstractions of everyday experiences that involve generalizations of classes of objects and influences. Reasoning primitives were introduced to, "reduce the huge number of p-prims and [show] how it creates knowledge elements that exist
at the same level of abstraction" (Tuminaro, 2004, p. 41). An example of reasoning primitives is given, where a " p-prim like force as mover results from mapping an abstract reasoning primitive like agent causes effect into a specific situation that involves forces and motion" (p. 46). Further examples of reasoning primitives and their descriptions are listed in table 3.2 below (Tuminaro, 2004, p. 46):

Table 3.2: List of Abstract Reasoning Primitives

| Abstract Reasoning Primitives |  |
| :--- | :--- |
| Blocking | The abstract notion that inanimate objects are not active Agents in any <br> physical scenario. |
| Overcoming | The abstract notion that two opposing influences attempt to achieve <br> mutually exclusive results, with one of these influences beating out the <br> other. |
| Balancing | The abstract notion that two opposing influences exactly cancel each other <br> out to produce no apparent result. |
| More is more | The abstract notion that more of one quantity implies more of a related <br> quantity. |

## Symbolic Forms

Symbolic forms are a framework that explains the way physics students and some physics experts view and apply physics equations. They are models that express individuals' understanding of physics equations. Tuminaro (2004) acknowledges that the symbolic forms he identified in the mathematical resources are the same as those introduced by Sherin (1996, 2001).

Symbolic forms are characterized as consisting of two elements; being the symbol template and the conceptual schema. The symbolic template explains the virtual structures through which mathematical expressions are seen, whereas the conceptual schema is the idea to be expressed in the equation. The schema is invoked when a student is given a problem; this schema then specifies equations and drives the solution. The symbolic forms model offers a moderately large vocabulary of simple ideas that successful physics students have to learn to express in, as well as read out of equations.

Extensive comparisons have been made between the symbolic forms and phenomenological primitives (p-prims). Symbolic forms mediate the connection between di Sessa's p-prims and equations. The origin and development of symbolic forms may be experiences working with physics equations as well as early mathematical experiences (Sherin, 2001). Tuminaro (2004)
agrees and notes that symbolic forms describe students' intuitive understanding of physics equations. These symbolic forms are in fact models as proposed and advocated for by Hestenes $(1987,1992)$ through the modeling thesis. Sherin (2001) recommends that students may need to learn to invent at least some simple types of mathematical models and to express the content of those models, prior to physics instruction.

One of the eminent points from the Sherin discourse is that, "equations can be understood in terms of more basic and generic intuitions that cut across expert domains" (p. 8). Part of the argument is that students have knowledge for constructing equations that cuts across and is not directly associated with physical principles. Physics equations are partly understood in ways that may be generic to other disciplines and in some instances in ways that are physicsspecific. Some other important observations made are that; a correct use of physics equations maybe misleading as the solver may thus be said to understand equations in the sense of knowing where and how to use them, when in fact the problem solver does not understand why individual expressions have their particular makeup. In a follow-up study Sherin (2006) still puts forth as one of the hallmarks of expert physics practice; "its ability to quantify the entirety of the physical world; everything is described in terms of numbers and relations between numbers, and equations may have the same form independent of whether the quantities that appear are forces or velocities"(p. 552). That equations may have the same form independent of quantities is essentially what symbolic forms are about.

Symbolic forms type of analysis is exemplified in a study of students' process of separating variables in algebraic equations, treating mathematical terms as physical objects, and moving these objects in a landscape of the surface the equation is written on (Wittman, Flood \& Black, 2013). Wittman et al. explains that this is evidenced when students rearrange and transforms equations without any indication of formal mathematical operations.

Another of the MPERG group, Jones (2010) extended the symbolic forms framework and applied it to the integral. Jones was able to demonstrate context-dependence on the activation of symbolic forms and an alternative way of effectively teaching the integral. Kuo et al. (2013) notes that symbol templates blend with a conceptual schema to derive meaning from symbolic forms. The symbolic forms model has since emerged as a fundamental pivot to subsequent studies on students' use of mathematics in physics (Tuminaro, 2004; Jones 2010).

Since students' use of mathematics in physics generally involves the use of equations, it is expected that the symbolic forms model will be the "root" of the theoretical framework to be designed and used to illuminate the role of mathematics in students understanding of physics. Their utility for this study will thus be demonstrated within the context of the theoretical framework.

## Interpretive Devices

Interpretive devices on the other hand are described as reasoning strategies used in interpreting physics equations, Sherin (1996; 2001). Tuminaro (2004) categorizes them into formal interpretive devices and intuitive interpretive devices, where the former relies on the formal properties of equations and the latter "are abstracted from everyday reasoning and applied to physics equations" (p. 53). An example of formal interpretive devices Kieran (2007) cites transposing - a mathematical operation that entails changing signs when changing sides of the equality and carrying out the same operation on both sides of an equation. An example of intuitive interpreted devices on the other hand may involve feature analysis; where the relative sizes of physical quantities may be compared in the absence of numerical values.

### 3.5 Design of the conceptual framework

A suitable framework is developed by combining the two widely used frameworks namely GST (see section 3.2) and Extended Semantic Model (ESM) (see section 3.3), interspersed with mathematical resources. The GST is used to guide the arrangement of electricity subtopics while the ESM is used to explain students' patterns of understanding. Mathematical resources depict students 'mathematical thinking.

### 3.5.1 Electricity layer

Starting with the physics topic of electricity as a unit of focus, this first year topic may then be pragmatically divided further into the subtopics: electric force, electric field and electric circuits as per the GST (see section 3.2). These sub - topics would then constitute the different sub - units of a core segment in an evolving framework (Fig. 2) carving out and existing in the physics world. In this layout, the three sub-topics should be distinguishable as distinct sub - units, but also connectable as the first year electricity topic.


Figure 2: Foundation of conceptual framework

### 3.5.2 Mathematical Resources layer

According to the GST categorization, "the role of mathematics" would be another unit of focus after the "physics topic of electricity" (first unit of focus) (see section 3.2.) A relevant theory of use and understanding of mathematics is adaptable to the development of the ensuing logic. Thus, mathematical resources as elaborated by Tuminaro (2004) are proposed to occupy the mathematical realm for this study. Mathematical resources represent the spectrum of mathematics knowledge elements which should be activated when students are solving electricity problems. These will be adopted as earlier explained (see section 3.4.2.2) to constitute another segment of the evolving framework (fig. 3).


Figure 3: Mathematical Resources

The development of the above two segments and their inherent subsections are in line with the GST's philosophy of units, connections and systems thinking. The segments are a plausible way of presenting the first two units of focus being; the physics topic of electricity and mathematics.

In addition to the use of mathematical resources for the stated purpose, this study will add another dimension that may portray the nature of students' understanding of physics, which is coming out narrowly in the previous mentioned studies. A two - pronged analyses of students mathematical approaches and patterns of understanding will be combined to uncover a reasonable relationship between the two. Earlier on (see section 2.3) Koichu (2010) was cited emphasizing the need to establish the interplay between students' mathematical knowledge and their strategic behaviors.

### 3.5.3 MATHRICITY

The third unit of focus, "students' understanding of physics", will be analyzed through Greeno's (1989) Extended Semantic Model (ESM). The ESM will be used to identify patterns of understanding as a result of the mathematics that students use in solving the different electricity sub-topics (see Figure 4). It will be critical for the purpose of this study to be able to discern which mathematical resources lead to what knowledge domain. That observation will help address the research question, "What patterns of understanding emerge when students use certain mathematical approaches to solve electricity problems?

Merging the three segments; electricity sub-topics, mathematical resources and the ESM, results in figure 4 below. The segments are connected such that they access a common "axle" about which they rotate; allowing variable permutations. A single, two or all the layers could move to indicate; mathematical resources activated in a particular electricity problem and the particular learning experience gained. This will be the conceptual framework for this study. The framework will use a composite of resources to model the role of mathematics in students' understanding of electricity problems. Since it is about mathematics in the physics topic of electricity, this framework will be named MATHRICITY; combing the first part of the word MATHematics and the last part of the word electRICITY.


Figure 4: MATHRICITY

A valid expectation is that as students will be working on a particular electricity subtopic, say electric field, a particular mathematical resource will be activated. The mathematical resource indicates a mathematical approach. As a result of the mathematical approach applied, a particular type of understanding will ensue. This pattern of understanding is indicated by the domain or combination of ESM domains discernable.

An activated mathematical resource like Symbolic Forms may link with the Symbolic Domain of the ESM, since both are about symbols. Symbolic Forms may also link in the Model Domain since as Sherin (2001) puts it, they (symbolic forms) are "some simple types of mathematical models".

The Intepretive Devices are well suited for the Model Domain. Models are representation of that which is physically or cognitively discernable, and assist in offering interpretation of phenomena.

The Reasoning Primitives may go with the Abstract Domain since "reason " is an abstraction that could simply be a congruence of "concepts, laws or principles".

The Intuitive Mathematics Knowledge resource is unique. This type is about "immediate" activation of basic mathematics that could result from a variety of contexts. Thus intuitive mathematics knowledge should be activated in any of the four areas of focus (domains) when solving physics problems. With respect to the concrete domain it is further noted that, students must "develop intuition" (p. 52) to make physical meaning from physical problems. This adds credence to the versatility of Intuitive Mathematics Knowledge resource.

Applying this whole framework to the investigation should reveal the knowledge domains that emerge when specific mathematical resources are activated by particular electricity problems. The electricity problems provide the context, the mathematical resource indicates the mathematical approach and finally the ESM domains describe students' understanding. MATHRICITY was pilot tested on a typical first year problem to demontrate its utility.

### 3.6 Application of MATHRICITY through analysis of a typical first year electricity question

To illustrate the feasibility of the framework, a typical first year electricity textbook question and solution are presented and then analysed by means of two components of the framework; Mathematical Resources and Extended Semantic Model (see Table 3.3).

## Sub unit - Electric force

Example 23.1 The Hydrogen Atom ( Serway \& Beichner, 2000, p. 715)

The electron and proton of a hydrogen atom are separated (on the average) by a distance of approximately $5.3 \times 10^{-11} \mathrm{~m}$. Find the magnitudes of the electric force and gravitational force between the two particles.

Solution (only the electric force component,which is of interest to us is presented)
From Coulomb's law, we find that the attractive electric force has the magnitude

$$
\mathrm{F}_{\mathrm{e}}=\mathrm{k}_{\mathrm{e}} \mathrm{e}^{2} / \mathrm{r}^{2}=\left(8.99 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right)\left(1.60 \times 10^{-19} \mathrm{C}\right)^{2} /\left(5.3 \times 10^{-11} \mathrm{~m}\right)^{2}=8.2 \times 10^{-8} \mathrm{~N}
$$

Table 3.3: Template for Using the Developed Conceptual Framework (MATHRICITY)

| Step | Activity | Description of <br> activity | Activated <br> mathematical <br> resource | Awarenes/ <br> translation <br> between ESM <br> domains |
| :--- | :--- | :--- | :--- | :--- |
| 1 | From Coulomb's law, <br> we find that the <br> attractive electric force <br> has the magnitude | Gives a simplified <br> statement of the problem <br> in words, highlighting <br> that the force to be <br> found is attractive | Reasoning <br> primitives | Symbolic, <br> layer $b$ |
| 2 | $\mathrm{~F}_{\mathrm{e}}=\mathrm{k}_{\mathrm{e}} \mathrm{e}^{2} / \mathrm{r}^{2}$ | Writes Coulomb's <br> equation | Interpretive <br> devices (Formal) | Symbolic, layer <br> $b$ |
| 3 | $=\left(8.99 \times 10^{9}\right.$ <br> $\left.\mathrm{N}^{2} / \mathrm{m}^{2} / \mathrm{C}^{2}\right)\left(1.60 \times 10^{-19}\right.$ <br> $\mathrm{C})^{2} /\left(5.3 \times 10^{-11} \mathrm{~m}\right)^{2}$ | -Substitutes numbers for <br> variables and constants <br> -Gives units for <br> variables and constants | Interpretive <br> devices (formal) $)$ | Symbolic, <br> layer $b$ |
| 4 | $=8.2 \times 10^{-8} \mathrm{~N}$ | Writes final numerical <br> answer with the unit of <br> force, $\mathbf{N}$ | Interpretive <br> devices(formal) | Symbolic, <br> layer $a$ |

The breakdown into steps for a solution to a problem as indicated in the table above, allows mathematical approaches to be identified through the activated mathematical resources. Awareness or translation between ESM domains illuminates patterns of understanding. An in-depth analysis of sequencing of units, variables and constants,-adds a further dimension.

### 3.7 Chapter summary

The General Systems Theory (GST) was used to reconstitute the topic electricity into the units: electric force, electric field and electric circuits. This was adopted as a segment in the evolving framework. Another segment of the framework was comprised of mathematical resources, being the activated conceptual models that students access when applying mathematics in physics. These are: intuitive mathematics knowledge, reasoning primitives, symbolic forms and interpretive devices. The Extended Semantic Model (ESM) was the third segment of the framework. It is about describing students' understanding of mathematics in physics. The ESM offers the domains; concrete, model, abstract and symbolic as cardinal in examining this understanding.

The complete framework was thus named MATHRICITY, on the basis that it is mathematics in the physics topic of electricity that is being investigated. The feasibility of the framework was presented. The results of applying the framework to students' work will be presented in
chapter 6 to demonstrate its utility. The results will be further thrashed-out in chapter 7 to identify common threads among the different students' approaches with respect to the different electricity subtopics.

The next Chapter, 4 details methodological, validity and reliability as well as ethical considerations in conducting the study.

## Chapter 4 Research Method

### 4.1 Introduction

Data collection was organized in three phases. The first phase established students' expectations of the role of mathematics in physics; the second determined what mathematical approach $1^{\text {st }}$ year students use when solving electricity problems; and the third determined types of understanding that emerge when $1^{\text {st }}$ year students solve electricity problems.

To provide data to answer the first research question, two instruments were used namely: a survey administered to students, and focus group interviews, both of which students' expectations on the use on mathematics in physics were explored. The survey was given preinstruction with respect to the topic of interest (electricity) and focused on students' views on their use of mathematics-in-physics, in general. The interviews were conducted during the period when the topic of electricity was being taught. The interviews focused on students' views on their use of mathematics in physics; starting broadly and eventually narrowing to the specific topic of electricity.

Copies of students' scripts were made to answer the second and third research questions. Previous studies on and related to students' use of mathematics in physics were coalesced to come up with a theoretical framework for this study (MATHRICITY) (see section 3.5). MATHRICITY was subsequently used to explain students' use of mathematics in the physics topic of electricity, when analyzing their test scripts. The analysis was also done in the context of what emerged from the survey and interviews.

Table 4.1 on the following page outlines the research process followed in this study.

### 4.2 Research Design

This study is based on an intepretivist research paradigm. Interpretation is about giving meaning to data, developing insight, making inferences, refining understanding and offering explanatory lessons (Hatch, 2002). The study approach is mainly qualitative but also makes use of quantitative methods in analysing some of the data. Qualitative research has been chosen because of the richness of the data that it produces as well the in-depth information
that results from the analysis (Hoepfl, 1997). Qualitative research focuses on the idiosyncratic as well as the pervasive, with an attempt to find the uniqueness of each case (Chenail, 2000) and emphasizes open - mindedness and curiosity of both the participants and the researcher.

The specific nature of this qualitative study is both descriptive and explanatory. Descriptive studies focus on what is going on, while explanatory studies focus on why something is going on (Otero \& Harlow, 2009). Qualitative studies are appropriate for this research since there is need to uncover, characterize and interpret what is observed. How students view the use of mathematics in physics; how students with those types of views eventually apply mathematics in physics; and the meaning (why) that may be derived from all these results, are fertile qualitative research fields.

Table 4.1: Study design

| Aim | Objectives | Research questions | Research Sub <br> Questions | Instruments | Methodology |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Establish students' expectations of the role of mathematics in physics | What are the students' expectations of role of mathematics in physics? |  | Survey <br> Focus group interviews protocol | Administer a survey questionnaire <br> Conduct focus group interviews |
|  | To determine what mathematical approach1 ${ }^{\text {st }}$ year students use when solving electricity problems | What are the mathematical approaches students' use in solving electricity problems? | Are there different mathematical approaches when students solve electric circuit problems? <br> Are there different mathematical approaches when students solve electric field problems? <br> Are there different mathematical approaches when students solve electric force problems? | Students' test scripts | Categorize students' mathematical approaches in terms of mathematical resources activated and students' use of units, variables and constants |

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\text { problems? }\end{array} & \text { Students test scripts }\end{array}
$$ \quad \begin{array}{l}Categorize students <br>
types of <br>
understanding by <br>
means of the <br>
Extended Semantic <br>

Model (ESM)\end{array}\right]\)|  |
| :--- |

### 4.3 Instruments

The three instruments that were used to collect all the data for this study are; the expectation survey, students' test scripts as well as focus group interviews. Data from these three were deemed appropriate to explore the boundaries of the research problems.

### 4.3.1 Expectation survey

Surveys are important evaluation tools that help in understanding a social or cognitive practice like pedagogy. Redish (2003) states that a cost-effective way to determine the approximate state of class knowledge is to use a carefully designed research-based survey. As baseline, an expectation survey was designed and administered to students, to find out "What are the students' expectations of role of mathematics in physics?" The survey was conducted at the end of the first semester, in the form of a questionnaire.

The survey was developed by coalescing selected items from three established science education questionnaires, namely: Maryland Physics Expectation - MPEX (appendix A) developed by Redish, Saul and Steinberg (1998); Views Assessment Student Survey -VASS (appendix B) developed by Halloun and Hestenes, (1998) and Epistemological Belief Assessment Physics Survey - EBAPS (appendix C) developed by Elby, Frediksen, Schwartz and White (1998).The chosen items from these three questionnaires were those deemed relevant to the study as they addressed particularly the first objective of the study; Establish students' expectations of the role of mathematics in physics.

MPEX could not be chosen as a whole as most items addressed students' expectations on introductory physics in general. Item (19) from MPEX, "the most crucial thing in solving a physics problem is finding the right equation to use" is an example of an item used in the construction of SERMP item 11 where it reads exactly the same. Item 18 from SERMP, "the first thing that I do when solving a physics problem is to search for formulae that relate givens to unknowns" was derived from VASS, from where it was item 35 and initially read "The first thing I do when solving a physics problem is: (a) represent the situation with sketches and drawings (b) search for formulas that relate givens to unknowns". EBAPS contributed to the construction of SERM item 25 "I treat equations as representations of reality" where the original item (12) was "when learning science, people can understand the material better if they relate it to their own ideas".

The resultant questionnaire was then named Student Expectation of the Role of Mathematics in Physics (SERMP) (appendix D).SERMP consisted of thirty (30) items put along a 5 point Semantic scale of; strongly agree, agree, neutral, disagree and strongly disagree. The items were parsed from categories that would depict students' perceptions on the "interrelationships, connections and generalities" between mathematics and physics.

Surveys only measure what students think they think (Redish, 2003). To really see how students think about mathematics in physics, their actual work during problem solving will provide that critical information. Accordingly, copies of students' test scripts when working on electricity questions were made for analysis.

### 4.3.2 Test scripts

A key source of data was the students' work in their test scripts. Two sets of students' test scripts were collected for the duration when the students were doing the electricity topic, which was the second semester. The first test consisted of questions mainly from the electric force and electric field subtopics while the second test covered the electric circuits subtopic.

Copies were made of students' scripts submitted for marking, with their informed consent. Students work from the electric force; electric field and electric circuit subtopic were evaluated. The particular students' solutions identified for even more detailed analysis were scanned and stored to make up this report. Data from the documents were analyzed in a bid to answer the two research questions; "What are the mathematical approaches students use in
solving electricity problems", and "What patterns of understanding emerge when students use certain mathematical approaches to solve electricity problems"?

### 4.3.3 Focus group semi - structured interviews

According to Wilson (1997) focus group discussions in educational research are normally employed in concurrence with other qualitative methods. The major advantage of focus group interviews is their capacity to produce "concentrated data on precisely the topic of interest" Mogan (1996). Being interviewed in a group gives informants a sense of security and comfort that may lead to more candid and reflective responses than in individual interviews (Gorrad, 2001).

Focus group semi - structured interviews were conducted with the students. The interviews were audio recorded and later transcribed (appendix H). Each tutorial group was interviewed about 2-3 times during the semester. Overall, 7 episodes of interviews covering approximately 7 hours were conducted. This was a period when the topic of electricity was being taught. The time interval between interviews of the same group was about 2-3 weeks. GST concepts were used in aiding the design of the interview questions. Questions were also framed along the continuing analyses filtered from students' responses to the SERMP as well as from their work on tests scripts. The interviews intended to further elicit "students' expectations of the role of mathematics in physics", with particular emphasis to the topic of electricity.

### 4.4 Validity and Reliability of the Instruments

Validity, an essential quality in research data, has to do with whether the data are, in fact, what they are believed or purported to be (Bless \& Higson - Smith 1995; Charles, 1998). Reliable information in qualitative studies simply means that the information has to be consistent, that similar results will be obtained in a similar environment. This is done with the understanding that individuals are different, and that a student's state of mind may be influenced by many external factors, but also that the fluctuations in individual responses tend to average out over a large enough class.

### 4.4.1 Validity and Trustworthiness of SERMP

The original MPEX instrument (appendix A) was validated through discussion with faculty and physics education experts, student interviews, and by giving the survey to a variety of
"experts". It was also given repeatedly to groups of students. It was refined after testing it through more than 15 universities and colleges in the USA (Omasits \& Wagner, 2006).

The VASS (appendix B) originally developed at the Arizona State University (ASU), has been administered to over ten thousand US high school and university students and in many countries around the world. It has been validated for surveying student views about knowing and learning science and for evaluating science or mathematics instruction and related reforms (Redish, 2003).

The EBAPS (appendix C) on the other hand was validated after making two sets of revisions based on pilot subjects and informal feedback, and getting about one hundred students on whom it was administered write down their reasons for responding as they did to each item (Redish, 2003).

The SERMP survey, derived from items in the above three, was expected to have a good measure of validity as the original items were validated. However, to obtain construct validity the SERMP was in addition given to two lecturers from the Science Education Department at the UB and two other lecturers from the Physics Department at the same institution for validation. The science education lecturers focused mostly on the ability of the questionnaire items to communicate, as well as the individual and holistic structure of the questionnaire items, face validity. The physics lecturers knew how well the students may interpret the items since they were the ones teaching them. Therefore theirs was both face and content validity.

Some of their overall comments included; aligning the items with the research questions and objectives, getting rid of negatively structured questions, and having only one statement in an item. All their suggestions were subsequently incorporated. A notable comment from one science education lecturer (who was not yet aware of all the instruments used in the study) was that "SERMP alone was not adequate to conduct the whole study", adding that, "Actually taking students scripts and analysing them would add more value to the study". Another science education lecturer had suggested, "Why don't you go and literally sit in the lecture to see how and what the students are being taught?" This was however not done, since being aware of the course content and analysing students' test script was considered adequate. Teaching methodologies, though important, are not part of this study.

### 4.4.2 Validity and Trustworthiness of the focus group interview

Interview questions were influenced by students' response to the SERMP as well what obtained from the continuing script analysis. The questions were shared and discussed with a colleague prior to interviews, who advised on keeping the questions as open as possible, and on allowing where possible, the interview to progress based on what the students were saying. The first interview was deliberately structured as general, with students asked to discuss the overall physics experience. This was so as to build rapport and establish proper context. Taking note of the context enhances validity and the right questions to be asked. Rapport ensures reliability as students will discuss without any form of bias. That one researcher was involved in all the interviews; and that there were at least two interviews conducted per group are other measures of reliability.

### 4.4.2.1 Focus group participants

The three (3) tutorial groups from which focus groups were chosen were identified. Students form those groups volunteered to participate in the interviews. Students chose a group leader who would communicate with the researcher on the convenient time to hold the interviews. They all gave their cell phone numbers to the researcher who sent messages to all of them to remind them of the agreed interview time.

### 4.4.2.2 Interview time schedule

The interviews were scheduled for one (1) hour. They were conducted during the day in between lessons. While the researcher suggested the week when he would like to conduct the interviews, the students were the ones who agreed on the right day and time for the interview.

### 4.4.2.3 Focus group moderator

The researcher was the focus group moderator and has more than 10 years of teaching physics at tertiary level. He has teaching experience across four (4) different tertiary institutions in total; two (2) universities and two (2) colleges. This breadth of experience and the knowledge of physics and physics teaching gained enabled him to guide the discussion with the UB first year physics students with a good measure of credibility.

### 4.4.3 Validity and Trustworthiness of the test Scripts

Being aware of the course plan, the researcher was sure that the test scripts were valid, as the questions asked in the tests were from the same content reflected in the course plan.

The UB physics department moderates all first year test questions. The course instructor sets the test, and then a team of physics lecturers converge to assess and adjust the suitability, level and the timing that each question may require.

### 4.4.4 Pilot study

SERMP was piloted midway through the first semester to three (3) tutorial groups ( $\mathrm{N}=40$ ) who would not be part of the groups that the questionnaire was given to for further analysis. Students' responses were checked for consistency and were also found to be giving the required information. A recurring comment from more than one student was that they did not understand the meaning of the word "intuitive " which was used in item16 that initially read, " a mathematical solution to a physics problem must make intuitive sense to me". The item was changed to, "a mathematical solution to a physics problem must be meaningful to me". The amount of time (at most 20 minutes) that it took students to complete the questionnaire was found to be both practical and fair.

### 4.5 Participants

Diverse trends in the background of students entering their physics degree courses, as well as the decreasing familiarity with mathematics, exacerbate the problem of use and understanding of mathematics in physics (Tinkers et al., 1999). In Botswana, since the University of Botswana (UB) is the only institution currently offering physics degrees, all high school completers from urban, rural, resourced and under resourced schools converge at the university to offer a rich and interesting population for investigating the topic. Race, nationality, and ethnic mix also contribute to the diversity of this study population.

The population was also chosen because the course is algebra-based physics, which is considered appropriate and adequate for the purpose of this study. This is so considering that most first year students would be more proficient in algebra than for example calculus (Martinez -Torregrosai et al., 2006).

Six (6) tutorial groups of the 2011/12 cohort of the UB responded to the questionnaire. Each of the tutorial groups consisted of about 30 students [ $\mathrm{N}=193$ ]. It is from these same groups that copies of tests scripts were obtained for analysis. Three groups of ten students per group, each group coming from a separate tutorial group participated in the focus group interviews. The interview groups were from the same tutorial groups whose test scripts were copied for analysis.

### 4.6 Analytical Framework

Remly (n.d.) quotes Stake (1995) declaring that, "Analysis in qualitative studies concentrates on the instance, trying to pull it apart and put it together more meaningfully - analysis and synthesis in direct interpretation" (p. 75). Gorrad (2001) says it is a systematic search for meaning.

### 4.6.1 Survey and interview analysis

The survey was administered during the tutorial sessions towards the end of the first semester to 193 first year physics students from six (6) tutorial groups. The tutorial groups chosen were different from those involved in the pilot study $(\mathrm{N}=40)$.

Initially SERMP comprised of five (5) Semantic scale options (Strongly Disagree, Disagree, neutral, Agree, Strongly Agree) which were then coalesced into three (3) options (Disagree, Neutral, and Agree). Responses to the "strongly agree-agree" and the "strongly disagree disagree" options were brought together to form the "agree" and "disagree" options respectively, because in retrospect, the options were found to offer no noticeably different responses.
The analysis of the SERMP involved first noting students' frequency response to individual items. Students' responses to similar items were then put together into categories, in a bid to systematically search for meaning, and give a more organized and coherent view of students' thinking. Outstanding responses were also noted and their significance evaluated. These are worth noting because in qualitative studies, even "the point out of the graph" is important, as it may sometimes offer very valuable insight (Ritchie \& Lewis, 2003).

With regards to analysis of interview, the first step involved transcription of the audio-taped data. The transcription involved listening to the tapes several times, back and forth to pick all the important details. Cues such as gestures and tone were also taken note of during the time of the interview, as these are important aspects of communication as well (Gorrad, 2001).

The analysis of the interviews was juxtaposed with that of the survey. Both means of data were addressing the research question, "What are the students' expectations of role of mathematics in physics?" Themes were drawn from students' discussion during interviews. These themes are similar to the categories used in the surveys. Points of emphasis as well as recurring comments during the discussion were also noted.

### 4.6.2 Scripts analysis

Thirty (30) scripts (10 from each tutorial group) were copied for analysis. Fifteen (15) students test scripts, five (5) from each of the tutorial groups $\mathbf{M}, \mathbf{V}$ and $\mathbf{H}$ were purposefully selected from the original 30 scripts for more detailed analysis. A comprehensive scan was done on each of the five per group for variation in terms of students' approach and use of mathematics when solving the problems.

For each of the three questions chosen for analysis, a single script from each group and one that offered noticeable variation in students' approach to problem solving and the inherent use of mathematics for the chosen problems was analyzed. ${ }^{1}$ The selection was based on the amount of detail that could be derived from the script as well as the mathematics that was used in solving the problem. Variation in approach by the different students was another factor guiding the selection criteria.

The analyzed scripts are from the only 2 tests for the semester when the topic electricity was done. Both tests are divided into section A ( 25 marks) and Section B ( 75 marks). Section A was divided into 5 "short" questions: A1; A2; A3; A4; A5 which accounted for five (5) marks each; students had to answer all questions in this section. Section B had 5 "long" questions: B1; B2; B3; B4; B5 which carried twenty five (25) marks each; students had to answer 3 of the 5 questions in this section.

Students' scripts were analysed through MATHRICITY. The framework applied the GST (see section 3.2) to organize the electricity topics. Students' mathematical approaches were assessed for the mathematical resources (see section 3.4.2.2) that are activated as students work on the physics problems. Students' awareness and translation between the different knowledge domains (Concrete; Model; Symbolic; Abstract) as described through the ESM (see section 3.3) was used to evaluate patterns of understanding.

In addition to the evaluation of students mathematical approaches in solving different electricity problems, students' use of units, variables and constants also augmented the analysis. The focus was on detecting how soon students' substitute numbers for variables and

[^2]constants and at what stages students drop and put back units when they are working on a particular mathematical - electricity physics problem (appendix J). This was expected to put into context and further clarify students' mathematical approaches as well as the type of understanding that emerge.

### 4.6.3 Integrating all the analyses

The use of three (3) data sources was so as to give more credence to the findings of the study. The different sources complement and corroborate each other. Depth would be achieved through triangulating the various data sources (survey, student's scripts and interviews). These three data sources were considered adequate to provide all the information required to answer the research questions. The various sampling sites: different tutorial groups (different tutors); different lecture streams (different lecturers); multiple tests (different electricity topics and questions); group interviews (multiple views) led to greater breadth.

What the students wrote in the survey, as well as what they said in the interviews about the role that mathematics plays in physics, was corroborated with the emerging trends when analysing their mathematical approach to electricity physics problems in tests; correlating their affective and the cognitive domains. A consistent and coherent formulation was expected to:

## 1. Validate MATHRICITY

2. Offer a plausible explanation on the role of mathematics in students' understanding of electricity problems in physics.

### 4.7 Ethics

The students were told what the purpose of the study is. They were then requested to sign consent forms (appendix E) to acknowledge their willingness to participate, and that they could withdraw from the study anytime they wish. The students were also informed that their names will be concealed from the scripts that may be used in the report.

The consent form, as well as the proposal was sent to the Unisa Ethics Committee to be cleared for use (appendix F). The same sets of documents were also sent to the University of Botswana office of Research and Development who subsequently gave permission for the research to go ahead (appendix G).

### 4.8 Summary

This study method was anchored on a qualitative framework as described in this chapter. The methods of data collection used in this study include: a survey, students scripts and focus group interviews. The participants were UB first year physics students. To tease out meaning and address the main objectives of the study, data analyses from the different sources was integrated. Survey and interviews analysis were combined to address the objective on students' expectation on the use of mathematics in physics. Analysis of students' scripts was to address the objectives on the varying mathematical approaches as well as types of understanding that students exhibit when solving electricity physics problems.

Trustworthiness of the instruments used was established by giving them to science education and physics lecturers. The use of multiple instruments as well the piloting also ensured trustworthiness and validity. The UNISA as well as the UB research review boards gave the go - ahead for the study after they were satisfied that ethical and scholarly requirements were in place.

The findings and analysis of the survey and interview responses are discussed in the next chapter (5).

# Chapter 5 Students' Expectations on the Use of Mathematics in Physics 

### 5.1 Introduction

This chapter established a baseline on students' use and understanding of the relationship between mathematics and physics. The baseline framed the entire study. A frame, which in this context refers to a mental frame, can be a belief, expectation or a mindset existing at a particular time that influences students to adopt a particular learning or problem solving strategy (Tuminaro, 2004). The baseline addresses the first research question, "What are students' expectations of the role of mathematics in physics?" Two instruments, an expectation survey and focus group interviews were used to acquire the baseline data.

The expectation survey (SERMP) is a pre - frame; where students indicate what they think about the use of mathematics in physics in general, basing on their first semester's experience. The semi - structured focus group interviews is a post -frame; where students are expected to reflect on their actual work on the electricity problems in the second semester and relate their mathematics experience. Data from the two instruments (SERMP and Interviews) will be corroborated to strengthen a particular frame or the resultant sub categories.

### 5.2 Students' response to the SERMP

The SERMP (appendix D) was used to extract students' expectations in order to establish context for analysis of students' use and application of mathematics in the physics topic of electricity (see chapter 6).

Students' responses were analysed in two main steps. The first step was to note emergent responses (see section 5.3). The second step was to categorise similar items and students' response frequencies to the items, and further delineate these categories as epistemological frames (see section 5.4). Data is presented by means of a frequency distribution of students’ response to the SERMP (see Table 5.1).

Table 5.1: Students' frequency response to SERMP questionnaire ( $\mathbf{N}=193$ )

| Item no | Item | Disagree (\%) | Neutral (\%) | Agree (\%) |
| :---: | :---: | :---: | :---: | :---: |
| 1 | I solve mathematical physics problems in order to learn physics. | 6.7 | 23.9 | 69.3 |
| 2 | Problem solving in physics means finding the right equation to use. | 6.4 | 12.8 | 80.9 |
| 3 | I understand the physical meaning of equations used in this course. | 14.3 | 43.9 | 41.8 |
| 4 | A necessary skill in this course is being able to memorize all the mathematical equations that I need to know. | 49.2 | 9.5 | 41.3 |
| 5 | Learning physics is a matter of acquiring knowledge that is specifically located in the laws and equations. | 8.8 | 21.1 | 70.1 |
| 6 | Physics laws relate to what I experience in real life. | 9.3 | 17.6 | 73.2 |
| 7 | I am able to solve a mathematical physics problem that I have never seen before. | 40.4 | 31.3 | 28.3 |
| 8 | I understand physics equations as relationship among variables. | 8.4 | 32.9 | 58.6 |
| 9 | Solving mathematical physics problems in the physics class is the same as doing so in the mathematics class. | 26.5 | 18.9 | 54.6 |
| 10 | Physical relationships can be explained using mathematics. | 6.9 | 17.7 | 75.4 |
| 11 | The most crucial thing in solving a physics problem is finding the right equation to use. | 8.9 | 7.4 | 83.7 |
| 12 | In solving a physics problem, I sometimes get a correct mathematical solution whose meaning I do not understand. | 23.8 | 26.4 | 49.7 |
| 13 | I take symbols in physical equations as representing numbers. | 19.5 | 26.8 | 53.7 |
| 14 | The use of mathematics in problem solving makes physics easier to understand. | 9.9 | 16.7 | 73.4 |
| 15 | Formulae describing physical relationships are "out there" to be discovered. | 13.9 | 28.3 | 57.8 |
| 16 | A mathematical solution to a physics problem must be meaningful to me. | 2.6 | 12.5 | 84.9 |
| 17 | It is necessary for lecturers to explicitly discuss with students, how mathematics is used in physics. | 2.6 | 8.8 | 88.5 |
| 18 | The first thing that I do when solving a physics problem is to search for formulae that relate givens to unknowns | 4.2 | 4.2 | 91.1 |
| 19 | To be able to use an equation in a problem, I need to know what each term in the equation represents. | 1.5 | 2.1 | 96.3 |
| 20 | I would prefer to learn physics with no mathematics. | 80.7 | 8.3 | 10.9 |
| 21 | I learn physics in order to solve problems. | 10.4 | 24.5 | 65.1 |
| 22 | I spend a lot of time figuring out the physics derivations in the text. | 15.3 | 32.8 | 51.9 |
| 23 | There can be no physics without mathematics. | 21.5 | 8.4 | 70.2 |
| 24 | The main skill to learn out of this course is to solve physics Problems. | 10.4 | 11.9 | 77.6 |
| 25 | I treat equations as representations of reality. | 12.5 | 31.3 | 56.3 |
| 26 | I always see symbols as representing physical | 14.2 | 31.6 | 54.2 |


|  | measurements. |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
| 27 | The mathematics that I learned in the mathematics class is <br> useful when solving physics problems. | 7.9 | 11.7 | 80.3 |
| 28 | When I solve most physics problems, I think about the <br> concepts that underlie the problem. | 3.2 | 17.0 | 79.8 |
| 29 | If I do not remember a particular equation needed for a <br> problem, in a test there is nothing much I can do. | 47.7 | 19.7 | 32.6 |
| 30 | There should be more physics problems involving the use <br> of mathematics than those where students just explain. | 19.9 | 21.9 | 58.1 |

### 5.3. Emergent responses from the SERMP

In order to note emergent responses, response frequencies were used to indicate the highest response per option and/ or responses of over $85 \%$ frequency. Students' responses to items accruing such frequencies were treated as emergent as students were indicating strong sentiments about the items.

Analysing the response frequencies along each of the three options (agree, neutral, disagree), the following items surface as emergent:

### 5.3.1 Students agree

When the items to which students agree were analyzed from the response frequency table, the three items ranked highest were:

- Item 19 - with $96.3 \%$

To be able to use an equation in a problem, I need to know what each term in the equation represents

- Item18 - with $91.1 \%$

The first thing that I do when solving a physics problem is to search for formulae that relate givens to unknowns

- Item 17 - with 88.5 \%

It is necessary for lecturers to explicitly discuss with students, how mathematics is used in physics

These are the items that students agree with and feel strongest about. They form part of a developing baseline with regards to the role of mathematics in students' understanding of physics. Closer attention will be given to these three items when students' actual use of mathematics when solving electricity problems is contextualized. Whether students know what the various "terms" in the problem they will be working on represent; and whether the first thing students do when solving physics problems is "to search for formulae that relate given to unknown", will be important considerations on which to base the analysis.

There is no direct way of validating students' response to item 17 since in this study, there is no direct observation of what happens in lectures. However in the analysis students express a desire for some means that enable them to explicitly understand "how mathematics is used in physics".

### 5.3.2 Students are neutral

As to which items students were neutral, the highest response in this category was for item 3 with 43.9\%: I understand the physical meaning of equations used in this course. This is not regarded as a strong response to base students' position on an issue.

However, since for this item, it is the highest response, and it is neutral, a likely conclusion could be that students are not sure whether they understand the physical meaning of equations used in the course.

### 5.3.3 Students disagree

The highest response in the category of 'disagree' was for item 20: I would prefer to learn physics with no mathematics.

This response frequency is notably important in the sense that, though indirect, it indicates what students consider to be critical in learning physics, mathematics. Students are suggesting it may not be possible to learn physics without mathematics. Students' use of mathematics will therefore be closely monitored to validate this claim.

### 5.3.4 Summary of emergent responses

All the above items are those where students' responses along the three options (agree, neutral, disagree) were notable. Due consideration was given to these items when students' work on electricity problems is analysed in the next chapter (6). A categorisation process of the SERMP items, with the intention of extracting meaning along epistemological frames follows.

### 5.4. Students' Epistemological Frames

Following the presentation of students' overall responses to the SERMP as well as the emergent responses, similar SERMP items were joined for a more coherent analysis. A unilateral theme was derived from the group of similar items, and this was subsequently
labeled as a distinctive epistemological frame ${ }^{2}$ (EF) (see sections $5.5 \& 5.6$ ). Focus group interview excerpts were used to uncover matching modes of thought. Extracts from the various interviews (see Appendix H) were combined with the delineated SERMP items with synchronous ideas, to corroborate an EF.

As noted earlier (section 5.2) EF's are specific groupings of similar SERMP items. To come up with an EF, the answers to the SERMP items were vigilantly read through, repeatedly, to try and get an underlying meaning from each. For all the 30 items, two underlying themes emerged as encompassing their core message. These themes are what have been referred to as epistemological frames (EF) in this study.

The EF's were labeled as: (1) What students think it takes to learn physics (2) What students think about the use of mathematics in physics. SERMP items which cumulatively encompassed and conveyed the same and bigger idea were assigned to either of these frames.

From within these frames, a closer analysis led to even further categorizations. These latter categories were delineated as codes. The codes are SERMP items within an epistemological frame deemed to be closer to each other, conveying an even more specific idea and corroborated by interviews with students. Extracts from the transcribed interviews echoing similar sentiments to the a priori coded SERMP items were added to the code to validate, or refute a claim. The interviews were structured on a reflective premise. Students were urged to think about what they do in their use of mathematics when solving physics problems. The discussions lead students to openly talk about what influences their approaches, and why. In addition, the interviews were focused on students' use of mathematics in physics, specifically concerning the topic of electricity. Analysis of the interviews combines well with that of the responses to SERMP in addressing the first research question, "What are students' expectations of the role of mathematics in physics". This combination of the two (2) instruments lent credence and validity to the frames as the consistency of what students were communicating through both inductive (SERMP) and deductive (interviews) approaches was established.

[^3]While the two frames made the two sets within which all the SERMP items could be grouped, the codes were the sub-sets within the two main sets. The first EF; what students think it takes to learn physics was sub-categorised into the codes: use of equations in learning physics, memorization in learning physics and conceptualization in learning physics (see section 5.5.1; 5.5.2 \& 5.5.3). The second EF; what students think about the use of mathematics in physics was further categorised into codes: the meaning of mathematical answers and the relationship between mathematics and physics (see section 5.6.1 \& 5.6.2).

Thus, SERMP frequency responses and interview extracts were categorised in the next two subsections into epistemological frames (EF) of what students think: it takes to learn physics, and about the use of mathematics in physics.

### 5.5. Epistemological frame: What students think it takes to learn physics

In forming this EF, selected SERMP items, as well as selected excerpts from the semistructured overarching interviews were used. The three items below are extracted from the SERMP and presented to demonstrate how this epistemological frame was composed.

Item 2: Problem solving in physics means finding the right equation to use. Whatever response students give about the meaning of problem solving, the implicit meaning is that, they think finding the right equation in problem solving is what it takes for them to learn physics.

Item 5: Learning physics is a matter of acquiring knowledge that is specifically located in the laws and equations. This item explicitly implores students to say what "learning physics" involves. Students' response to the item will be a direct statement about what they think it takes for them to learn physics.

Item 17: It is necessary for lecturers to explicitly discuss with students, how mathematics is used in physics. This item has in fact two aspects. One is about what lecturers should do for students to learn physics, "explicitly discuss". The other aspect is about what should be discussed, "how mathematics is used in physics". Both of these combined solicit students' ideas about what they think it takes to learn physics.

From the SERMP questionnaire, and by the same reasoning as indicted for these three items above, items in Table 5.2 below were selected as being part of the epistemological frame; what students think it takes to learn physics.

Table 5.2: SERMP items relating to what students think it takes to learn physics

| Item <br> no | Item | Disagree <br> $(\boldsymbol{\%})$ | Neutral <br> $(\boldsymbol{\%})$ | Agree <br> $(\boldsymbol{\%})$ |
| :--- | :--- | :--- | :--- | :--- |
| 1 | I solve mathematical physics problems in order to learn <br> physics. | 6.7 | 23.9 | 69.3 |
| 2 | Problem solving in physics means finding the right equation <br> to use. | 6.4 | 12.8 | 80.9 |
| 4 | A necessary skill in this course is being able to memorize <br> all the mathematical equations that I need to know. | 49.2 | 9.5 | 41.3 |
| 5 | Learning physics is a matter of acquiring knowledge that is <br> specifically located in the eaws and equations. | 8.8 | 21.1 | 70.1 |
| 7 | I am able to solve a mathematical physics problem that I <br> have never seen before. | 40.4 | 31.3 | 28.3 |
| 9 | Solving mathematical physics problems in the physics class <br> is the same as doing so in the mathematics class. | 26.5 | 18.9 | 54.6 |
| 11 | The most crucial thing in solving a physics problem is <br> finding the right equation to use. | 8.9 | 7.4 | 83.7 |
| 17 | It is necessary for lecturers to explicitly discuss with <br> students, how mathematics is used in physics. | 2.6 | 8.8 | 88.5 |
| 18 | The first thing that I do when solving a physics problem is <br> to search for formulae that relate givens to unknowns | 4.2 | 4.2 | 91.1 |
| 19 | To be able to use an equation in a problem, I need to know <br> what each term in the equation represents. | 1.5 | 2.1 | 96.3 |
| 21 | I learn physics in order to solve problems. | 10.4 | 24.5 | 65.1 |
| 22 | I spend a lot of time figuring out the physics derivations in <br> the text. | 15.3 | 32.8 | 51.9 |
| 24 | The main skill to learn out of this course is to solve physics <br> Problems. | 10.4 | 11.9 | 77.6 |
| 28 | When I solve most physics problems, I think about the <br> concepts that underlie the problem. | 3.2 | 17.0 | 79.8 |
| 29 | If I do not remember a particular equation needed for a <br> problem, in a test there is nothing much I can do. | 47.7 | 19.7 | 32.6 |
| 30 | There should be more physics problems involving the use <br> of mathematics than those where students just explain. | 19.9 | 21.9 | 58.1 |

Based on these SERMP items and interviews, sub - categories were identified as: the use of equations in learning physics, memorization in learning physics and conceptualization in learning physics.

### 5.5.1 Use of equations in learning physics

One of the requirements to learning physics is the ability to solve problems (Redish, 2005). Solving physics problems in most cases involve students' use of equations and this can be analyzed to infer its impact on learning.

SERMP items 2, 11 and 19 all include the use of the term "equation", invoking students to indicate through a collective response frequency how they use equations in problem solving. In the analysis of the survey, more than $80 \%$ of the students agree with each of the three items. In this they affirm their perception of the link between equations and problem solving, which leads to learning physics. Students' positive response to the items indicate understanding of a relationship which according to the GST is hierarchical (see section 3.2); where in solving physics problems, equations are perceived as fundamental.

Item 2 explicitly invites students to say what they consider to be the meaning of problem solving with regard to equations. Item 11 emphasizes the same point by suggesting that finding the right equation is the key thing for students when solving problems. Item 19 refers to the fact that students need to understand each term in the equation. In order to understand each term students would break the equation into its constituent parts; which could be variables, units and constants. Students' positive response to the item and its extent can be explained through the GST as demonstration of their quest to understand "the properties or behavior of the unit in focus as part or function of the system" (see section 3.2). Students are saying; it must be clear to them how the individual terms come to be part of the equation (structural configuration), or how the terms function in the equation for meaningful learning to occur.

The interview extracts below corroborate four students' views on the use of equations in learning physics.

## Student H5:

H5: Starting even with capacitors... but there is some correlations and not the same as last semester. Again first semester, eish it was a bit tougher than now, now as long as you can understand how the formula work like it will be easy for somebody to pass, it's not like last semester where even if you knew the equations, you may not be able to integrate it properly.

Student $\mathbf{H 5}$ states that for him, understanding equations (formula) is all that he needs to do well in the course. This still links to the GST explanation of some knowledge being perceived hierarchical.

## Student M3:

M3: If you are somebody, you are just coming and being told about Coulomb's law. It quite confuses you the first time. But once you do the calculations and see, you will get it.

## Student M2:

M2: Yes it does help, but sometimes ahh, I only use the equation and get the answer and say ahh here I don't understand. I just got the answer. I know how to find... I know how to use the equation and find the answer. Not necessarily meaning I understand the concept.

Student M2 notes that he does use equations to get the answers but points out that this does not necessarily mean that he understands the concepts.

## Student M1:

R: So if you get a question in electricity and magnetism that you have not seen before, can you solve it, or has it ever happened before?

M1: I just apply the equations.
(R laughs, M1 laughs too)
Student M1's response is spontaneous, brief and precise. This student did not speak during the whole interview (refer to appendix H ). For him to give such a spontaneous response could be an indication that he did not have to think about the answer. It's a "knee jerk reaction". Webster's Revised Unabridged Dictionary describes a "knee jerk reaction" as, "an immediate unthinking emotional reaction produced by an event or statement to which the reacting person is highly sensitive; - in persons with strong feelings on a topic, it may be very predictable".

M1's response is an indication of the strong feeling he has with regards to what he may do when presented with a physics question. Also, the use of the word "just" in the statement, as opposed to if the student had said, "I apply the equations", could indicate a casual approach in the student's attitude. It is as if the student is saying, I will "just" use equations, even though I don't know whether they will be helpful.

### 5.5.2 Memorization in learning physics

Two items in the SERMP questionnaire refers to this sub-category namely item 4 and 7. For item $4,49.2 \%$ of the students disagree with the statement that a necessary skill in this course is being able to memorize all the mathematical equations that I need to know.

For item 7, 40.4 \% disagree with the statement $I$ am able to solve a mathematical physics problem that I have never seen before. Only $28.3 \%$ agree that they can solve a physics problem that they have not seen before. The fact that most students (40.4\%) need to have seen a problem for them to solve it, indicates some tendencies towards memorization.

The interview extracts below illustrate students' views on memorization in learning physics.

R: When you go for a physics test, how much memorization do you do?

## A lot... a lot (at least 4 voices at the same time)!!

At least 4 students agree that cramming (memorization) plays a big part in their learning of physics. Memorization means storage in the short term memory. Some of the indications that information is stored in short term memory is that it is quickly forgotten after engagement with the task. This should be dismissed as an ineffective learning approach.

## Student M2

R: Tell me when you solve this problem; be it in a test or tutorial, can you solve a problem that you have not seen before, Slim?

M2: I need to have seen an example of the exact question.

## Student M6

M6: Yaa same here...aah its quite difficult to solve because we expect like aah...an example of each type so that we know what to do... we know that if it is like this we do this.

R: Yes.
M6: If you have not seen it's going to be a bit, a little hard to, unless maybe... it's your good day...you woke up on the good side of the bed (chuckles) or something.

M6: Without that no way!!
R: Yes!!
This could suggest that students memorize problem solving steps per individual questions. For them, memorization plays a big part in learning physics. This approach to learning is
counter to what is proposed through the GST. The GST rather advocates for students "to have long-term and integrated understanding of science content" (see section 3.2).

### 5.5.3 Conceptualization in learning physics

Only one item refers to this sub-category. The response to item 28 , when I solve most physics problems, I think about the concepts that underlie the problem, where $79.8 \%$ of the students agree with the item, indicate that students consider conceptualization an important part of learning physics. The interview extract below expands students' thinking on this.

## Student M3

M3: In our tutorial session, eeh...we should review those things. Like what we did from Monday up to Friday, not necessarily doing all the questions...we make take one or two from the tutorial script and concentrate on the concept that we learned.

R: Нтт...
M3: Because you find that those answers that are written on the board are.... are meaningless to most of us. So we need only to review what we learnt.

For student M3, the purpose of tutorials, where students solve problems should be to
"Concentrate on concepts" because without that, according to the student, the answers are "meaningless".

## Student M4

M4: From my experience here with tutorials...I don't know about other classes. From our class is generally the same problem which they just mentioned. Of which I feel that there is not much enough explaining of the key concepts.

R: Yeeh!
M4: Yeeh... they have already mentioned that we are just given the solutions. And there is no much explaining of the key concepts, of which is very vital. If you don't get something from the lecture, you are hoping to get it from the tutorials. And with our case that's not how it is.

Student M4 says that explaining key concepts [by the tutor] is "very vital". This could be interpreted to mean students think that for them to effectively learn physics, it is very vital for them to understand key concepts.

## Student M2

M2: Because sometimes I get a question, ok fine, I look for the correct mmm... the right formula to use, I use that formula, I check the answer at the back of the book. Ok the answer is correct but not necessarily understanding the concept...so I do have a problem sometimes.

## Student M4

M4: Yaa, most of them yaa, you feel that the answer is in line with the concept. But sometimes yaa you do feel that yaa here I just got the answer but you don't know what the meaning of the answer is.

The terms "meaning", "concept" and "understanding" appear as key in these students' conversations. Students mention and link these terms to describe what they think is an effective learning approach.

## Student H6

H6: The thing is if you don't understand the concepts, you will have problems throughout. So for that part I think it is very important to understand the concepts.

Students H6 states what she thinks could really be fundamental in learning physics. "... if you do not understand the concepts, you will have problems throughout". The student is arguing that conceptualization is a pre- requisite to all other forms of teaching and learning.

After establishing what students think it takes to learn physics, in general, the next sub section ushers the second and more specific epistemological frame, on; what students think about the use of mathematics in physics?

### 5.6. Epistemological frame: What Students think about the Use of Mathematics in Physics

This subsection crystallizes an epistemological frame where selected SERMP items and interview excerpts are interpreted to mean, what students think about the use of mathematics in physics. As explained through the GST (see section 3.2), the items could be collated to "generally" fit in this category.

The three items below were extracted from SERMP, and are used to demonstrate how an item qualified for this category:

Item 10: Physical relationships can be explained using mathematics. Students' response to this item is an indication of how they think mathematics is used in explaining relationships between physical entities. In a way, that mathematics acts as a tool.

Item 23: There can be no physics without mathematics. Students' response to this item indicates the extent to which students see the intrinsic nature of mathematics in physics; that mathematics is part of physics and the two are inseparable.

Item 26: I always see symbols as representing physical measurements. Symbols are part of equations which are mathematical in nature. If students are able to make a connection between them and physical measurements, then this implies how and the extent to which students think about the use of symbols (mathematics)in physics.

From the SERMP questionnaire, and by the same reasoning as demonstrated for three items above, items in table 5.3 below were selected as being part of the epistemological frame:
What students think about the use of mathematics in physics.

Table 5.3: SERMP items relating to what students think about the use of mathematics

| Item <br> no | Item | Disagree <br> $(\boldsymbol{\%})$ | Neutral <br> $(\boldsymbol{\%})$ | Agree <br> $(\boldsymbol{\%})$ |
| :--- | :--- | :--- | :--- | :--- |
| 3 | I understand the physical meaning of equations used in this <br> course. | 14.3 | 43.9 | 41.8 |
| 6 | Physics laws relate to what I experience in real life. | 9.3 | 17.6 | 73.2 |
| 7 | I am able to solve a mathematical physics problem that I <br> have never seen before. | 40.4 | 31.3 | 28.3 |
| 8 | I understand physics equations as relationship among <br> variables. | 8.4 | 32.9 | 58.6 |
| 10 | Physical relationships can be explained using mathematics. | 6.9 | 17.7 | 75.4 |
| 12 | In solving a physics problem, I sometimes get a correct <br> mathematical solution whose meaning I do not understand. | 23.8 | 26.4 | 49.7 |
| 13 | I take symbols in physical equations as representing <br> numbers. | 19.5 | 26.8 | 53.7 |
| 14 | The use of mathematics in problem solving makes physics <br> easier to understand. | 9.9 | 16.7 | 73.4 |
| 15 | Formulae describing physical relationships are "out there" <br> to be discovered. | 13.9 | 28.3 | 57.8 |
| 16 | A mathematical solution to a physics problem must be <br> meaningful to me. | 2.6 | 12.5 | 84.9 |
| 20 | I would prefer to learn physics with no mathematics. | 80.7 | 8.3 | 10.9 |
| 23 | There can be no physics without mathematics. | 21.5 | 8.4 | 70.2 |
| 25 | I treat equations as representations of reality. | 12.5 | 31.3 | 56.3 |
| 26 | I always see symbols as representing physical <br> measurements. | 14.2 | 31.6 | 54.2 |
| 27 | The mathematics that I learned in the mathematics class is <br> useful when solving physics problems. | 7.9 | 11.7 | 80.3 |

Based on these SERMP item and interviews, sub - categories were identified as; the meaning of mathematical answers as well as the relationship between mathematics and physics.

### 5.6.1. The meaning of mathematical answers

Students agree with item 12 with $49.7 \%$ that in solving a physics problem, I sometimes get a correct mathematical solution whose meaning I do not understand. When nearly half the class expresses this observation, one is made to ask the question; why do students solve problems in the first place; shouldn't a correct solution to a problem indicate that students have understood?

While it is only $14.3 \%$ of the students who disagree with Item 3, (I understand the physical meaning of equations used in this course), this it still suggests that there are students in the course who only focus on the manipulation of variables. This could be because symbols may be incomprehensible to them, or that students may have found understanding of equations to be an unnecessary inconvenience (Woolnough, 2000).

The interview extract below further explains students' understanding of the meaning of answers:

## Student M3

R: Mmm.....now tell me from your experience of learning physics both from lectures and tutorials, when you solve physics problems, do you get an idea that the act of solving a physics problem and getting it correct, does it help you understand the physics concepts and principles you are talking about?

M3: I think actually getting a correct answer boosts your morale towards physics.
R: Yaa.
M3: Because it proves that what they said is, the principle what they said about it is right. It applies.

M3 thinks that the use of mathematics in problem solving is done to "prove". This student will use mathematics in problem solving not necessarily to learn but to prove laws, concepts and theories. Student M3 brings another aspect to the use of mathematics in solving physics problem and getting a correct answer. He says it "boosts your morale towards physics". This could be interpreted to mean that getting a correct answer makes the student more confident and even makes them like the course. Further still, when student M3 says "But once you do the calculations and see, you will get it", this could be interpreted to mean
students view mathematical calculations as aiding in their understanding of physics (Kuo et al., 2013).

## Student M2

M2: Yes it does help, but sometimes ahh, I only use the equation and get the answer and say ahh here I don't understand. I just got the answer. I know how to find... I know how to use the equation and find the answer. Not necessarily meaning I understand the concept.

R: $M m m$.
M2: Because sometimes I get a question, ok fine, I look for the correct mmm... the right formula to use, I use that formula, I check the answer at the back of the book. Ok the answer is correct but not necessarily understanding the concept...so I do have a problem sometimes.

Student M2, Fizo ${ }^{3}$ differs on what solving a physics problem and getting a correct answer, sometimes mean to him. His sentiments about problem solving, the use of equations, what answers mean to him and how that leads to understanding are all explicit. He can do the job; sometimes does it as well as it should be done, but with no concomitant understanding of what he is doing. His description of what he is doing is like of someone engaged with a puzzle.

In applying the concept of mapping scientific knowledge as espoused by the GST (section 3.2), this meaning of mathematical answers portray an obscure and distorted map. Students are either contradicting themselves, or contradicting substantiated information from empirical studies. While the GST supports the idea of the mathematics that I learned in the mathematics class is useful when solving physics problems; as it demonstrates that "knowledge that is fragmented across subjects is harmonized", this is still contradictory to literature from empirical studies (Basson, 2002; Redish, 2005).

### 5.6.2. Relationship between mathematics and physics

The $80.7 \%$ of students who disagrees with item 20, "I would prefer to learn physics with no mathematics", is a telling statistic. Firstly, this is the only item in the whole questionnaire where more than $50 \%$ of the students disagree with a statement. Like all items in the questionnaire, item 20 challenges students to state a position that will be used to infer their understanding of the role of mathematics in physics. Item 20 however is much straighter. It

[^4]explicitly probes students to say whether they think physics can be learned without mathematics.

An 80.7 \% response is a significant degree to leave room for uncertainty, in case one was to generalize. For students to dispute that I would prefer to learn physics with no mathematics is indicative of their unwavering conviction of a close relation or near oneness between mathematics and physics. Uhden et al. (2012) has also demonstrated how at some level in problem solving, the distinction between mathematics and physics become blurred. Students' response to item 23, where $70.2 \%$ agree with the statement "There can be no physics without mathematics" corroborates both the above statements.

Hewitt (2010) has invested over two decades of work on how and why students should first do and appreciate physics with minimal mathematics. He emphasizes conceptual understanding as an obligatory precept especially at introductory level. Hewitt notably goes by the tagline "comprehension before computation" to buttress his conviction that students have to understand physics concepts before they can use mathematics to explore relationships amongst them. Hewitt will be intrigued by the above response.

The interview excerpts below show students discussing what they think about the relationship between mathematics and physics.

## Student M3

R: Alright, now the mathematics does it simplify or makes physics easier, or more difficult.
M3: Well it makes it easier because mathematical illustrations, they tend to make you understand or believe because they are proved.

R: They are proved.
M3: Yes, you know in physics there is a lot of proofing and you tend to get it more quickly when there is mathematics involved.

## Student M1

M1: Yaa... physics, maths, yaa when you are taught concepts and then you might not get, but then when you apply maths then ... it makes you believe, then you understand.

R: Maths makes you believe?
M1: It compliments.

## Student M4

M4: I feel that maths is simpler than physics.
R: It's simpler than physics.
M4: Yaa of which the problem now is in most of the physics problems, you have to understand the physics part of the problem first, before you get to solve with maths. Of which I don't think...... ahh it makes me feel that it doesn't make any difference, with maths.

R: It doesn't make any difference.
M4: Yaa, because you have to go through the physics first before you go to the maths part of it which is the easy part?

## Student H1

H1: You know sometimes you can get the physics, your physics maybe right but your maths is wrong.

R: Ooh.... so either way.
H1: Yaa sometimes it's the physics and then the maths which is wrong.
Students think that mathematics is not only easier than physics, but it also makes physics simpler. They observe that in solving physics problems, there are two distinct aspects; the physics aspect, and the mathematics aspect.

## Student M2

R: Ok...Mr. Fizi, can you learn physics without mathematics?
M2: Aah I don't think so.
$\mathbf{R}$ : It's impossible?
M2: It's impossible, it's very impossible. You need to apply maths in order to understand the physics

## Student M5

M5: I think that it is possible but there is a lot of maths.
$\mathbf{R}$ : There is a lot of maths. So you can learn physics without maths?
M5: Yes.

## Student H2

H2: Yes I do but I don't think it will be as fun or it will be as interesting because answering questions, from senior school, answering theory questions proved to be more difficult than the mathematical part because the theory you have to read. Physics you know we don't usually re...ad physics. We just, I don't re...ad physics, I just find the question and see how they relate.

R: You solve problems.
H2: Yes, the structured ones I know it can be taught using the structured but, I think, a lot of people fail it. Again I think there are other chapters or parts of physics which is impossible to teach without the mathematics.

## Student M1

M1: Yes, physics you have to apply and that application is...is related somehow to mathematics. It links mathematics with physics.

Fizi (refer to M2) boldly states that one cannot learn physics without mathematics. He explains that one needs to apply mathematics in order to learn physics. Tracy (give student number) is unsure, and says it's because there is a lot of mathematics in physics. The students say mathematics makes them solve problems "quicker"; makes them "believe" and that believing leads to understanding.

Fizi thinks it's "impossible" to learn physics without mathematics because "You need to apply maths in order to understand the physics". For this student, physics will be almost meaningless without mathematics since one cannot understand physics without mathematics. Students in this group indicate a deliberate preference for learning physics that involves the use of mathematics.

### 5.7 Summary

Students' expectations were solicited when they had just begun the first semester of the first year physics course. Interviews on the other hand were conducted when students were in the middle and towards the end of the second semester. With regard to the role of mathematics in students' understanding of physics, both means of data collection can be summarized as being in agreement on that students...

- are accustomed to solving problems they have seen before
- use a lot of memorization
- can get correct answers which they do not understand
- think that problem solving should make them understand concepts
- do not believe that physics can be learnt without mathematics

While they were conducted at different times reflecting varying students experiences, the corroboration factor among the two sets of data could be regarded as a measure of reliability of students' responses. Corroboration strengthens a particular frame and helps put in context students' actual use of mathematics when solving physics problems in the topic of electricity.

Having established students expectations; the premise from which students engage with physics and the use of mathematics in problem solving, this study then delves into the practice, to parse it, and eventually relate students' expectations with how they actually engage with physics problems. The baseline so established built context that helped put into perspective students' work when their scripts were analysed by means of the developed conceptual framework.

With the baseline now in place, the analysis of students' test scripts is presented in chapter 6 to address the second and third research questions, namely:
b. What mathematical approaches do students use when solving electricity problems?
c. What types of understanding emerge when students use certain mathematical approaches to solve electricity problems?

Analysis based on these research questions, and expounded through application of MATHRICITY should yield which types of mathematical resources are activated, and the domains of knowledge that emerge, presumably influenced by the established baseline.

## Chapter 6 Students' test scripts

### 6.1. Introduction

In the previous chapter (5) a baseline of students' expectations on the use of mathematics in physics was established. Students indicated some strongly held conceptions about; what it takes to learn physics in general, and specifically about the use of mathematics in physics. Through the use of the survey and interviews, students indicated that in their learning of physics, equations are important and that they also use them even when they may not understand the meaning of their actions, and that memorization plays a big part. Students also explained why they use mathematics in physics; through the use of words and phrases such as, to "prove", "boost their morale towards physics", and "simplify". While they claim that mathematics and physics are inseparable, and they overwhelmingly respond to the survey that one cannot learn physics without mathematics, in part of their interview discussions, students still talk of "the mathematics part" and "the physics part" (see section 5.6.2). This gives a contradicting message to their view that mathematics and physics are inseparable.

In this chapter data will be presented and analyzed using the conceptual framework (see chapter 3) to answer the second and third research questions:
2. What mathematical approaches do students use when solving electricity problems?
3. What types of understanding emerge when students use certain mathematical approaches to solve electricity problems?

In order to answer both these research questions, three sub-questions were developed for each of the questions.

Sub questions for research question 2 :
a) Are there different mathematical approaches when students solve electric circuit problems?
b) Are there different mathematical approaches when students solve electric field problems?
c) Are there different mathematical approaches when students solve electric force problems?

Sub questions for Research question 3:
a) What types of understanding emerge when students solve electric circuit problems?
b) What types of understanding emerge when students solve electric field problems?
c) What types of understanding emerge when students solve electric force problems?

The intentional approach to learning and meaning-making developed in research on learning (Hallde'n, Scheja \& Haglund, 2008) guided the analysis of the students' scripts. The analysis based on such an approach focuses on the students' activities in terms of intentional action. Intentional action is when particular actions can be explained in terms of the motive behind. Students' use of mathematics in solving electricity problems is viewed in terms of some underlying motivation or a desire to achieve a goal. By analyzing how students use mathematics in solving electricity problems, with a focus on their approaches, and the resultant understanding from the particular learning tasks, one would be able to infer the role that mathematics play in their understanding of physics. Whether the role of mathematics is any different in the topic of electricity compared to other physics topics in general is an important consideration. In particular, the analysis sought to describe students' mathematical approach in the topic electricity, and how that leads to what type or level of understanding.

The analysis focused on three (3) questions from the two tests on electricity that students wrote (see section 4.5.2). Two (2) questions were from test 1 (electric force - question $1 \mathrm{~A}_{1}$ and electric field - question $1 \mathrm{~B}_{2}$ ). The third question, $2 \mathrm{~B}_{2}$ from test 2 was on electric circuits. The prefixes 1 and 2 were used to distinguish a question chosen from test 1 and 2 respectively, whereas letters A and B showed that the questions were from section A or B of the test (see section 4.5.2). The subscript showed the question number ( $1,2,3 \ldots$ ) in a particular test that is being analyzed. The questions were deliberately chosen so that they spread across the electricity subtopics; electric force, electric field and electric circuits as established by means of the GST in chapter three (see section 3.2).

For each of the three (3) selected questions, three (3) different students' solutions are presented. Nine (9) different solutions involving the use of mathematics in the physics topic of electricity were analyzed. The students' solutions will be referred to as, H2, M3, V1 etc. depending on the tutorial group ( $\mathrm{H}, \mathrm{M}$ or V ) to which the student belonged, and as indicated in their test scripts. The subscripts 1, 2, 3 indicate whether it is the first or second student's script to be analyzed and presented in that tutorial group.

For each student's solution; the use of mathematics is mainly analyzed by means of the theoretical framework (MATHRICITY) depicting the activated mathematical resources, as well as the knowledge domains described through the ESM (See section 3.5). In addition the different stages for dropping and using units, substituting numbers for variables, and substituting numbers for constants further compounds the analysis.

### 6.2 Analysis of students' work on Electric Force

Electric force is a sub-topic of electricity that was identified by means of the GST as distinctively contributing to the first year physics topic of electricity (see section 3.5).

Question $1 \mathrm{~A}_{1}$ (from test 1 , section A , question no. 1) involves electric force between two point charges placed some distance apart and was chosen for analysis as it was from the electric force sub-topic.

## SECTION A <br> (Answer ALL parts of this Section - Each Question carries 5 marks)

A1. Force of attraction between two point electric charges placed at a distance $d$ in a medium is $F$. What distance apart should these be kept in the same medium, so that force between them becomes $\frac{F}{3}$ ?

## Figure 5: Question 1A $\mathbf{A}_{1}$

[A model answer from the instructor's marking guide is shown on appendix I1]

### 6.2.1 Analysis of Student V1's work on Q1A ${ }_{1}$



Figure 6: Student V1's solution to Q1A $\mathbf{1}_{1}$
The student realized that this question requires the use/application of Coulomb's law and thus writes down the equation that represents this law. The student then substitutes the Coulomb constant $k_{e}$ with the numerical value $9 \times 10^{9}$ in the second line of the solution. In this step, the student also replaced the symbol for distance $r$ used as the common symbol for distance between two point charges in the Coulomb's law equation in the first step with $d$. This is despite that $d$ is stated in the problem as an absolute value for distance.

Table 6.1 is a step-by-step portrayal of student V1's solution as analysed by using the conceptual framework.

Table 6.1: A step-by-step description of student $V 1^{\prime}$, work on Q1A $_{1}$

| Step | Activity | Description of activity | Activated mathematical resource | Awareness/ translation between ESM domains |
| :---: | :---: | :---: | :---: | :---: |
| 1 | $F_{e}=k_{e} q_{1} q_{2} / r^{2}$ | Wrote Coulomb's law | Unidentified resource 1 | Abstract domain, layer $b$ |
| 2 | $F=\left(9 \times 10^{9} x q_{1} x q_{2}\right) / d^{2}$ | Substituted numbers for the constant $k_{e}$, and expressed the distance as $d$ (it was expressed as $r$ in step 1) | Interpretive devices (formal) | Symbolic domain, layer $b$ |
| 3 | $\frac{F}{3}=\left(9 x 10^{9} x q_{1} x q_{2}\right) / x$ | -Wrote equation for, "so that the force between them becomes F/3". <br> -Constant $k_{e}$ is substituted and unknown distance expressed as $x$ | -Interpretive Devices (formal) <br> -Interpretive Devices (formal) | -Symbolic domain, layer $b$ <br> -Symbolic domain, layer $b$ |
| 4 | $F=1 / d^{2}$ | The numerator from step 2 disappears | Interpretive devices(formal ) | Symbolic domain, layer $b$ |
| 5 | $F=\frac{1}{x}$ | The numerator from step 3 disappears | Interpretive devices | Symbolic domain, layer $b$ |
| 6 | $\frac{1}{x}=1 / d^{2}$ | Equations on steps 4 and 5 are added simultaneously | Interpretive devices (formal) | Symbolic domain, layer $b$ |
| 7 | $d^{2}=x$ | Items on equation in step 6 are cross-multiplied | Symbolic form | Symbolic domain, layer $b$ |
| 8 | It should be kept at $d^{2}$ | A worded statement is given as an answer to the question | Interpretive devices (formal) | Symbolic domain, layer $b$ |

## a) Mathematical Resources activated

It is unknown, how upon reading the question, the student arrived at $\mathrm{F}_{\mathrm{e}}=\mathrm{k}_{\mathrm{e}} \mathrm{q}_{1} \mathrm{q}_{2} / \mathrm{r}^{2}$ in step 1 . This cannot be explained through activation of known mathematical resources (see section 3.3.2.2). This step is therefore designated as an unidentified resource 1. It will be discussed further at the end of the chapter.

Student V1substituted $r^{2}$ with $d^{2}$, and $k_{e}$ with the numerical value $9 \times 10^{9}$ from step 1 to step 2. Substitution in problem solving is a technique that is applied with a realization that one entity can be used in place of another. The activated mathematical resource here is intuitive interpretive devices (see section 3.4.2.2).The mathematical technique of substitution can be described as activation of intuitive interpretive devices because it is "abstracted from everyday reasoning and applied to physics equations" (see section 3.4.2.2). Substitution is
commonly used in everyday reasoning outside physics when there is need for a simpler, alternative, or more pragmatic explanation.

With regard to the equation in step $3 ; F / 3=9 \times 10^{9} q_{1} q_{2} / x$ the student generates the symbol $x$ for an unknown quantity which is distance. Generating a new symbol (especially $x$ ) for an unknown quantity is a standard mathematical procedure and therefore indicates the activation of formal interpretive devices.

While not correctly done, the steps 2 to 4 and steps 3 to 5 for both equations appeared to be an attempt at the mathematics of cancellation. Here the symbolic forms (see. section 3.3.2.3) type of mathematical resources is activated. The student is looking at the structural form (symbol template) of the two equations ( $F=\frac{9 \times 10^{9} \times q 1 \times q 2}{d^{2}}$ and $\frac{F}{3}=\frac{9 \times 10^{9} x q 1 \times q 2}{x}$ ) and notices the similarity in part of their form (numerators). The partial symbol template is ■ $=\frac{\Delta}{d^{2}}$ and $\frac{\square}{3}=\frac{\Delta}{x}$. The student cancels the " $\Delta$ " in both equations and remains with numerators of 1 .

Step 6 shows cross-multiplication while step 4 and 5 then shows addition of simultaneous equations. Both simultaneous equations and cross-multiplication are standard mathematical procedures, thus activates formal interpretive devices.

Neither intuitive mathematics knowledge nor reasoning primitives type-of-mathematical resources were activated anywhere as this student was solving this problem. The student did not use any basic everyday mathematics, nor indicate any "abstractions of everyday experiences".

## b) Awareness of ESM domains

The ability to translate the physical situation by stating Coulomb's law in the first line of the solution places the student's cognition in the abstract domain; a law is being used to "explain the physical or concrete aspect" (see section 3.3). This could be that the student converts the worded physical description into the physical equation of Coulomb's law, or since the student is aware of the context, question $1 \mathrm{~A}_{1}$ cues in his mind a mental note/image of Coulomb's law. The student proceeds to operate in the symbolic domain as he: substitutes the value of the Coulomb's constant, and assigns the unknown distance the symbol $x$ on the equation in step 3. The subsequent "mathematics of algebra" that leads to the solution from step 4 to line 8 (though incorrect) is still indicative of a symbolic approach.

The model domain (see section 3.3) where diagrams could be used to represent phenomena is absent from the student's entire work.

From the solution, the student may have construed the physical meaning (concrete domain) of the problem. This would be so if the student had realized that; for the force between two Charges to decrease (from F to F/3) the distance should increase (from d to $\mathrm{d}^{2}$ ). However the mathematics leading to $d^{2}$ (step 8) does not indicate that. It is incorrect and therefore $d^{2}$ cannot be deduced as physically meaningful, even to the student. In addition, the phrase "it should be kept at" (step 8) rather than "it should move to" also indicates that the student may actually be thinking that the original distance is the same as the final distance. There is thus no indication of the concrete domain (see section 3.3).

With respect to the two layers $a$ and $b$ that according to the ESM should constitute each of the knowledge domains, student $\mathbf{V} 1$ appear to be working in layer $b$ only. Only meaningful combinations of items appear in delineated problem solving steps. No items are presented on their own, independent of one another (layer $a$ ).

## c) Use of units, variables and constants

In student V1's solution to this problem:

- No units were used
- No variables have been substituted
- Constant $\left(\mathrm{k}_{\mathrm{e}}\right)$ was substituted in the two equations in step 2 and 3

Since only the constant ( $\mathrm{k}_{\mathrm{e}}$ ) was used in this solution, there is no order of substitution to be discussed with respect to units and variables.

### 6.2.2 Analysis of Student M3 on Q1A



Figure 7: Student M3 solution to Q1A ${ }_{1}$
Student M3 realized the question as requiring the use/application of Coulomb's law and writes it down in the first step. In step 2, the student writes the mathematical expression for when the force becomes $F / 3$. At this point, the student incorrectly puts $d$ as the distance between the charges. In step 3 , the constant $k_{e}$ and the charge $q$ are substituted and $F$ disappears inexplicably. Proceeding to calculate for $d$ becomes a futile effort.

Table 6.2 is a step-by-step portrayal of the application of the conceptual framework to student M3's solution to the question.

Table 6.2: A step-by-step description of Student M3' work on Q1A $\mathbf{A}_{1}$

| Step | Activity | Description of activity | Activated mathematical resource | Awareness/ translation between ESM domains |
| :---: | :---: | :---: | :---: | :---: |
| 1 | $F_{e}=k_{e} q_{1} q_{2} / r^{2}$ | Writes Coulomb's law | Unidentified resource 1 | Abstract domain, layer $b$ |
| 2 | $1 F / 3=k_{e} q^{2} / d^{2}$ | writes the expression for, "so that the force between them becomes $\mathrm{F} / 3^{\prime \prime}$, with $d$ as the distance between the charges. | Interpretive devices (formal) | Symbolic domain, layer $b$ |
| 3 | $\frac{1}{3}=\left(9 \times 10^{9}\right) \frac{\left(-1.6 \times 10^{-19}\right)^{2}}{d^{2}}$ | -the symbol $F$ on the left side of the equation in step 2 disappears - constant $k_{e}$ and the variable $q_{1}$ are substituted | -Interpretive devices (formal) <br> -Interpretive devices(formal) | Symbolic domain, layer b <br> Symbolic domain, layer b |
| 4 | $\begin{aligned} & d^{2} \\ & =3\left(9 \times 10^{9}\right)\left(1.6 \times 10^{-19}\right)^{2} \end{aligned}$ | Equation in step 3 is rearranged so that $d^{2}$ is alone and on the left side | Interpretive devices(formal) | Symbolic domain, layer $b$ |
| 5 | $d^{2}=6.912 \times 10^{-28}$ | Numbers on the right side of the equation in step 4 are computed | Interpretive devices(formal) | Symbolic domain, layer $b$ |


| 6 | $d=2.63 \times 10^{-14} \mathrm{~m}$ | -The square root is performed <br> on both sides on the equation in <br> step 5 | -Interpretive <br> devices(formal) | -Symbolic domain <br> layer b |
| :--- | :--- | :--- | :--- | :--- |
| $-m$ is given as the units | -Unidentified <br> resource 2 | -concrete domain, <br> layer $b$ |  |  |

## a) Mathematical Resources activated

In step 1 , the unidentified mathematical resource 1 is activated as this student, like student V1 above simply states Coulomb's law.

Once the equation is stated, the student's focus is solely on manipulation of the equation and involves substitution (step 3), cross multiplication (step 3 to 4 ) multiplication (step 4 to 5) and finding the square root (step 5 to 6 ). All these steps are standard mathematical procedures thus require activation of interpretive devices type of mathematical resources.

This student, also like V1, has not activated intuitive mathematics knowledge through use of any basic everyday mathematics knowledge nor reasoning primitives to indicate any intuitive sense of physical mechanism. Symbolic forms, where the structural forms of equations guide the student's work, are also absent.

A second unidentified mathematical resource, designated unidentified resource 2, is activated in the last step (6) when, from nowhere the student assigns the solution units $m$ (metres). The nature of this mathematical resource will, together with unidentified resource 1 be discussed at the end of this chapter.

## b) Awareness of ESM domains

Stating Coulomb's law in step 1 is indication of the abstract domain. The rest of the steps from 2 to 6 involving; substitution of the constant and variables; cross multiplication and finding the square root are an indication of awareness of the symbolic domain (see section 3.4).

Though the solution is incorrect, the use of $m$ for units of distance shows the student's awareness of the concrete domain (see section 3.3). He understands distance as physical quantity that is being determined, and so the solution should have units of distance, metres.

Awareness of the model domain, where the student could have used diagrams to demonstrate relationship between variables (say charges and distance) is absent.

Student M3 appears to be working on layer $b$ only, as no independent terms (layer $a$ ) were presented or considered on their own in any of the steps.

## c) Use of units, variables and constants

In student M3's solution to this problem:

- Units for distance (m) were used only in the last step ( $6^{\text {th }}$ )
- Variable $\mathrm{q}_{1}$ was substituted in the $3^{\text {rd }}$ step
- Constant $\left(\mathrm{k}_{\mathrm{e}}\right)$ was substituted in the $3{ }^{\text {rd }}$ step

The order of use or substitution in this solution was; variables and constant first (both $3^{\text {rd }}$ step) and units last ( $6^{\text {th }}$ step).

### 6.2.3 Analysis of Student M5'swork on Q1A



Figure 8: Student M5 solution to Q1A $\mathbf{1}_{1}$
This student realized that the question requires the use of Coulomb's law and writes it (though incorrectly with the $d$ not squared) in the first step. He then writes let the new distance $=x$ for when the force is $F_{e} / 3$ in step 2 .

In step 3 of the solution, student M5 equates the mathematical expressions for the statements; so that the force between them becomes $F / 3$ with when the distance between two point charges is $d$. In the next line, the student cancels the constants $k_{e}$ on both sides on the equation. The student cancels $q_{1}$ and $q_{2}$ in step 5 . In step 6 , the student performs crossmultiplication and proceeds with the mathematics to get the correct answer.

Table 6.3 is a step-by-step portrayal of the application of the conceptual framework to student M5's work.

Table 6.3: A step-by-step description of student M5's work on Q1A

| Step | Activity | Description of activity | Activated mathematical resource | Awareness/ translation between ESM domains |
| :---: | :---: | :---: | :---: | :---: |
| 1 | $F_{e}=k_{e} q_{1} q_{2} / d$ | Writes an equation similar to Coulomb's law; with the distance $d$ not squared | Unidentified resource 1 | Abstract domain, layer $b$ |
| 2 | $\frac{F_{e}}{3}=$ the force due to the new distance, <br> let the new distance $=x$ | -writes a statement to explain $F_{e} / 3$ <br> - assigns the new distance a variable | -Interpretive devices(formal) <br> -Interpretive devices (formal) | Symbolic domain, layer $b$ |
| 3 | $1 / 3\left[\frac{k_{e} q_{1} q_{2}}{\left(d^{2}\right)}\right]=k_{e} q_{1} q_{2} /\left(x^{2}\right)$ | -Writes the expression for $F_{e}$ in step 1 multiplied by $1 / 3$ (now, with $d$ squared) <br> - the expression is equated to Coulomb' law on the right, with $x$ as the distance | -Interpretive devices(formal) <br> -Interpretive devices(formal) | -Symbolic domain, layer $b$ <br> -Symbolic domain, layer $b$ |
| 4 | $k_{e} q_{1} q_{2} / k_{e} 3 d^{2}=k_{e} q_{1} q_{2} / k_{e} x^{2}$ | Divides both sides of the equation in step 3 by $\mathrm{k}_{\mathrm{e}}$ | Interpretive devices(formal) | Symbolic domain, layer $b$ |
| 5 | $q_{1} q_{2} / 3 d^{2}=q_{1} q_{2} / x^{2}$ | The $\mathrm{k}_{\mathrm{e}}$ 's in step 4 are cancelled | Interpretive devices(formal) | Symbolic domain, layer $b$ |
| 6 | $1 / 3 d^{2}=1 / x^{2}$ | The $\mathrm{q}_{1}$ and $\mathrm{q}_{2}$ in step 5 are cancelled | Interpretive devices(formal) | Symbolic domain, layer $b$ |
| 7 | $x^{2}=3 d^{2}$ | Items in step 6 are crossmultiplied. | Symbolic form | Symbolic domain, layer $b$ |
| 8 | $x=\sqrt{3 d}$ metres | -the square root is performed on both sides of the equation in step 7 <br> -units, metres are given | -Interpretive devices(formal) <br> -Unidentified resource 2 | Symbolic domain, layer $b$ <br> -Concrete domain, layer $b$ |

## a) Mathematical Resources activated

The unidentified resource 1 observed in the first step for the two previous a solution is activated again in step 1, when this student simply states Coulomb's law.

Equating mathematical expressions (steps 2 and 3); cancellation (steps 4 and 5) and finding the square root (step 7 to 8 ) are all examples of activation of interpretive devices. Cross
multiplication (step 6) is an example of activation of the symbolic form type-ofmathematical resource.

Neither intuitive mathematics knowledge nor reasoning primitives were activated as the student was solving this problem.

The assigning of the units metres (step 8) is activation of unidentified mathematical resource 2, as explained for student M3's solution previously.

## b) Awareness of ESM domains

The student has shown awareness of the abstract, symbolic and concrete domains (see section3.4) in their solution. Awareness of the abstract domain is indicated by stating Coulomb's law. Manipulating variables, constants and the mathematical computations that follow are an indication of awareness of the symbolic domain.

The student stating of the units of metres in the last line of his solution, though inappropriate in this question, may still be interpreted as indication of awareness of the concrete domain (see section 3.3) - where the student indicates awareness that distance is the physical quantity that is under consideration, and is measured by metres.

Awareness of the model domain, where the student could have used diagrams to demonstrate relationship between variables (say charges and distance) is absent.

All the distinct steps for student M5's work are in layer $b$ only, since it is relationships between items that is presented.

## c) Use of units, variables and constants

In student M5's solution to this problem:

- Units for distance (m) were used only in the last step ( $8^{\text {th }}$ )
- Variables were not substituted (cancelled out)
- Constant not substituted (cancelled out)

Only units were substituted in this solution. Variables and constants were not substituted but rather both were cancelled out. The constant $k_{e}$ was cancelled in the $5^{\text {th }}$ step while the variables $q_{1}$ and $q_{2}$ were cancelled out in the $6^{\text {th }}$ step.

### 6.2.4 Summary of the three students' work on Q1 $A_{1}$

All the three students started their solution by stating the Coulomb's equation; $F_{e}=k_{e} q_{1} q_{2} / r^{2}$. There is no indication as to how they reasoned that Coulomb's equation was needed. This observation could not be identified with the existing mathematical resources and was therefore designated as unidentified mathematical resourcel.

Both students M3 and M5 haphazardly assigned units at the end of their solutions, without any trace. This particular step in the two students' solutions was also noted as another unidentifiable mathematical resource and designated, unidentified mathematical resource 2 .

The activated mathematical resources when students solve this problem are mostly interpretive devices, where formal mathematical rules are applied; and seldom symbolic forms), where mathematical expressions may be seen through virtual structures of the equation (only twice; step 7 for both students V1 and M5). Reasoning primitives and intuitive mathematics knowledge resources are not activated in all the three students' solution.

In all the three students' solutions, awareness of the abstract, symbolic and concrete knowledge domains is prevalent. However the symbolic domain comes out as the most favored. The model domain does not appear in any step. If anything, they may have drawn diagrams on the question paper. This would still be an indication that they think it is not important to show diagrams on the work that is graded.

What also emerges as the three students solve the same question on electric force between point charges is that:

For Units:

- Either units are not used at all (V1), or that the two students who used them (M3 and M5) only did so in the last step on their solution.

For Variables:

- They are substituted in the $1^{\text {st }}$ or $2^{\text {nd }}$ line after stating Coulomb's law.

For Constants:

- Just like variables, they are substituted in the 1st or 2nd line after introduction of formula, or they are not be substituted at all (student M5).


### 6.3 Analysis of students work on Electric Field question

Electric field is another sub-topic of electricity that was identified by means of the GST as distinctively contributing to the first year physics topic of electricity (see section 3.5)

Question 1B $\mathrm{Ba}_{2 \mathrm{a}}$ (from test 1, section B, question no. 2a) involves calculation of electric field at some distance due to two point charges.

Figure B1.
B2. $\mathbf{A B D}$ is an equilateral triangle of side 2 m . Point $E$ is the center of the equilateral triangle (see Fig. B2). Point charges $Q_{1}=(+) 5 \mu \mathrm{C}$ and $\mathbf{Q}_{2}=(+) 5 \mu \mathrm{C}$ are placed at $\mathbf{B}$ and $\mathbf{D}$, respectively.

(a) Calculate net electric field at point F

Figure B2

Figure 9: Question 1B 2a
[A model answer from the instructor's marking guide is shown on appendix I2]

### 6.3.1 Analysis of student V1's work on Q1B 2a

$$
\begin{array}{ll}
\text { B2. } \\
\text { (a) }
\end{array}
$$

## Figure 10: Student V1 solution on Q1B 2a

The student starts by calculating the magnitude of the electric field $E_{1}$ and $E_{2}$ separately. The student also calculated the distance(r) from the point charges $\left(\mathrm{Q}_{1}\right.$ and $\left.\mathrm{Q}_{2}\right)$ to the center of the equilateral triangle (point E ). In this instance; the student realized that $r$ is the same for $\mathrm{Q}_{1}$
and $\mathrm{Q}_{2}$ with respect to point E . It is a possibility that the student may have noticed that he needs $r$ only after starting on the electric field equation, and then calculates it on the right side of the page.

Table 6.4 below portrays a step-by-step analysis of student V1's solution, using the conceptual framework.

Table 6.4: A step-by-step description of student V1's work on Q1B $\mathbf{2 a}^{\text {a }}$

| Step | Activity | Description of activity | Activated mathematical resource | Awareness/ translation between ESM domains |
| :---: | :---: | :---: | :---: | :---: |
| 1 |  | -Draws triangle showing the electric point charges <br> -Shows electric field vectors <br> -Geometrically determines the mid-point between the two charges as $1 m$ | -Interpretive devices(formal) <br> -Interpretive devices(formal) <br> -Reasoning primitives | -Model domain, layer $b$ <br> -Model domain, layer $b$ <br> -Model domain, layer $b$ |
| 2 | $\begin{gathered} c^{2}=a^{2}+b^{2} \\ 2^{2}=a^{2}+1^{2} \\ a^{2}=4-1 \\ \sqrt{a^{2}}=\sqrt{3} \\ a=\sqrt{3} \\ \therefore \frac{\sqrt{3}}{2}=0.86 m=0.9 m \\ r^{2}=0.86^{2}+1^{2} \\ r=1.32 m \end{gathered}$ | -Uses the sums of squares rule to get the distance from one base of the triangle to the other corner <br> -Divides the result above to get the length to point $E$. <br> - Assigns the solution units of $m$ | -Interpretive devices (formal) <br> -Interpretive devices (formal) <br> -intuitive mathematics resource <br> -Unidentified resource 2 | -Symbolic domain, layer $b$ <br> -Symbolic domain, layer $b$ <br> -Concrete domain, layer $b$ |
| 3 | $\begin{gathered} E_{1}=k_{e} q_{1} / r^{2} \\ =\left(9 \times 10^{9} \times 5 \times 10^{-6}\right) /(1.32)^{2} \\ =2.5862 .5 \mathrm{C} / \mathrm{m}^{2} \end{gathered}$ | -States the electric field equation <br> -Calculates electric field $E_{I}$ due the point charge $\mathrm{Q}_{1}$ <br> -Assigns the solution units of $\mathrm{c} / \mathrm{m}^{2}$ | -Unidentified resource 1 <br> -Interpretive devices(formal) <br> -Unidentified resource 2 | -Abstract domain, layer $b$ <br> -Symbolic domain, layer $b$ <br> -Concrete domain, layer $b$ |
| 4 | $E_{2}=k_{e} q_{2} / r^{2}$ | -States the electric field equation | -Unidentified resource 1 | -Abstract domain, layer $b$ |


|  | $=\frac{\left(9 \times 10^{9} \times 5 \times 10^{6}\right)}{(1.32)^{2}}=25862.5 \mathrm{C} / \mathrm{m}^{2}$ | -Calculates electric field $E_{2}$ due the point charge $\mathrm{Q}_{2}$ <br> - Assigns the solution units of $\mathrm{c} / \mathrm{m}^{2}$ | -Interpretive devices(formal) <br> -Unidentified resource 2 | -Symbolic domain, layer $b$ <br> -Concrete domain, layer $b$ |
| :---: | :---: | :---: | :---: | :---: |
| 5 | $\begin{gathered} \frac{X-\text { components }}{} \\ E_{1 x}-E_{2 x}=E_{x} \\ 25862.5 \cos \theta-25862.5 \cos \theta=0 \\ \therefore E_{x}=0 \end{gathered}$ | Calculates the x component of the electric field by subtracting the x component of $\mathrm{E}_{2}$ from the x component of $\mathrm{E}_{1}$ | Interpretive devices(formal) | Symbolic domain, layer $b$ |
| 6 | $\begin{gathered} \frac{Y-\text { components }}{} \\ E_{y}=E_{1 y}+E_{2 y} \\ =25862.5 \sin \theta+25862.5 \sin \theta \\ =2(25862.5 \sin \theta) \\ E_{y}=44732.8 \mathrm{C} / \mathrm{m}^{2} \end{gathered}$ | Calculates the $y$ component of the electric field by adding the y component of $\mathrm{E}_{2}$ to the $y$ component of $\mathrm{E}_{1}$ | Interpretive devices(formal) | Symbolic domain, layer $b$ |
| 7 | $\begin{aligned} & \text { Net electric field }=\sqrt{E_{x}^{2}+E_{y}^{2}}= \\ & \sqrt{44732.8} \end{aligned}$ | Adds the x component to the y -component to get the net electric field at point E | Interpretive devices(formal) | Concrete domain, layer $b$ |

## a) Mathematical Resources activated

In step 1, student V1 sketches the triangle showing electric point-charges as well as the midpoint between the electric point charges. This is an application of the mathematics of geometry and thus activates interpretive devices. Calculating $r$ (step2); calculating $E_{l}($ step3); calculating $E_{2}$ (step4); calculating the $x$ and $y$ components of E (step 5and 6); and finally calculating the net electric field (step 7) are all further examples of activation of interpretive devices, as formal mathematical rules are applied.

For the student to determine that the mid-point between the two point charges is $\operatorname{lm}$ (step 1) involves noticing that two halves equal a whole. What is activated here is a reasoning primitive type - of - mathematical resource similar to the whole is equal to the sum of its parts.

The student calculates a single $r$ for the distance from corners B and D to the centre point E in step 2. Realizing that $r$ is the same from both corners of the triangle indicates activation of the
pairing type - of - intuitive mathematics knowledge. The student immediately grouped the two distances "for collective consideration" (see section 3.4.2.2.).

This student does not indicate anywhere that from the problem statement; $\mathrm{Q}_{1}=5 \mathrm{uC}$ and $\mathrm{Q}_{2}=$ $5 u C$; they immediately (intuitively) realize that $\mathrm{Q}_{1}=\mathrm{Q}_{2}$. This would have happened through activation of intuitive mathematics of pairing, but it is not activated by the student. The student proceeds to calculate the electric fields at point E due to $\mathrm{Q}_{1}$ and $\mathrm{Q}_{2} ; \mathrm{E}_{1}$ and $\mathrm{E}_{2}$ separately (steps 3 and 4). That the student still does not indicate realization at this stage that $\mathrm{E}_{1}=\mathrm{E}_{2}$ verifies unavailability of the intuitive mathematics resource of pairing.

The student does not show realization that, if two objects (point charges $\mathrm{Q}_{1}$ and $\mathrm{Q}_{2}$ ) with the same magnitude are placed the same distance away from a point (E), then their effect at that point (electric field) should be the same. This would be reasoning primitive comparable to the more is more (see section 3.4.2.2). In this case the reasoning primitive would have been same is same.

The student uses formal mathematics to show that $\mathrm{E}_{\mathrm{x}}=0$ (step5). Failure to realize without using mathematics that $\mathrm{E}_{1 \mathrm{x}}$ and $\mathrm{E}_{2 \mathrm{x}}$ are equal is further indicative of failure to activate intuitive mathematics knowledge of pairing. Similarly, failure to realize that $\mathrm{E}_{1 \mathrm{y}}$ and $\mathrm{E}_{2 \mathrm{y}}$ are equal without the use of formal mathematics indicates that the pairing type - of - intuitive mathematics knowledge is not activated.

Had the pairing intuitive mathematical knowledge resource been activated, the next cognitive level would have been to realize without the use of formal mathematics that $\mathrm{E}_{1 \mathrm{x}}$ and $\mathrm{E}_{2 \mathrm{x}}$ are "opposing influences exactly cancelling each other out to produce no apparent result" (section 3.4.2.2). This would be the reasoning primitive of balancing (section 3.4.2.2). The same level of thinking would be applied to indicate $\mathrm{E}_{1 \mathrm{y}}$ and $\mathrm{E}_{2 \mathrm{y}}$ as adding influences which produces twice the effect. This would be reasoning primitive which I call doubling.

## b) Awareness of ESM domains

Sketching of the diagram in step 1 indicates that the student's awareness starts on the model domain. The student's awareness then moves toward the abstract domain when stating the electric field equation at the beginning of steps 3 and 4 . The rest of the steps that involve algebraic manipulation until the end indicate awareness of the symbolic domain.

The student's use of units $\left(\mathrm{C} / \mathrm{m}^{2}\right.$ and m$)$ in the last line of the solutions in the various steps is an indication of awareness of the concrete domain. The student is aware that he is dealing with physical quantities that should have units.

This student's entire work only shows relationships between different variables, with no variable being considered on their own. This is therefore an indication of the awareness throughout, of the ESM layer b only.

## c) Use of units, variables and constants

In student V1's solution to this problem;

- Units $C / m^{2}$ and $m$ were used in the last step of the various parts of the solution
- Variables $q_{1}$ and $q_{2}$ were substituted in the $2^{\text {nd }}$ line immediately after introduction of formula
- Constant $k_{e}$ was substituted in the $2^{\text {nd }}$ line immediately after introduction of formula

The order of use or substitution in this solution was; variables and constants first (both in the $2^{\text {nd }}$ line) and units last.

### 6.3.2 Analysis of student M4 on Q1B $_{2 \mathrm{a}}$



Figure 11: Student M4 solution to Q1B $_{\text {2a }}$

Student M4 sketches the diagram and even shows vectors at the beginning of his work (faint arrow sketches). The student then calculates the distance from the corners of the triangle to
the mid-point (r). He must have either worked on the side or mentally recalled the electric field equation to realize that he needs $r$. He then proceeds to calculate the electric field $\mathrm{E}_{1}$ and then notices that $E_{1}$ is the same as $E_{2}$. The student then resolves the electric field vector into its component vectors, and proceeds on to get the question right.

Table 6.5 shows a step-by-step portrayal of student M4's solution viewed through the conceptual framework.

Table 6.5: A step - by - step description of student M4's work on Q1B $\mathbf{2 a}^{2}$

| Step | Activity | Description of activity | Activated mathematical resource | Awareness/ translation between ESM domains |
| :---: | :---: | :---: | :---: | :---: |
| 1 |  | Draws triangle showing the angles and designates the length to point E , as $r$ | -Interpretive devices (formal) <br> -Intuitive mathematics | Model domain, layer $b$ |
| 2 | $\begin{gathered} \cos \theta=\text { adjacent/hypotenuse } \\ \cos 30^{\circ}=1 / r \\ r=1 / \cos 30^{\circ} \\ r=1.15 \mathrm{~m} \end{gathered}$ | -States part of the SOHCAHTOA rule <br> -Substitute the numerical values for the angle and the adjacent side, as well as the symbol for the hypotenuse $r$ <br> - Calculates $r$, and assigns it the units m | -Interpretive devices (formal) <br> -Interpretive devices (formal) <br> -Interpretive devices (formal) | Symbolic domain, layer $b$ |
| 3 |  | -Sketches electric field vectors at point E , due to point charges $\mathrm{Q}_{1}$ and $\mathrm{Q}_{2}$ | Interpretive devices(formal) | Model domain, layer $b$ |
| 4 | $\begin{gathered} \overrightarrow{E_{1}}=k_{e} q_{1} / r^{2} \\ =\frac{9 \times 10^{9}\left(5 \times 10^{-6}\right)}{(1.15)^{2}} \\ =45000 / 1.3225 \\ \overrightarrow{E_{1}}=34026.5 \mathrm{~N} / \mathrm{C} \\ \overrightarrow{E_{1}}=\overrightarrow{E_{2}} \end{gathered}$ | -States the electric field equation -Calculates the electric field at point $E$, due to $\mathrm{Q}_{1}\left(\mathrm{E}_{1}\right)$ <br> -Assigns it the units N/C <br> -Equates $\mathrm{E}_{1}$ to $\mathrm{E}_{2}$ | -Unidentified resource 1 <br> -Interpretive devices <br> -Unidentified resource 2 <br> -Reasoning primitive | -Abstract domain, layer b <br> -Symbolic domain, layer $b$ <br> -Symbolic domain, layer $b$ -Concrete domain, layer a and b |


| 5 | $\underline{X}$ - component $\begin{aligned} \overrightarrow{E_{1}} \operatorname{Cos} \theta & =34026.5 \operatorname{Cos} 30^{\circ} \\ & =29467.8 \mathrm{~N} / \mathrm{C} \end{aligned}$ $\begin{aligned} \overrightarrow{E_{2}} \operatorname{Cos} \emptyset & =34026.5 \operatorname{Cos} 30^{\circ} \\ & =29467.8 \mathrm{~N} / \mathrm{C} \end{aligned}$ | Calculates the x component of $\mathrm{E}_{1}$ and $\mathrm{E}_{2}$ and assigns both the units $\mathrm{N} / \mathrm{C}$ | Interpretive devices (formal) | Symbolic domain, layer $b$ |
| :---: | :---: | :---: | :---: | :---: |
| 6 | $\underline{Y-c o m p o n e n t}$ $\begin{gathered} \overrightarrow{E_{1}} \operatorname{Sin} \emptyset=34026.5 \operatorname{Sin} 30^{\circ} \\ =17013.25 \mathrm{~N} / \mathrm{C} \\ \overrightarrow{E_{2}} \operatorname{Sin} \emptyset=34026.5 \operatorname{Sin} 30^{\circ} \\ =17013.25 \mathrm{~N} / \mathrm{C} \end{gathered}$ | Calculates the y component of $\mathrm{E}_{1}$ and $E_{2}$ and assigns both the units N/C | Interpretive devices (formal) | Symbolic domain layer $b$ |
| 7 | $\begin{aligned} & \overrightarrow{E_{r}}=(29467.8-29467.8 N / C) i \\ &+(17013.25) \\ &+17013.25) j N \\ & / C \\ &=(34026.5) N / C j \end{aligned}$ | -Calculates the resultants electric field at point $E\left(E_{r}\right)$ by subtracting the x -components and adding the y components | Interpretive devices (formal) | Symbolic domain, layer $b$ |
| 8 | $\begin{aligned} \overrightarrow{E_{r}}= & \sqrt{\left(0^{2}+34026.5^{2}\right)} \\ & =34026.5 \mathrm{~N} / \mathrm{C} \end{aligned}$ | Calculates the numerical value for the $\quad$ resultant electric field | Interpretive devices (formal) | Concrete domain layer $b$ |
| 9 | $\begin{gathered} \operatorname{Tan} \varnothing=y / x \\ \emptyset=\tan ^{-1}\left(\frac{34026.5}{0}\right) \\ =90^{\circ} \end{gathered}$ | Calculates the angle due the resultant electric field, $\varnothing$ | Interpretive devices (formal) | Concrete domain layer $b$ |

## a) Mathematical Resources activated

Application of the mathematics of geometry in the sketched triangle (step 1); calculating $r$ (step2); sketching electric field vectors at point E , due to point charges $\mathrm{Q}_{1}$ and $\mathrm{Q}_{2}($ step3); calculating (step4); calculating the $x$ and $y$ components of electric field at point E (step 5and 6);calculating the resultant electric field ( steps 7 and 8); and finally calculating the angle due the resultant electric field (step 9), are all examples of activation of interpretive devices, as formal mathematical rules are applied.

In addition to interpretive mathematical resources, the following type of mathematical resources is also activated:

The sketch on the triangle by the student (step 1) indicates that he has realized that the distance(r) from both corners B and D to the centre is the same. This is further validated when the student calculates a single value for $r$. Both the above steps indicate activation of pairing type-of-intuitive mathematics knowledge.

The student immediately writes $E_{1}=E_{2}$ after calculating $E_{1}$. This indicates activation of the reasoning primitives - same is same - as explained in the analysis of Student V1's work on Q1B $_{2 \mathrm{a}}$ above. The student realizes immediately, without the use of formal mathematics that if $Q_{1}=Q_{2}$, then the effect of both charges at some point ( $\mathrm{E}_{1}$ and $\mathrm{E}_{2}$ ) must be the same.

In steps 5 and 6, the student resolves the electric fields $E_{1}$ and $E_{2}$ into their $x$ and $y$ components. Like student $\mathbf{V} 1$ above, the student's failure to realize without the use of formal mathematics that $E_{I x}$ and $E 2 x$ are the same, opposite and thus cancel each other out, indicates inability to activate reasoning primitives - balancing. Furthermore, failure to realize without the use of mathematics that $E_{1 y}$ and $E_{2 y}$ are the same, and thus should be multiplied twice is testament of failure to activate at least two mathematical resources. $E_{1 y}=E_{2 y}$ would be realized through activation of the intuitive mathematics knowledge of pairing - the ability to group two objects for collective consideration. That the $y$ component of the electric field on point E is a combined effect of $E_{1 y}$ and $E_{2 y}$ requires activation of the reasoning primitive doubling.

## b) Awareness of ESM domains

Student M4 started with the awareness of the model domain when drawing the diagram in step 1. Sketching of electric field in step 3 is another indication of awareness of the model domain.

The stating of the electric field equation in step 4 is an awareness of the abstract domain. The rest of the steps involving calculating of the electric field; calculating the x and y components; calculating the resultant electric field; and the angle due the resultant electric field are all indicative of awareness of the symbolic domain.

The use of units ( $m$ and $N / C$ ) is an indication of the awareness of the concrete domain (section 3.3).

The student had not given any item for consideration individually, therefore only the ESM $b$ layer is evident in the entire student' steps.

## C) Use of units, variables and constants

In student M4's solution to this problem:

- Units ( $m, N / C$ ) are used in last line of the various parts of the solution
- Variables $(q, r)$ are substituted in the $2^{\text {nd }}$ line of the various parts of the solution
- Constant $\left(k_{e}\right)$ is substituted in the $2^{\text {nd }}$ line of the various parts of the solution

The order of use or substitution in this solution was variables and constants first (both in the $2^{\text {nd }}$ line) and units last.
6.3.3 Analysis of student M5's work on Q1B ${ }_{2 a}$


Figure 13: Student M5 solution to Q1B $_{2 \mathrm{a}}$

The student starts by drawing the diagram from which he is able to geometrically determine the length to the centre BE. Student M5 realizes immediately that the electric field at the centre due to $Q_{1}\left(\mathrm{E}_{1}\right)$ is the same as the electric field at the centre due to $Q_{2}\left(\mathrm{E}_{2}\right)$. He then calculates the resultant electric field at point E due to both point- charges as requested.

Table 6.6 below gives a step-by step portrayal of the student's solution, viewed through the conceptual framework.

Table 6.6: A step-by-step description of student M5's work on Q1B $\mathbf{2 a}^{2}$

| Step | Activity | Description of activity | Activated mathematical resource | Awareness/ translation between ESM domains |
| :---: | :---: | :---: | :---: | :---: |
| 1 |  | Draws triangle showing electric field vectors $E_{1}$ and $E_{2}$ about the center | Interpretive devices (formal) | Model domain, layer b |
| 2 | $\text { Length } B E ; \cos 30=\frac{1}{B E}$ $B E=\frac{1}{\sqrt{\frac{3}{2}}}=2 / \sqrt{3 m}$ | -Calculates the length from the corner of the triangle the center B.E <br> - Assigns units $m$ | -Interpretive devices (formal) <br> -Unidentified resource 2 | Symbolic domain, layer b |
| 3 | $E_{1}=E_{2}$ | States that electric fields at point E due $\mathrm{Q}_{1}$ and $\mathrm{Q}_{2}$ are the same. | Reasoning primitives | Concrete domain, layer b |
| 4 | $\begin{aligned} & E_{1}=k q_{1} / B E^{2}=9 \times 10^{9}\left(5 \times 10^{8}\right) \prime(2 / \\ & \sqrt{3}) 2=33750 \mathrm{n} / \mathrm{c} \end{aligned}$ | - States the electric field equation <br> -Calculates electric field at E due to $\mathrm{Q}_{1}\left(\mathrm{E}_{1}\right)$ <br> - Assigns units N/C | -Unidentified resource 1 <br> -Interpretive devices (formal) <br> -Unidentified resource 2 | Abstract domain, layer b <br> Symbolic domain, layer b |
| 5 | $E_{2}=E_{1}=33750 \mathrm{~N} / \mathrm{c}$ | Give the numerical value for $E_{2}$ and $E_{1}$ | -Interpretive devices (formal) | Symbolic domain, layer b |
| 6 | $\begin{gathered} -E_{1 i}=E_{1} \cos 30=33750 \sqrt{3 / 2} \\ =29228.4 \mathrm{n} / \mathrm{C} i \\ -E_{1 i}=33750 x^{1 / 2}=16875 \mathrm{~N} / \mathrm{C} \end{gathered}$ | -Calculates the xcompany of $\mathrm{E}_{1}$ <br> -Calculates the ycompany of $\mathrm{E}_{1}$ <br> - Assigns units N/C | -Interpretive devices (formal) <br> -Interpretive devices (formal) <br> -Unidentified resource 2 | Symbolic domain, layer b |


| 7 | $\begin{aligned} &-E_{2 i}=-E_{2} \cos 30^{\circ} \\ &=-33750 \sqrt{3 / 2} \\ &=29228.4 \mathrm{~N} / \mathrm{C} \end{aligned} \quad \begin{aligned} -E_{2 i}=E 2_{2} \sin 30 & =33750 \times 1 / 2 \\ & =16875 \mathrm{~N} / \mathrm{C} \end{aligned}$ | -Calculates the xcompany of $\mathrm{E}_{2}$ <br> -Calculates the ycompany of $\mathrm{E}_{2}$ <br> -Assigns units N/C | -Interpretive devices (formal) <br> -Unidentified resource 2 | Symbolic domain, layer b |
| :---: | :---: | :---: | :---: | :---: |
| 8 | $\begin{aligned} & E_{r}=\left[\left(\sum E_{i}\right)^{2}+\left(\sum E_{j}\right)^{2}\right]^{1 / 2} \\ = & {\left[0^{2}+33750^{2}\right]^{1 / 2} } \\ = & 33750 \frac{N}{C} \text { in the positive } j \text { direction } \end{aligned}$ | -Calculates the result electric field <br> - Assigns units N/C | -Interpretive devices (formal) <br> - Unidentified resource 2 | Symbolic domain, layer b |

## a) Mathematical Resources activated

Sketching the triangle showing electric point-charges at the centre (step1);calculating the length from the corner of the triangle the center BE (step 2); calculating the net electric field through all the steps from 4 to 8 are all examples of activation of interpretive devices, as formal mathematical rules are applied. The first part of step 4, stating the electric field equation $E_{1}=k q_{1} / B E^{2}$ is however activation of the mathematical resource, unidentified resource 1 .

Step 3 is different as the student did not use any apparent formal mathematics to arrive at it. The student would have reasoned that if the charges $Q_{1}$ and $Q_{2}$ are the same, then their effect (electric filed) at a similar distance apart should be the same. The mathematical resource activated here is the reasoning primitive same is same. An unidentified mathematical resource; unidentified resource 2 is activated when, without any trace, the student assigns units in the last line of the various steps.

Neither intuitive mathematics knowledge nor symbolic forms type of mathematical resources are activated.

## b) Awareness ESM Domains

The student demonstrates awareness of the model domain in step 1 and the concrete domain in step 3. The model domain is demonstrated by the use of the diagram to represent (model)
the electric field. The concrete domain on the other hand surfaces when the student shows understanding of the physical phenomena without the use of mathematics.

All other steps involving manipulation of mathematical equations show the student's awareness of the symbolic domain.
c) Use of variables, constants and Units

In student M5's solution to this problem;

- Units ( $m, N / C$ ) used in last line of the various steps of the solution
- $\quad$ Variables $(q, B E)$ were substituted in the $2^{\text {nd }}$ line after stating the formula (step 4)
- $\quad \operatorname{Constant}\left(k_{e}\right)$ substituted in the $2^{\text {nd }}$ line after stating the formula (step 4)

The order of use or substitution in this solution was; variables and constants first (both in the $2^{\text {nd }}$ line) and units last.

### 6.3.4 Summary of the three students' (V1, M4, M5) work on Q1B ${ }_{2 \mathrm{a}}$

All the three students had to draw a diagram to show the distance from the point charges $Q_{l}$ and $Q_{2}$ to the mid - point $E$, as well as to show the electric field vectors $E_{1}$ and $E_{2}$.

One student $\mathbf{V} 1$ had to calculate the electric fields $E_{1}$ and $E_{2}$ first, to realize that they are the same. While student M3 realized that $E_{1}=E_{2}$ after the calculating $E_{1}$, this was not done through the use of mathematics. Student M5 did not have to use any formal mathematics to realize that $E_{1}=E_{2}$.

Interpretive devices and reasoning primitives are the only two (2) mathematical resources activated when students solve this problem. Reasoning primitives have only been activated once, in showing that $\mathrm{E}_{1}=\mathrm{E}_{2}$. Since the use of formal mathematics is common in all the three students work, interpretive devices are the predominantly activated mathematical resource. Both Unidentified resource 1 and unidentified resource 2 are activated in all the three students' solution.

What also emerges from the three students as they were solving the electric field problem is that;

For units:

- They were only used in the last line on a step.

For variables:

- They were only substituted in the $1^{\text {st }}$ line immediately after introduction of formula. For constants:
- They were only substituted in the $1^{\text {st }}$ line after introduction of formula


### 6.4 Analysis of students' work on Electric Circuit question

Electric circuits is the third sub-topic of electricity that was identified by means of the GST as distinctively contributing to the first year physics topic of electricity (see section 3.5).

Question $2 \mathrm{~B}_{2}$ (from test 2, section B, question no. 2) is on currents entering and leaving a junction.

B2. Use the Figure B2.1 given below to answer the questions on the circuit.


Figure B2.1
(a) Write the Kirchoff's current law (KIL) for the junction C in the circuit. [3]

## Figure 14: Question 2B $\mathbf{2}_{2}$

[A model answer from the instructor's marking guide is shown on appendix I3]

### 6.4.1 Analysis of Student M6 on Q2B $\mathbf{2}_{2}$



Figure 15: Student M6 solution to Q2B $_{2}$
The student starts by writing an equation where the three different currents are added to get a zero. The two currents $I_{l}$ and $I_{2}$ are denoted as negative, while $I_{3}$ is assigned a positive sign. In
the second step, the student applies a correct Kirchhoff's law at junction C (the correct answer).

Table 6.7 is the step-by-step portrayal of the application of the conceptual framework to student M6's solution.

Table 6.7: A step-by-step description of student M6's work on Q2B $\mathbf{2}_{\mathbf{2}}$

| Step | Activity | Description of activity | Activated <br> mathematical <br> resource | Awareness/ <br> translation <br> between ESM <br> domains |
| :--- | :--- | :--- | :--- | :--- |
| 1 | $-I_{1}-I_{2}+I_{3}=0$ | Equates the sum of the three <br> current (with $\mathrm{I}_{1}$ and $\mathrm{I}_{2}$ negative ) <br> to a zero | Interpretive <br> devices (intuitive) | Symbolic <br> domain, layer $b$ <br> dern |
| 2 | $\therefore I_{3}=I_{1}+I_{2}$ | Expresses $\mathrm{I}_{3}$ in terms of $\mathrm{I}_{1}$ and $\mathrm{I}_{2}$ | Interpretive <br> devices (formal) | Symbolic <br> domain, layer $b$ |

## a) Mathematical Resources activated

In the equation that the student writes in the first step, intuitive interpretive devices are activated in assigning negative signs to currents $I_{1}$ and $I_{2}$ to show a reverse direction to the one shown in the diagram. This is a very common mathematical resource in physics used to show opposite direction. Rearranging the variables from step 1 to step 2 involves activation of formal interpretive devices.

No intuitive mathematics knowledge, reasoning primitives, nor any symbolic forms is indicated by the student in the two lines used to get the question right.

## b) Awareness of ESM domains

Despite the question being presented in the model domain (diagram), the student by-passes that and engages in the symbolic domain to write a mathematical equation expressing the relationships between the three currents, in the first step. The student then uses this mathematical equation to arrive at the correct expression for Kirchhoff's current law at junction C, which is still the symbolic domain, in step 2.

Since both the steps involve relationships between the three currents in the circuit, only the ESM layer $b$ is evident in this solution.

## c) Use of units, variables and constants

In student M6's solution to this problem, this was observed:

- No units used
- No variables substituted
- No constant substituted

Since no units, variables nor constant were used or substituted in this solution, there is no order of use or substitution to be discussed.

### 6.4.2 Analysis of Student H1's work on Q2B $\mathbf{2}_{2}$



Figure 16: Student H1 solution to Q2B $_{2}$
Student H1 starts by writing a general equation with an incorrect expression for Kirchhoff's $1^{\text {st }}$ rule. Current $\left(\mathrm{I}_{\mathrm{c}}\right)$ is erroneously used in place of potential difference $(\Delta \mathrm{V})$ around a closed loop ( $2^{\text {nd }}$ rule). In the second step the student correctly writes Kirchhoff' rule at junction C. Student H1 continues to step 3 where she works in reverse and attempts to present the equation in step 2 in a similar manner to the one in step 1.

Table 6.8 portrays the application of the conceptual framework to student H1's solution, in steps.

Table 6.8: A step-by-step description of Student H1' work on Q2B $\mathbf{2}_{2}$

| Step | Activity | Description of activity | Activated <br> mathematical <br> resource | Awareness/ <br> translation <br> between ESM <br> domains |
| :--- | :--- | :--- | :--- | :--- |
| 1 | $\sum I_{c}=0$ | Gives mistaken expression <br> for Kirchhoff's 1 st $c u r r e n t ~$ <br> rule | Unidentified <br> resource 3 | Abstract, layer b |
| 2 | $\mathrm{I}_{3}=\mathrm{I}_{1}+\mathrm{I}_{2}$ | Equates $\mathrm{I}_{3}$ to the sum of $\mathrm{I}_{1}$ <br> and $\mathrm{I}_{2}$ | Reasoning <br> primitives | Model, layer a and <br> $b$ |
| 3 | From $\mathrm{I}_{3}-\mathrm{I}_{1}-\mathrm{I}_{2}=0$ | Subtracts $\mathrm{I}_{1}$ and $\mathrm{I}_{2}$ from $\mathrm{I}_{3}$ <br> to get zero | Interpretive <br> devices (formal) | Symbolic, layer $b$ |

## a) Mathematical Resources activated

An unidentifiable mathematical resource is activated when the student, upon reading the question writes the equation in step 1 . This resource is similar to the ones described in previous solutions (see section 6.2 and 6.3 ) and therefore designated unidentified resource 1 .

In step 2 the student notice from the diagram the relation that; current $I_{3}$ breaks into $I_{1}$ and $I_{2}$ and writes is mathematically as $I_{3}=I_{1}+I_{2}$. Step 2 could not be immediately derived from step 1 , which is an incorrect mathematical expression of the Kirchhoff's rule around a closed loop. Step 2 thus involves activation of reasoning primitives that I shall call sum of parts is whole. Sum of parts is whole is an intuitive sense of physical mechanism (reasoning primitive) with the abstract notion that a whole can be divided into its individual parts. The same reasoning could be used for a river breaking into two streams, to say the water in the two streams is the same as the water in the river, for example. Even though the student is not aware that step 2 is the correct solution, the equation was motivated by a sense of physical mechanism; where students use a form of intuitive knowledge about physical phenomena and processes (Tuminaro, 2004; p. 45).

Formal interpretive devices are used in the rearrangement of variables from step 2 to step 3 . Neither intuitive mathematics knowledge resources nor do symbolic forms appear to be activated in this student's solution.

## b) Awareness of ESM domains

Despite the question being presented in the model domain, student $\mathbf{H 1}$ starts on the abstract domain by writing the incorrect Kirchhoff's rule. The student awareness of the model domain helps him to come up with the expression $I_{3}=I_{1}+I_{2}$ in step 2 . While step 2 could be interpreted as indication of the model domain - demonstrating understanding of the relationship of the three currents from the diagram, it also could appear accidental. Step 3 indicates awareness of the symbolic domain, and is further proof of the accidental nature of step 2 as the student appears to be working in reverse.

Step 1 gives a mistaken mathematical expression for current around a loop. It is therefore ESM layer b since an expression is a relation, or a comparison. Since step 2 is not motivated
by step 1 , it can only be due to the awareness of first; the currents $I_{3}, I_{2}$ and $I_{1}$ as separate entities (ESM layer a) and then their combined relation (ESM layer b). Step 3 is an awareness of ESM layer $b$ resulting from step 2 .

## C) Use of units, variables and constants

In student H1's solution to this problem, the following was observed;

- No units are used
- No variables are substituted
- No constant are substituted

Since no units, variables or constant were used or substituted in this solution, there is no order of use or substitution to be discussed.

### 6.4.3 Analysis of student H2's work on Q2B $\mathbf{2}_{2}$



Figure 17: Student $\mathbf{H} 2$ solution to Q2B $_{2}$

Student H2 writes the expression for Kirchhoff's law at junction C, at once. Application of the conceptual framework to the student's solution is tabulated in table 6.9 below.

Table 6.9: A step-by-step description of Student H2's work on Q2B $\mathbf{2}_{2}$

| Step | Activity | Description of activity | Activated <br> mathematical <br> resource | Awareness/ <br> translation <br> between ESM <br> domains |
| :--- | :--- | :--- | :--- | :--- |
| 1 | $K I L \rightarrow I_{3}=I_{1}+I_{2}$ | Writes an abbreviation for <br> Kirchhoff's current law <br> and continues to write an <br> expression for Kirchhoff's <br> current law at junction C | Reasoning <br> primitive | Model, <br> layer $a$ and $b$ |

## a) Mathematical Resources activated

Reasoning primitives sum of parts is whole are the type of mathematical resources activated here. The student visually observes that one (1) thing led to two (2) other things (the whole as the sum of individual parts) and simply writes the equation.

No intuitive mathematics knowledge resources, symbolic forms nor interpretive devices are activated in this student's solution.

## b) Awareness of ESM domains

This student could also be said to have immediately discerned the relationship of the currents from the diagram (model domain), thus understanding the physical meaning of the solution that he puts forward. The student may have looked at the diagram (model domain) and notices a physical situation where one entity $\left(\mathrm{I}_{3}\right)$ breaks into two other entities ( $\mathrm{I}_{1}$ and $\mathrm{I}_{2}$ ) at junction C .

This single step indicates awareness of the model domain in both layers $a$ and $b$. Since the summation of the two currents $I_{1}$ and $I_{2}$, and their equating to current $I_{3}$ are not motivated by any prior written equation, it is reasonable to suggest that the student noticed the currents as separate entities first (layer $a$ ), and then constructed a mathematical expression that shows their relationship (layer $b$ ).

## C) Use of units, variables and constants

In student H2's solution to this problem, the following was observed:

- No units used
- No variables were substituted
- No constants were substituted

Since no units, variables or constant were used or substituted in this solution, there is no order of use or substitution to be discussed.

### 6.4.4 Summary of the three students' (M6, H1, H2) work on Q2B $\mathbf{2}_{2}$

Solutions from the first two students (M6 and H1) show students trying to get to the answer by starting from some known or general equation. In both cases students state an incorrect equation. These students appear to be motivated by "working towards" a solution. While their first steps do not necessarily help them, they end up getting the correct answer in subsequent steps. Student $\mathbf{H 2}$ on the other hand did a visual inspection to arrive at his single step solution.

While the activation of reasoning primitives alone proved sufficient to lead to the correct answer, interpretive devices are the commonly activated mathematical resource.

The first two students missed the opportunity to immediately translate from the model domain to the concrete domain looking at the way the currents are represented in the diagram. They had to traverse the symbolic domain even when it proved unnecessary.

The answer to this question is correctly expressed without the use or substitution of units and constants. Only variables are used.

### 6.5 Chapter Summary

Various mathematical approaches emerged as the students were solving the electricity questions. While interpretive devices where students used formal mathematical procedures appeared to be the most common approach in all the three questions, there was variation in the way the students started their solutions to the questions.

Students started the electric force question $\left(\mathrm{Q}_{1} \mathrm{~A}_{1}\right)$ the same way - by stating Coulomb's law. A different set of students also all started the electric field question $\left(\mathrm{Q} 1 \mathrm{~B}_{2 \mathrm{a}}\right)$ the same - by drawing a diagram that depicts electric field vectors. For the third question on electric current in a circuit $\left(\mathrm{Q}_{2} \mathrm{~B}_{2}\right)$, the students started the question in three different ways. The first one started with an incorrect Kirchhoff's rule. The second one started with a correct Kirchhoff's rule which they were not aware of its correctness. The third student answered the question correctly by stating at once, Kirchhoff's rule at junction C.

All the domains of knowledge (concrete, model, abstract, symbolic) appear in a sporadic manner as students are answering the questions. The symbolic domain, where students indicate awareness of "symbolic ways of representing a problem" is the most predominant domain.

Two of the students' various approaches in solving the electricity questions could not be identified with mathematical resources as described in section 3.4.2.2. These were designated unidentified resource 1 and unidentified resource 2. Table 6.10 presents a summary of students' solutions along the three electricity subunits indicating the resultant mathematical resources activated, and the ESM domains.

Table 6.10: Summary of application of MATHRICITY on students' solutions

| Sub unit | Student | Mathematical resource activated | ESM domain |
| :---: | :---: | :---: | :---: |
| Electric force | V1 | -Unidentified resource 1 <br> -Symbolic form <br> -Interpretive devices(formal) | -Abstract domain, layer $b$ <br> -Symbolic domain, layer b |
|  | M3 | -Unidentified resource 1 <br> -Interpretive devices(formal) <br> -Unidentified resource 2 | -Abstract domain, layer $b$ <br> -Symbolic domain, layer b <br> -Concrete domain, layer b |
|  | M5 | -Unidentified resource 1 <br> -Interpretive devices(formal) <br> -Symbolic form <br> -Unidentified resource 2 | -Abstract domain, layer $b$ -Symbolic domain, layer b <br> -Concrete domain, layer b |
| Electric field | V1 | -Interpretive devices(formal) <br> -Reasoning primitives <br> -Intuitive mathematics resource <br> -Unidentified resource 2 <br> -Unidentified resource 1 | -Model domain, layer b -Symbolic domain, layer b <br> -Concrete domain, layer b -Abstract domain, layer b |
|  | M4 | -Interpretive devices(formal) <br> -Intuitive mathematics <br> -Unidentified resource 1 <br> -Unidentified resource 2 <br> -Reasoning primitives | -Model domain, layer $b$ -Symbolic domain, layer -Abstract domain, layer b -Concrete domain, layer a and b |
|  | M5 | -Interpretive devices(formal) <br> -Unidentified resource 1 <br> -Unidentified resource 2 <br> -Reasoning primitives | -Model domain, layer b -Symbolic domain, layer b <br> -Concrete domain, layer b -Abstract domain, layer b |
| Electric circuit | M6 | -Interpretive devices (intuitive) <br> -Interpretive devices (formal) | -Symbolic domain, layer <br> b <br> -Symbolic domain, layer b |
|  | H1 | -Unidentified resource 3 | -Abstract domain, layer b |


|  |  | -Reasoning primitives <br> -Interpretive devices <br> (formal) | -Model domain, layer a <br> and $b$ |
| :--- | :--- | :--- | :--- |
|  | H2 | -Reasoning primitives | -Model domain, <br> layer $a$ and $b$ |
|  |  |  |  |

## Chapter 7 Summary of Study and Findings

### 7.1. Study summary

The use of mathematics in physics must be understood for the role its serves. In students' learning of physics, it is even more important that students understand why they use mathematics the way they do. Do students' use mathematics so that mathematics helps them understand the physics, or is students' "efficient" use of mathematics when solving physics problems simply an indication of their understanding of the subject - mathematics? Is students' use of mathematics in physics much like solving a puzzle? More so, does students' use of mathematics in the physics topic of electricity bring out any unique approaches or notable types of understanding?

A comprehensive coverage of the relevant literature indicated that students' effective use of mathematics in physics is still a contentious issue (see sections 2.1; 2.2; 2.3; $2.4 \& 2.5$ ). There are those researchers who reckon that students do badly in physics because students do not have requisite mathematical preparedness (Ayene et al., 2012). Others argue that even if students did have the required level of mathematics, the issue of transfer of knowledge across different domains is really the problem (Basson, 2002; Redish, 2005). Researchers have described how mathematics and physics are ontologically different types of knowledge, which also require different epistemological energies (Pettersson \& Scheja, 2008). These differences include descriptions of knowledge as procedural as opposed to conceptual, or objective as opposed to subjective. Inevitably, these varying and at times conflicting descriptions also affect the way students perceive mathematics in physics.

Salaam (2007) and Quale (2011) both maintain that students' perceptions with regard to their understanding of the purpose of problem solving are polarized. This, they say results from their view of physics as objective knowledge, and mathematics as subjective knowledge. Physics is viewed as representing real physical objects, while mathematics relates to human imagination. Quale (2011) recommends for some kind of middle ground between the positions of realism (physics) and relativism (mathematics), since he says even "so called" objective objects are perceived by the human mind.

Two other dichotomous views on mathematics - in - physics that emerged from literature are with regards to the way physics should be taught. One school of thought is the conceptual approach which proposes that concepts be taught before mathematical computation (Hewitt, 2010). The other school of thought argues that mathematics is fundamental to understanding physics and therefore mathematical skills must be taught before or concurrently with physics content (Mulhall \& Gunstone, 2008).

The above cited literature and similar ones discussed in detail in the introductory chapters dwelt largely on the use on mathematics in physics in general, or on the topic of mechanics. This one-dimensional approach in particular is what led the current study to carve out the investigation in a different dimension - the physics topic of electricity. The role of mathematics in students' understanding of physics, regardless of the level of instruction or the extent of use, appeared invaluable. However, there was scarcity of literature with regards to students' use of mathematics in the physics topic of electricity and how that may be influenced by their expectations. The preceding background led the current study to be formulated on the basis of the following three research questions:
a. What are students' expectations of the role of mathematics in physics?
b. What mathematical approaches do students use when solving electricity problems?
c. What types of understanding emerge when students use certain mathematical approaches to solve electricity problems?

To answer these research questions, a qualitative study was designed. The study made use of a survey, focus group interviews and student scripts. Data from the survey and interviews was used to address the first research question while students' scripts were analyzed to address the next two research questions.

A three-tier conceptual framework (MATRHICITY) was presented in chapter three (3). The first part of MATHRICITY was to identify and constitute important first year physics, electricity topics. The topics were constituted as guided by the General Systems Theory (GST) to be; electric force, electric field and electric circuits (see section 3.5).

The second part of MATHRICITY was informed largely by prior work on cognitive approaches in understanding students' use of mathematics in physics. The development of
these cognitive approaches could be traced to the phenomenological primitives (di Sessa, 1993); through symbolic forms (Sherin, 1996; 2001) up to mathematical resources (Tuminaro, 2004). These three approaches are largely informed by each other. The last one mathematical resources - was specifically chosen as it emerged from a study that claimed to have "synthesized previous studies on students' use of mathematics in physics" (see section 2.7.4.2) and also "purports to have developed a vocabulary and grammar as useful tools for understanding the nature and origin of students' mathematical thinking in physics" (see section 2.7.4.2).The mathematical resources constitute intuitive mathematics resources, reasoning primitives, symbolic forms and interpretive devices.

The third component of MATHRICITY was the Extended Semantic Model (ESM). It is a framework developed by Greeno (1989) and could be used to describe distinct areas of focus when solving physics problems, which are called knowledge domains. The ESM advocates for idealized problem solving that incorporates four domains of knowledge. These four are the concrete, model, abstract and symbolic domains. The ESM was used in this study to delineate patterns of understanding from students' mathematical approaches in electricity questions.

Students' use of units, variables and constants was an additional dimension of analysis. This perspective was to validate students' mathematical approaches towards the electricity questions. It was as well, also expected to indicate any understanding that would result from student engagement with the questions.

Students expectations, their use of mathematics in physics, the understanding that result from their use of mathematics, together with analysis of their use of units, variables and constants composite, were thus collectively used to thrash out the role of mathematics in students' understanding of the physics topic of electricity.

### 7.2 Discussion of findings

Students' overall responses to the survey and interviews were grouped into two main categories of what they think it takes to learn physics and what they think about the use of mathematics in physics. In these groupings, interview excerpts were used to corroborate emergent views.

How students actually use mathematics was explored through application of, mainly MATHRICITY.

### 7.2.1 What they think it takes to learn physics

The data analysis of students' responses to the survey could be curtailed into a map of students' expectations on physics. The terrain of the map depicts a non-uniform and wobbly field where students' expectations are: uncertain, incoherent with expert knowledge, and even contradicts evidence from empirical studies in some instances.

Students were adamant about the role of mathematics in physics from the onset, when $80.7 \%$ of them disagreed with a survey item (20); I would prefer to learn physics with no mathematics. Students' responses are consistent with what obtains in literature when $91 \%$ agree that, the first thing that I do when solving a physics problem is to search for formulae that relate givens to unknowns. Mulhall and Gunstone (2008), Redish (2005), and Van Heuvelen (1991) have all pointed to this and also demonstrated how it is an ineffective approach to learning (see section 2.5.2).

When $96.3 \%$ of the students agreed with the survey item (19); To be able to use an equation in a problem, I need to know what each term in the equation represents; this was contrary to existing literature (Dawkins et al., 2008; Redish, 2005). Dawkins et al. observes that in fact "students are asked to apply these basic mathematics operations in physics with minimal consideration given to whether students know what they represent" (see section 2.5.2).

In addition to the above observed contrasts between students' responses and literature, one (1) emergent view was on students' perception on the use of equations in learning physics. $83.7 \%$ of the students agree with survey item (11) that stated, the most crucial thing in solving a physics problem is finding the right equation to use. When probed about a similar issue one student commented, "...now as long as you can understand how the formula work like it will be easy for somebody to pass" (see section 5. 5.1).

Students also indicated an inclination towards memorization in their learning of physics. During the interviews when students were asked about the extent of memorization that they do, four students responded at once, "a lot" (see section 5.5.2).

Students also generally felt conceptualization was a very important aspect of their learning of physics. One student said during the interviews, "the thing is if you don't understand the
concepts, you will have problems throughout. So for that part I think it is very important to understand the concepts" (see section 5.5.3.). This was in agreement with students' response to a survey item (28), where $79.8 \%$ agreed that when I solve most physics problems, I think about the concepts that underlie the problem.

One thing that the students demonstrated when their scripts were analysed in the electric force question is simply that they recall Coulomb's law. An alternative but similar question probing introductory students' conceptual understanding of physics would have been; Explain what happens to two electric point charges placed a distance $d$ apart if the force between them is reduced by a third? Here a student who understands Coulomb's law will demonstrate understanding that if electric force is reduced, its effect is reduced as well. The suggested question, as Hewitt (2010) has observed will make first year students understand the physics "before they go in to computation". Evidently, introductory students hardly encounter these type of questions even at high school as stated by one student (S2) in the interviews when she said, "... from senior school, answering theory questions proved to be more difficult than the mathematical part because the theory you have to read. Physics you know we don't usually re ...ad physics. We just, I don't re...ad physics, I just find the question and see how they relate" (see section 5.6.2). The "re...ad" in this text suggests that the student wants to differentiate reading normal text from re...ading and interpreting mostly formulae or mathematical notation.

### 7.2.2 What students think about the use of mathematics in physics

Students' expectations on the use of mathematics in physics also came to the fore. Students expressed their views in two broad areas; the meaning of mathematical answers (section 5.6.1) as well as the relationship between mathematics and physics (section 5.6.2).

In the survey, students' views were not convincing. Only about half (49.7\%) of the students agreed with the item (12) in solving a physics problem, I sometimes get a correct mathematical solution whose meaning I do not understand. The interviews however precipitated with this comment, "Because sometimes I get a question, ok fine, I look for the correct mmm... the right formula to use, I use that formula, I check the answer at the back of the book. Ok the answer is correct but not necessarily understanding the concept...so I do have a problem sometimes" (see section 5.6.1). This statement could be interpreted to mean, the student can get a correct answer even though they may not understand the physics.

When probed on the relationship between mathematics and physics, students' views were unambiguous. Students disagree ( $80.7 \%$ ) with the survey item (20) that states I would prefer to learn physics with no mathematics. Students agreed (70.2\%) with the survey item (23) there can be no physics without mathematics. The following interview excerpt substantiates students’ thinking:

R: Ok...Mr. Fizi, can you learn physics without mathematics?
S2: Aah I don't think so.
R: It's impossible?
S2: It's impossible, it's very impossible. You need to apply maths in order to understand the physics.

The student here says it is "very impossible" to learn physics without mathematics. 'Very impossible' expresses strong feelings and could be interpreted that he implies that physics and mathematics are one.

Generally, in students' conversations during interviews the words understand, formula, mathematics, concepts, apply, and the phrases, understand concepts, understand formula, concept behind, and key concept, recur with noticeable regularity. These words and phrases depict students' mindsets when engaged with physics tasks. They are the contours through which physics is mapped in students' minds.

### 7.2.3 How students used mathematics in the physics topic of electricity

Analysis of students' test scripts through MATHRICITY shows students' varying mathematical approaches in terms of the mathematical resources activated (see sections 6.2; 6.3 \& 6.4). Some approaches, especially those inclined to pure mathematical manipulation were more prevalent. With regards to the domains of knowledge that emerged, the symbolic domain where students dwell on symbolic ways of representing a problem or through the mathematics of algebra, was predominant (see sections 6.2, $6.3 \& 6.4$ ). Analysis of students' use of units, variables and constants also brought out a discernible pattern.

### 7.2.3.1 Activated Mathematical Resources

According to literature, Intuitive mathematical resources, Reasoning primitives, Symbolic Forms and Interpretive Devices were the mathematical resources that are activated when students solve physics problems (see section 3.4.2.2). Two more mathematical resources, unidentified resources 1 and unidentified resources 2 emerged in this study as students'
work was analysed for the use of mathematics when solving electricity problems (see sections $6.2,6.3 \& 6.4)$.

## Unidentified resources 1 -Retrieval cues

Unidentified resources 1 are mathematical resources activated at the beginning of problem solving, just after students complete reading a physics question. Students appear to be automated to start with a formula. This first step is very important as it signifies what the student immediately makes out of the question that they read. Problem solving however does not start with the equation already there. In all the solutions where they emerged, unidentified resources 1 appeared at the beginning of the problem solving exercise.

These were activated in six (6) different students' solutions as students wrote down both Coulomb's law and the electric field equation as the first step in their problem solving (see section $6.2 \& 6.3$ ). The two questions (electric force and electric field) cued in students mind, the respective equations, and students were able to retrieve those and write them down.

Since the mathematical resource literature (see section 3.4.2.2) only describes resources that are activated once the equation is stated, I notice this as an omission of the first step from which other resources can be meaningfully evaluated. I therefore designate this step Retrieval Cues. When students read a question, and the first thing they do is to write an equation, it is because the question "cued" in students' mind to "retrieve" that particular equation. Numerous studies (Reif \& Heller, 1982; Heller et al., 1992) have observed and described this step as part of the problem solving process. None have treated it explicitly, and in isolation. Step 1 was noted and described in this study as consequent of activation of retrieval cues.

## Intuitive mathematical resources

Intuitive mathematical resources are basic every day mathematics activated without the use of formal mathematical rules or operations (see section 3.4.2.2). Intuitive mathematical resources were activated in solving the electric field question ( $\mathrm{Q} 1 \mathrm{~B}_{2 \mathrm{a}}$ ) when students ( V 1 and M4) noticed that the distance from $Q_{1}$ and $Q_{2}$ to the pointe $E(r)$ is the same (see section 6.3). This type of mathematical resource was not activated when students were solving both the electric force question $\left(\mathrm{Q}_{1} \mathrm{~A}_{1}\right)$ and the electric circuits question $\left.\mathrm{Q}_{2} \mathrm{~B}_{2}\right)$.

The electric field question in particular and how students approach it brings out some notable observations with regard to activation of intuitive mathematical knowledge resource. Once it was stated in the problem that $\mathrm{Q}_{1}=5 \mu \mathrm{C}$, and $\mathrm{Q}_{2}=5 \mu \mathrm{C}$; then intuitively (without the use of
any formal mathematics) students should notice $\mathrm{Q}_{1}$ and $\mathrm{Q}_{2}$ to be the same; that $\mathrm{Q}_{1}=\mathrm{Q}_{2}$ can be discerned through activation of the pairing type-of-intuitive mathematics knowledge resources (see section 3.4.2.2). Similarly, that $r_{1}=r_{2}$ can be discerned by the same mode of thought. Once it was stated in the problem that the triangle is equilateral and that $E$ is at the centre, then intuitively, it should follow that the distance from the two corners of the triangle $B$ and $D$, being $r_{1}$ and $r_{2}$ must be the same. This still demonstrates activation of the pairing type-of- intuitive mathematics knowledge resources.

When resolving the electric field vectors in their components; that both the x and y component of $E_{1}$ are the same in magnitude as that of $E_{2}$ would be realized through activation of the intuitive mathematics resource of pairing.

## Reasoning primitives

These mathematical resources deals with common sense reasoning about physical events and processes that involve generalizations of classes of objects and influences (see section 3.4.2.2). When solving the electric force question $\left(\mathrm{Q} 1 \mathrm{~A}_{1}\right)$ the reasoning primitives were not activated at all.

Reasoning primitives were activated but only once as two students (M4 and M5) were solving the electric field question $\left(\mathrm{Q}_{1} \mathrm{~B}_{2 \mathrm{a})}\right.$ (see section 6.3). This was when the students would have reasoned that "if two similar objects are placed the same distance away from a point, then their effect at that point must be the same", thus the $\mathrm{E}_{1}=\mathrm{E}_{2}$ ( step 4 for $\mathbf{M 4}$ and step 3 for M5). This is reasoning primitive comparable to the more is more as noted by Tuminaro (see section3.4.2.2). In this case the reasoning primitive is same is same.

Still with electric field question reasoning primitives would have been activated had students noticed that $\mathrm{E}_{1 \mathrm{x}}$ and $\mathrm{E}_{2 \mathrm{x}}$ are "two opposing influences exactly cancelling each other out to produce no apparent result" (see section 3.4.2.2). This is the balancing type of reasoning primitive - the abstract notion that two opposing influences exactly cancel each other out to produce no apparent result (see section 3.4.2.2). Students would then assign $\mathrm{E}_{\mathrm{x}}$ a zero (0) without the use of any formal mathematics.

Students would have also noticed that the $y$ - components of $\mathrm{E}_{1}$ and $\mathrm{E}_{2}\left(\mathrm{E}_{1 \mathrm{y}}\right.$ and $\left.\mathrm{E}_{2 \mathrm{y}}\right)$ are equal influences adding to get twice the amount of one ( $\mathrm{E}_{\mathrm{y}}$ ). One would then simply "double" whatever value they get for the $y$-component of either $E_{1}$ or $E_{2}$ to get their combined electric
fields at point $\mathrm{E}\left(\mathrm{E}_{\mathrm{y}}\right)$. There is no need to calculate the y-component of electric field due to the second point-charge. If activated, this is a reasoning primitive I will call doubling.

When working on the electric circuits question ( $\mathrm{Q}_{2} \mathrm{~B}_{2}$ ) two students ( $\mathbf{M 6}$ and $\mathbf{H 1}$ ) tried to employ interpretive devices in order to explain Kirchhoff's law at a junction (see section 6.4). This is despite the fact that the diagram linking the three currents ( $\mathbf{I}_{\mathbf{1}}, \mathbf{I}_{\mathbf{2}}$ and $\mathbf{I}_{3}$ ) visually shows how they relate, which is what the third student (H2) immediately realized. Reasoning primitives were however activated as two of the three students ( $\mathbf{H} \mathbf{1}$ and $\mathbf{H} 2$ ) reasoned that when a physical entity divides into two (in this case current) then, the whole is equal to the sum of its parts $\left(\mathrm{I}_{3}=I_{1}+I_{2}\right)$. An elementary school pupil, who is asked to demonstrate how the currents relate and not necessarily using Kirchhoff's law, will get this question right by following the obvious diagram!! Analysis of this type of question then becomes very crucial as it demonstrates how introductory university students forsake very basic but useful skills for more complicated approaches.

## Symbolic Forms

Symbolic forms are the type of mathematical resources whose activation is enhanced by the structural and conceptual forms of a physics equation (see section 3.3.2.3).

Symbolic forms were activated only once for two students (V1 and M5) as they were solving the electric force question $\left(\mathrm{Q} 1 \mathrm{~A}_{1}\right)$. This occurred when students noticed the similarity in the structural form of an equation and perform "cross-multiplication" (see section 6.2). Symbolic forms were not activated as students were solving both the electric field $\left(\mathrm{Q} 1 \mathrm{~B}_{2 \mathrm{a}}\right)$ and the electric circuits $\left(\mathrm{Q}_{2} \mathrm{~B}_{2}\right)$ questions.

With regard to the electric circuit question, all three students represented Kirchhoff's rule at junction $C$ as $I_{3}=I_{2}+I_{1}$ and none as $I_{2}=I_{3}-I_{1}$ or $I_{1}=I_{3}-I_{2}$. While the mathematics for the three expressions maybe slightly different, the physics is the same. Students operate on rigid symbolic templates (Sherin, 2001, 2006). This is an indication of a fixated cognition, limited understanding or could even be described as dogmatic behavior as students may be writing it exactly the way they were taught. They are using the symbolic template $(\boldsymbol{\square}+\boldsymbol{\square}=\boldsymbol{\square})$. None is observed to use[■=■-■]. One student (M6) is actually observed stating in a later step the solution which is expressed in the symbolic template as template $(\square+\square=\boldsymbol{\square})$ results from the symbolic template $(\square-\llbracket-\llbracket=0)$ (see section 6.3).

For students to be observed to want to make a "direct" translation from words to a symbolic (mathematical) equation, or from the semantic to the syntactic, points to an unhealthy dogmatic engagement. Sierpinska (1994) explains this to say semantic (meaning-based) reasoning methods are sometimes used when constructing proofs in order to identify and make sense of the mathematical properties and relationships they describe.

## Interpretive Devices

These mathematical resources primarily refer to formal mathematical procedures used in interpreting physics equations (see section 3.4.2.2).

When solving the electric force question $\left(\mathrm{Q}_{1} \mathrm{~A}_{1}\right)$ all the three students (V1, M4, M5) dwelt on the interpretive devices type-of-mathematical resources (see section 6.2). As they were solving the electric field question $\left(\mathrm{Q}_{1} \mathrm{~B}_{2 \mathrm{a}}\right)$ a different set of three (3) students also showed inclination towards interpretive devices (see section 6.3). When solving the electric circuit $\left(\mathrm{Q}_{1} \mathrm{~B}_{2}\right)$ question interpretive devices were activated for two (2) out of the three (3) students (M6 and H1) (see section 6.4). Interpretive devices were activated in a majority of the problem solving steps for all the students.

In the third student's (H2) solution (section 6.4), interpretive devices were not activated at all. This was the first time that interpretive devices were not activated in the entire analyses of the three questions by three different sets of students. Incidentally student H2 got the question correct.

## Unidentified resources 2 - Sense of instructional correctness

Unidentified Resource 2 is activated when students perform a mathematical activity (in this case assigning of units) simply because they know that it is the correct thing to use (see section 6.5). For all the students who used units, they did so only at the end of the solution. This activity, or problem solving step is evidently not preceded by anything that it could be linked to. It just appears, "from nowhere".

This last step, what the students do at the end of their problem solving is very important as this is more or less a conclusion. The students finish off their problem solving by saying in conclusion, "so and so" metres or "so and so" Coulombs. When the students do so after several steps ( in some cases 6), and for a quantity like electric field which is not very common, it is not apparent how the students would have known what units to use. The step
has also been observed in prior studies (Maloney, 1994; Dawkins et al., 2008), but none have given it in-depth investigation with regards to how it surfaces. In this study, this last step is described as activation of the mathematical resource Unidentified Resource 2 - sense of instructional correctness. I reason that students only put units because they know that it is the "correct thing to do". A "sense of instructional correctness" is activated in cases like these. Students use units $m$ for the distance between point charges; and $C / m^{2}$ for electric field all appeared in the last step of problem solving, without any trace (see section 6.3).

### 7.2.3.2 Awareness of Knowledge Domains

The Extended Semantic Model (ESM) was used to portray knowledge which students are aware of as they solve physics problems (see section 3.3). These domains depict students' nature of understanding.

## Concrete Domain

Awareness of the concrete domain means students being able to deduce the physical meaning from the physics problem they are solving. When solving the electric force question awareness of this domain was exhibited only once, when two students (M3 and M5) wrote units at the end of their solutions (see section 6.2). Several steps in students solutions to the electric field question also show units being given at the end (see section 6.3.). In the electric circuit question (Kirchhoff's $1^{\text {st }}$ rule), students' awareness of this domain did not surface as no units were used by any student (see section 6.4).

## Model Domain

Students' awareness of the model domain involves the use or interpretation of diagrams in part of their solution. In solving the electric force question, the model domain was absent in all the three students solutions. A different set of students however, all demonstrated awareness of the model domain when solving the electric field $\left(\mathrm{Q} 1 \mathrm{~B}_{2 \mathrm{a}}\right)$ question. All the three students did draw a diagram from which they were able to calculate the distance from the point charges $\left(\mathrm{Q}_{1}\right.$ and $\left.\mathrm{Q}_{2}\right)$ to the midpoint $(\mathrm{E})$; and calculate the angles, as well the electric field vectors at a point (E). It must be noted that the question for electric field was already presented as a diagram. With regard to electric circuit question, which was also presented as a diagram, it was precisely a result of students' awareness of the model domain that the last student (H2) solved the question without the use of any formal mathematics and with relative ease (see section 6.4.3).

## Abstract Domain

Awareness of the abstract domain involves students' use of concepts, laws and principles that explain the physical or concrete aspect. With regard to the electric force question it surfaced only once for each of the three students as they stated the Coulomb's equation $F_{e}=k_{e} q_{1} q_{2} / r^{2}$ (see section 6.2.). Awareness of this domain was demonstrated again by a different set of three students as they stated the electric field equation $E=k_{\mathrm{e}} \mathrm{q}_{2} / \mathrm{r}^{2}$ at some step during their problem solving (see section 6.3). In the electric circuit question, only one student (M6) indicated awareness of the abstract domain by stating "an incorrect Kirchhoff's rule" (see section 6.4.).

## Symbolic Domain

Students' awareness of the symbolic domain - use of symbols to represent a problem mostly through the mathematics of algebra - is the predominant knowledge domain. Awareness of this domain appears for eight of the nine students' solutions in all the three different electricity questions. In the electric force question (section 6.2) students dwelt on the use and manipulation of the symbols in the Coulomb's law to try a get a solution. Likewise, in the electric field question (section 6.3) students dwelt on the use and manipulation of symbols from the electric field equation. Awareness of the three symbols $\left(I_{1}, I_{2}\right.$ and $\left.I_{3}\right)$ also guided two of the three students' problem solving steps in the electric circuit question (see section 6.4).

Since a majority of the students in all the problems used symbols, it could be a reasonable conclusion that problem solving in physics involves primarily, awareness of symbols and their mathematical use (Gaigher et al., 2007).

## ESM Layers $a$ and $b$

The layers $a$ or $b$ refer to the awareness of an entity in its own right (layer a) or as existing in relation to others (layer b). In all the students' solutions to the electric force and electric field questions, there is only awareness of layer $b$. Awareness of layer $a$ is in only in two steps of two solutions to the electric circuit question (for student M6 and H2). This can be explained that students are mostly aware of relations between entities but show little awareness of the entities themselves, where the entities may be physics concepts and the relations are mostly relationships between the concepts which are expressed mathematically. Students' smallest unit of thought is mostly relations (equations) and not independent physics concepts as may be expected.

### 7.2.3.3 Use of Units, Variables and Constants

Students did use units when solving both the electric force and electric field questions. In all the cases where they are used, units appeared at the end. Students engage in shortcut habits such as dropping of units to clear the problem solving field and put them back when the field is less crowded. The use of units at the end of a solution could also be as, "just the right thing to do". The effect of this on students' conceptual growth and understanding, though not determined in this study, remains doubtful.

Students are observed to substitute variables and constants at earlier stages during problem solving (see section $6.2 \& 6.3$ ). Students could be doing substitution with the hope that it may "clear the bush" for the next stage in their solving of a problem and towards a solution.

The electric circuits question $\left(\mathrm{Q}_{2} \mathrm{~B}_{2}\right)$ and students work on it usher a different perspective to the role of mathematics in students' understanding of physics. In all the students' solutions to the question neither units, variables, nor constants are used. Students' only used symbols (see section 6.4).

Students' use of units, variables and constants, seems to be guided by pragmatism. Students believe in, and may always follow the sentiments they have expressed in response to the questionnaire; as long as they have proved useful, even if it could be for a short term, and as long as they may not be aware of better alternatives. While their sentiments may be temporarily fruitful, they are neither intelligible nor plausible.

### 7.2.4 Updated MATHRICITY

With the emerging of two (2) more mathematical resources from the analysis of electricity questions, the theoretical framework designed in chapter 3 (see section 3.5) had to change to encompass the two new resources. These resources are also unique from the prior stated resources in the sense that they are "position sensitive". Retrieval Cues only appear at the beginning of the problem solving process while sense of instructional correctness appears at the end. Thus the new theoretical framework for describing the role of mathematics is students' understanding of electricity problems in physics will appear as figure 18 in the following page.


Figure 18: MATHRICITY (UPDATED VERSION)

This version of MATHRICITY differs from the one developed in section 3.5 in that it has the two new mathematical resources - Retrieval Cues and Sense of instructional Correctness, which emerged through analysis of students solving electricity problems.

### 7.3 Conclusion

The conceptual framework developed and used in this study pulls together two descriptive theories; mathematical resources (Tuminaro 2004) and Extended Semantic Model (Greeno
1989), framed within the GST guided electricity sub-topics; electric force, electric field and electric circuits.

Establishing a baseline on students' understanding on the use of mathematics in physics which was a response to the first research question - provides valuable information about students' expectations towards their learning. Physics education research and other educational research areas are replete with studies on students' expectations and how they influence their learning. Jones (2010) when investigating students' application of mathematics to physics and engineering affirms that students' beliefs play a key role in the framing of a context, like the role of mathematics in their understanding of physics.

### 7.3.1 Students Expectations

Students' expectations at the beginning of the course during the first semester seemed not coherent with regard to the use of mathematics in physics. Students think that mathematics is used in problem solving so as to learn physics. Students still think that learning physics involves solving mathematical problems. This according to the GST indicates lack of "connection" in students' cognition, when the cause also becomes the effect. It is circular reasoning.

With abundant literature on students' formula-centred approach to problem solving or recursive plug and chug, and how ineffective the approach is in students learning of physics, results from the survey indicate that UB physics 1 students think that problem solving in physics means finding the right equation to use.

Still in response to the first research question, students' expectations on the role of mathematics in physics were further corroborated by interviews. There was a general agreement between interviews and the survey responses that students think that in learning physics: equations are very important; memorization is a common practice; conceptualization is very important and that mathematical answers could be meaningless.

When probed further on this issue from interviews students stuck to their conviction, with one student saying, "Again I think there are other chapters or parts of physics which is impossible to teach [learn] without the mathematics" (see section 5.6.2.). This is still consistent with a majority ( $70.2 \%$ ) of students echoing in item 23 of the questionnaire that there can be no physics without mathematics. Students think that they learn physics by
solving problems; and in so doing they do not differentiate between the mathematical skills and operations required to solve the physics problems, and the physics concepts (see sections 5.6.2); since to them mathematics and physics are inseparable (one). In this case students just see the system and no unit at all; contrary to the GST's recommending, "Identify the system of which the unit in focus is a part" (see section 3.2).

### 7.3.2 Mathematical Approaches

Students' mathematical approaches in solving electricity problems - a response to the second research question - appear varied. When analysed by means of the mathematical resources activated, students' mathematical approaches indicate activation of; of retrieval cues, intuitive mathematical knowledge resources, reasoning primitives, symbolic forms, interpretive devices and sense of instructional correctness.

Retrieval cues appear to be the mathematical resources "of choice" among all the students. Students are evidenced to at the earliest opportunity, state an equation that they associate with the type of question they are solving. This appears to be done even when the equation does not appear to help the problem solving process.

Students overwhelmingly activate formal mathematical rules (interpretive devices) in solving most of the questions analysed. Once a needed equation is stated, students were seen to dwell on formal mathematics until the end of the question. Students' expectations with regards to the use of mathematics in physics, where $80 \%$ of them disagreed with item 20 of the questionnaire I would prefer to learn physics with no mathematics is consistent with what they actually do as they solve problems as reflected from their scripts. In most of the analysed scripts, students dwell on mathematical manipulation whether it is the best approach or not.

The sense of instructional correctness, where students use units - mostly without trace - at the end of their problem solving is one of the three (3) (including the two stated above) most commonly activated mathematical resource when students solve the electricity questions. Students generally discard units as soon as they start problem solving, only to put them back at the end. It is evident that students view units as an extra load in problem solving. Thus it may be argued that students perceive units as; obscuring their working on the problem; populating the variables and symbols that they have to work with; but necessary to put in the solution so that one gets the question right. Students simply assign units at the end (mostly correct) without an indication of where the units come from. In one instant a student assigned units even when they were not needed.

Other mathematical resources appeared to be activated only sparsely. Intuitive mathematical resources, where students activate basic or everyday mathematics were activated only once in the electric field question when students realized that; the part of some distance divided by two is equal to half that distance. Symbolic forms type-of-mathematical resources were also only activated once in the electric force question when students applied an automated mathematical procedure - cross multiplication - that involves recognition of similarities in the structure of mathematical equation. Reasoning primitives were also rarely activated, in just two (2) instances in the entire analyses. In one instance they were activated when students had to reason that the electric field at some point is the same when two point - charges are equidistant from a point (E). In the other instance it was in the electric circuit problem; when one student reasoned that the sum of two current leaving a junction must be equal to the single current entering a junction.

From this study, it is a valid conclusion that focusing on the mathematics takes away the reasoning and the intuition, some of it very basic, from the students. Students go straight for the mathematical formalism which gives them correct answers; but devoid of basic understanding and reason. This reasoning and use of intuition would in most cases have left the student much more conceptually enriched about the physics involved than is the case where students dwell on mathematical formalism. Again, it is what every day problems involve. Making use of the extra mathematical formalism which has been evidently dismissed as necessary only indicates failure to activate both intuitive mathematical resources and reasoning primitives.

It remains an empirical question why intuitive mathematics and reasoning primitives, which involve use of basic mathematics and every day experiences, are overlooked in preference to interpretive devices, which requires activation of mathematical formalism? This same question was asked by Koichu (2010), "Why students at times miss mathematically simpler ideas in preference for more involving formal mathematical problem solving endeavors?" (see section 2.4.2).

### 7.3.3 Types of understanding

With regard to the third research question, students' emerging types of understanding as they were solving the electricity problems were indicated by students' awareness of the different knowledge domains. While students' awareness and translation across all the four domains of knowledge (concrete, abstract, model, symbolic) came to the fore, awareness of some
domains occurred more regularly and at specific problem solving steps than others. On analysis of students' work through the ESM, their focus appears to be concentrated on the abstract and symbolic domains, but mostly the symbolic domains. Though there are traces of the concrete domain, where students were expected to demonstrate whether meaning has been attained, this has appeared only superficially.

Awareness of the abstract domain appeared mostly at the beginning of students' problem solving, where students would simply write down an electricity equation. The symbolic domain in particular, where students indicated awareness of symbolic ways of representing the problem, mainly through algebra, was the most common. Students dwelt on the awareness of this domain during the numerous algebraic steps evidenced during their problem solving; for eight (8) out of the nine (9) different solutions analyzed.

Awareness of the model domain, where students would use diagrams to represent or solve a problem occurred with the questions on electric field and electric circuits. Students used a diagram already presented in the problem to further determine electric field vectors or the relationship between currents at a junction. Awareness of the concrete domain was indicated mainly when students used units to indicate that there is some physical meaning in the physics problem they are solving. This occurred mostly at the end of solutions when students were solving the electric force and electric field questions.

From the variation of approaches presented in the students' work, it could be a valid claim to suggest that, since the different subtopics of electricity require different epistemological energies, specifically with the use of mathematics, then a student could understand and do well in one subtopic than another. The Kirchhoff' rules approach used in solving electric circuit problems is one peculiarity about the topic of electricity. Problems involving electric circuits are mostly presented using diagrams. The analysis of this type of problems is unique. The ensuing logic could be extrapolated to other physics topics and subtopics

Students' habit of immediately searching for formulae/equations when faced with a physics problem garnered the greatest congruence among the three means of data collection (surveys, scripts and interviews). In all of students' work portrayed from their scripts, the first step is to state a formulae/equation. Students had from the onset agreed (91.1\%) with the survey item (18) the first thing that I do when solving a physics problem is to search for formulae that relate givens to unknowns, and maintained that during the interviews, "I just apply the
equations" (see section 5.5.1.). This is also in agreement with literature (Redish, 2004; Van Heuvelen, 1991).

That a student can get a physics question mathematically correct and fail where there is need to demonstrate understanding of the physics, points to discordance between the two. Thus while mathematical aptitude is necessary for students' understanding of physics and studies have over-elaborated its use, without an extensive analysis, its actual effect could be deceiving. Even in the physics topic of electricity, mathematics emerges as a handy tool, but also as an impediment that at times gives students a false sense of meaningful engagement with physics tasks. Students' understanding of the physical meaning of their mathematical manipulation is directly related to the role that mathematics plays in their understanding of physics. While the topic of electricity was identified from other physics topics as a knowledge gap as well as for expediency; results have indicated that students' use of mathematics vary according to the various subtopics. This finding has only been demonstrated in this study.

The more the number of knowledge domains traversed, the more likely will a meaningful learning experience be achieved. Working in one domain alone, even if it is the concrete domain, may give an impression that a student understands physics while in fact they just "landed" there by accident. When students operate in the symbolic domain, where the focus in on mathematics, it still cannot be claimed that they understand the mathematics either, least the physics. In some cases they are just following heuristics. Tuminaro (2004) says that students' mathematical difficulties in physics may not be with mathematics itself, "it lies in translation of their conceptual understanding into physics equations and expressions" (p. 67). According to Tuminaro students experience challenges both in "mapping mathematics to meaning" (p.67) and "mapping meaning to mathematics" (p. 62).

Analysis of the problem solving through the ESM shows a conspicuous scarcity towards meaning, understanding and by extension, learning. While $69 \%$ of students believe that they solve mathematical physics problems in order to learn physics (item 1 of the survey), looking at students' actual work on the scripts through the ESM shows that the concrete domain, which demonstrates that physical meaning is obtained, is rare. One student (M2) declares during interviews ".... but sometimes yaa you do feel that yaa here I just got the answer but you don't know what the meaning of the answer is" (see section 5.6.2). Students' expectations, what they actually do and eventually say basing on their problem solving
experience is in disagreement. The actual practice does not meet expectations. What students actually do satisfies the "problem solving as an end upon itself" observation made Maloney, 1994). The mathematics hoodwinks students solving physics problems to focus on itself (the mathematics). While students may know how to solve physics problems, it is evident from the analysis of the scripts that students do not know why, as demonstrated by their empirically meaningless solutions.

This study was modeled in a similar to what Hammeyer (2007) refers to as "specific subjectbased features of the text". Research needs to dwell on what problem solving strategies and learning approaches are applicable and suited for specific physics topics. The generalization where physics is understood and approached as one topic could be delusional. As demonstrated by the noticeable difference in the electricity subtopics; electric circuits, electric field and electric forces; different topics in physics may have to be approached differently for effective teaching and learning. The subsequent use and application of mathematics in physics emerges to be different as evidenced by the activated mathematical resources; intuitive mathematical reasoning; reasoning primitive, symbolic forms and interpretive devices. Even as this observation is drawn from the topic of electricity only, the number of mathematical permutations in the various physics topics and subtopics should be numerously overwhelming. As Tuminaro (2004) observed, even students do show varying mathematical abilities in physics problem solving. This evidently complicates the matrix of students- mathematics-physics-use-understanding even more. It is specifically this matrix that pedagogical content knowledge (PCK) is trying to address (Gess - Newsome \& Lederman, 2001).

Because the number of students in this study is small, the aim was not to make statistically significant claims about mathematical approaches or types of understanding. Instead, the data has been used to bolster arguments that; the use of mathematics in the physics topic of electricity offers students a variety of outcomes across the varied electricity subtopics, with an inclination towards mathematical formalism, and within an awareness of the symbolic domain of knowledge. Instruction that overly engages students in mathematics questions in a physics context, rather than physics questions in a mathematics context exacerbates this challenge, robbing students of meaningful engagement; for while the mathematics is definitely in the physics, the physics is not in the mathematics.

In conclusion, the framework that has evolved with the foregoing analysis describes the role of mathematics in students' understanding of physics as multivariate. The logic of the argument emerging is that students think mathematics is key to learning physics; students do use and dwell on the mathematics when solving electricity physics problems, even when it is not the best approach; and that what emerges as students' understanding is still the mathematics of manipulating symbols (symbolic domain). The central claim is that as students solve electricity problems, basic mathematics (intuitive mathematical resources and reasoning primitives) is overlooked in preference to extended mathematical formalism (interpretive devices). This result in a predominance of an awareness of the symbolic domain; where the focus is on symbolic ways of representing a problem through the mathematics of algebra, rather than an awareness of the concrete domain, where the focus would be on developing intuition that helps students make physical meaning from the physics problems they are solving.

### 7.4 Limitations of the study

That this study did not provide a quantitative analysis of students' response to the SERMP is is a significant limitation. This was a limitation as a quantitative approach would have provided a more extensive analysis. Analysing interview data with qualitative analysis software such as Nvivo, rather than by visual inspection would also have led to more trustworthy results. The choice of the brief electric circuit question that could be answered in one or two steps by the students emerged as another was limiting factor. A much more extended question would have provided more mathematical approaches and the types of understanding as useful data.

Finally, and notwithstanding the suggestions above it follows that in this study the first two objectives; Determine first year students' expectations of the role of mathematics in physics and Determine what mathematical approach first year students use when solving electricity problems have been comprehensively addressed, but less so with the third objective; Determine types of understanding that emerge when students solve electricity problems. The particular type of understanding has not been shown to result in different levels of understanding of the underlying physics concepts. This is identified as a limitation of the study and suggested for further research.

## 7. 5 Implications and further studies

This study has implications in so far as curriculum development, textbook writing and instruction that involve the use of mathematics for introductory physics courses. The use and application of mathematics in physics during instruction, textbook writing and curriculum development requires not a general, but a tactical and in some cases topic - specific approach. Whether the different physics topics calls for different epistemological and mathematical approaches needs further investigation. A unique use and application of mathematics in each physics topic, or subtopic may need to be considered, to anchor curriculum development, text book writing, or instruction.

While most introductory text books start with introducing measurement as a central topic in physics, and expand by related concepts such as use of units and dimensional analysis; the use of mathematics, and especially the various mathematical resources explained in this study, the conditions under which they may be activated, is another important chapter that should be added at the beginning. This approach should also be extended to instruction. When students learn through this approach, the debate will no longer be about "conceptual" or "mathematical" but rather about which mathematical resource to activate, and when.

When students are observed to use formal mathematics to arrive at solutions which can be obtained with very basic, everyday mathematics, the problem may be with how students understand the role of mathematics in physics. Future studies need to focus more on the use of basic mathematical resources such as intuitive mathematics knowledge and reasoning primitives, and the extent of their applicability. Students need to be able to use these resources where there is need, and only proceed to formal mathematical operations where the "basics" cannot be used, or do not apply. It also surfaces that mathematical resources involving basic, every day, or intuitive approaches, do inculcate conceptual understanding of physics. While it may not be immediately apparent with the use of more detailed formal mathematics, I argue that conceptual understanding of physics is more likely to occur if students have first learned meaningful application of basic, every day, intuitive mathematical resources.

The above observation could actually be broader, and noticeable even in other physics aspects other than use of mathematics. Students are inclined to look for formal, textbook definitions even for seemingly simple questions. This could be a result of a bigger culture of
science being perceived as objective knowledge or as "facts" and as being different from everyday experiences, which is partly perpetuated through pedagogy.

Task design is another challenge that exacerbates the problem of excessive mathematical formalism. In tests, students write and use the detailed mathematics being guided by the amount of marks allocated per question. In some of these cases, less number of mathematical steps that involve the use of "basic mathematics" may actually lead to the correct answer.

MATHRICITY could be further refined and developed into a computer model. This model should be validated with much larger data. The utility of the model should be in entering student mathematical approaches in the electricity subtopics as input data, and noting resultant activated mathematical resources as well as knowledge domains, as output data. If it works well, similar models to MATHRICITY should be applicable to other physics topics. In this manner we would be able address students' effective use of mathematics in physics.

The IUPAP need to look at symbols as used in physics and other sciences to come up with a nomenclature that is more consistent and one that will minimize the tendency towards surface learning approaches such as excessive use of much mathematical formalism by students especially at introductory level.

Symbols for most physical quantities are commonly assigned by the first letter of the alphabet. In some cases this does not happen, and students are left to figure this out all by themselves. Students may use the subscript $e$ in $F_{e}$ for the electric force; others just use a plain $F$. In teaching the topic of mechanics where the force law is commonly introduced, no subscript is used except where specific forces such as centripetal and frictional force are dealt with. While the present data does not suggest that; it may be argued from experience that students can hardly relate the Coulomb's law to Newton's second law. It could be a source of confusion for some students, as textbooks (and probably instruction hardly show (except in working examples) the relationship between the $\mathrm{F}_{\mathrm{e}}$ in $\left[F_{e}=k_{e} q_{1} q_{2} / r^{2}\right]$ and the $F$ in $[F=M a]$. Attaining this vital connection may call for a paradigm shift on the manner in which physics content is presented in introductory textbooks, and subsequently taught. Instead, or in addition to having different types of forces taught in the separate topics of mechanics, electricity, modern physics; an alternative approach could be to have Force as a topic, and a central one for that matter, presented as a whole with the different types of forces (frictional, tensional, centripetal, gravitational, electric, magnetic etc.) constituted in the topic (Chabay \& Sherwood, 2005). This approach may give a more logical, coherent and comprehensive
content. It may only be done for the concept of force because of its central nature to the discipline of physics or it may extend to other concepts where it appears practical.

Another source of students' confusion is to assign symbols small letters where as in some cases capital letters are used (r or R for radius and d or D for distance). Since there is no explanation for this; all symbols should either be capital or small letters, differentiated by the second or third letter where the preceding ones are the same. Criteria, on which quantity is assigned the first letter only, should also be developed. In solving the electric field question (section 6.3) for example, student $\mathbf{V 1}$ (section 6.3.1) assigns electric charge capital $Q$ whereas student M4 (section 6.3.2) assigns electric charge small letter $q$.

In physics different symbols are sometimes used to represent the same physical quantities. For example symbols; $S, d, r$ may all be used to represent the quantity distance in different contexts. In the case of the topic electricity, the symbol $r$ is commonly used for distance between point charges in Coulomb's law, while $d$ is the distance between dielectric charged plates. It is hardly explained in text books, or during instruction for that matter why what letter was chosen for which context. The confusion that this brings for students is demonstrated during analysis of students' scripts; where the symbols $r$ and $d$ are used interchangeably, and unintelligibly. A comparable observation by Tuminaro (2004) showed a student interpreting the $R$ in the equation [ $\mathrm{PV}=n R T]$ to represent radius (p. 88)! All these chaos, inconsistencies and disjunctions are left to the student to make sense of. Inevitably students resort to memorization as a practical approach. Students even end up confusing ad hoc symbols for specific values with general symbols that make conventional equations as reflected from student M3's working on the electric force question, step 2 ( section 6.2.2).

The observed physics knowledge disorganization as presented to students at introductory level suggests that knowledge structural issues may be the major concern here. While the focus of this study was not on the organization of physics knowledge, it succinctly comes out as inevitable. It is therefore conceivable that the way in which mathematical physics knowledge (formulae encompassing units, variables and constants) are structured is what encourages students to use mathematics as they do and as demonstrated broadly in this study. What if we are looking at student understanding of a branch of knowledge, rather than starting with the ontology of the knowledge domain and then the cognitive resources it activates? Thus studies on the role of mathematics in physics (Feynman, 1992) were on the right path to excavate this field, only that they fell short by not expanding on how that affects
students understanding of physics; hence this study. At times studies focus on some methods of instruction and lament that they encourage rote learning for example; other studies will focus on students understanding of content. The crux could be, the particular knowledge is best learnt by rote or through mathematical formalism. Studies that would step out of the paradigm (Kuhn, 1962) and focus on why physics instruction and students learning of physics have shown the touted inefficiencies, including a significantly automated inclination towards mathematical formalism could illuminate this jinx.

It is an inconsistent expectation that physics education research frequently decry lack of or no show of conceptual understanding by students, when few text book type - of - questions such those analysed in this study ever request for it. By giving mathematical solutions that mostly are noticeably without concomitant conceptual understanding, students are complying with what is required of them. Maybe the focus of research on students' use and understanding of mathematics in physics should in fact change from students but rather to the nature of the task or type of questions asked of them. While different students solve similar physics questions differently, different questions also bring about different approaches to problem solving in physics. I could not agree more with Hameyer (2007) who observes that, though there are a lot of studies on strategies students use to make sense of knowledge, the quality of text (clarity, design, specific subject-based features of the text, degree of inclusion and explicitness, language use, logical structure, quantity of concepts used etc.) is a very important area worth giving more attention to.

Knowing why one is engaged in a particular activity is vital. Students' challenges bedeviling introductory physics teaching and learning such as; motivation, attrition and lack of realitylink result could be a result of widely observed pedagogical deficiencies. These pedagogical deficiencies include: formula-centred approach, misconceptions, rote learning, disconnected knowledge and lack of conceptual understanding. A more effective way of addressing this could be if students know precisely, why they have to engage in certain activities like the use of mathematics in physics for problem solving; and when the problem solving they do in the physics class enables them to solve similar and more complex problems in real life. It is a plausible hypothesis that - if students are unable to see the mathematics imbedded in physics for what it is - this study has confirmed what Sherin (2001) found elsewhere that students lack a robust and coherent grasp of physics concepts.

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## APPENDICES

## Appendix A: MPEX

## Student Expectations in University Physics: The Maryland Physics Expectations Survey

Here are 34 statements which may or may not describe your beliefs about this course. You are asked to rate each statement by circling a number between 1 and 5 where the numbers mean the following:

## 1: Strongly Disagree 2: Disagree 3: Neutral 4: Agree 5: Strongly Agree

Answer the questions by circling the number that best expresses your feeling. Work quickly. Don't overelaborate
the meaning of each statement. They are meant to be taken as straightforward and simple. If you
don't understand a statement, leave it blank. If you understand, but have no strong opinion,

| 1 | All I need to do to understand most of the basic ideas in this course is just read the text, work <br> most of the problems, and/or pay close attention in class. | 12345 |
| :--- | :--- | :--- |
| 2 | All I learn from a derivation or proof of a formula is that the formula obtained is <br> valid and that it is OK to use it in problems. | 12345 |
| 3 | I go over my class notes carefully to prepare for tests in this course. | 12345 |
| 4 | "Problem solving" in physics basically means matching problems with facts or <br> equations and then substituting values to get a number. | 12345 |
| 5 | Learning physics made me change some of my ideas about how the physical world <br> works. | 12345 |
| 6 | I spend a lot of time figuring out and understanding at least some of the derivations <br> or proofs given either in class or in the text. | 12345 |
| 7 | I read the text in detail and work through many of the examples given there. | 12345 |
| 8 | In this course, I do not expect to understand equations in an intuitive sense; they <br> must just be taken as givens. | 12345 |
| 9 | The best way for me to learn physics is by solving many problems rather than by <br> carefully analyzing a few in detail. | 12345 |
| 10 | Physical laws have little relation to what I experience in the real world. | 12345 |
| 11 | A good understanding of physics is necessary for me to achieve my career goals. A <br> good grade in this course is not enough. | 12345 |
| 12 | Knowledge in physics consists of many pieces of information each of which applies <br> primarily to a specific situation. | 12345 |
| 13 | My grade in this course is primarily determined by how familiar I am with the <br> material. Insight or creativity has little to do with it. | 12345 |
| 14 | Learning physics is a matter of acquiring knowledge that is specifically located in <br> the laws, principles, and equations given in class and/or in the textbook. | 12345 |
| 15 | In doing a physics problem, if my calculation gives a result that differs significantly | 12345 |


|  | from what I expect, I'd have to trust the calculation. |  |
| :---: | :---: | :---: |
| 16 | The derivations or proofs of equations in class or in the text has little to do with solving problems or with the skills I need to succeed in this course. | 12345 |
| 17 | Only very few specially qualified people are capable of really understanding physics. | 12345 |
| 18 | To understand physics, I sometimes think about my personal experiences and relate them to the topic being analyzed. | 12345 |
| 19 | The most crucial thing in solving a physics problem is finding the right equation to use. | 12345 |
| 20 | If I don't remember a particular equation needed for a problem in an exam there's nothing much I can do (legally!) to come up with it. | 12345 |
| 21 | If I came up with two different approaches to a problem and they gave different answers, I would not worry about it; I would just choose the answer that seemed most reasonable. (Assume the answer is not in the back of the book.) | 12345 |
| 22 | Physics is related to the real world and it sometimes helps to think about the connection, but it is rarely essential for what I have to do in this course. | 12345 |
| 23 | The main skill I get out of this course is learning how to solve physics problems. | 12345 |
| 24 | The results of an exam don't give me any useful guidance to improve my understanding of the course material. All the learning associated with an exam is in the studying I do before it takes place. | 12345 |
| 25 | Learning physics helps me understand situations in my everyday life. | 12345 |
| 26 | When I solve most exam or homework problems, I explicitly think about the concepts that underlie the problem. | 12345 |
| 27 | "Understanding" physics basically means being able to recall something you've read or been shown. | 12345 |
| 28 | Spending a lot of time (half an hour or more) working on a problem is a waste of time. If I don't make progress quickly, I'd be better off asking someone who knows more than I do. | 12345 |
| 29 | A significant problem in this course is being able to memorize all the information I need to know. | 12345 |
| 30 | The main skill I get out of this course is to learn how to reason logically about the physical world. | 12345 |
| 31 | I use the mistakes I make on homework and on exam problems as clues to what I need to do to understand the material better. | 12345 |
| 32 | To be able to use an equation in a problem (particularly in a problem that I haven't seen before), I need to know more than what each term in the equation represents. | 12345 |
| 33 | It is possible to pass this course (get a "C" or better) without understanding physics very well. | 12345 |
| 34 | Learning physics requires that I substantially rethink, restructure, and reorganize the | 12345 |


|  | information that I am given in class and/or in the text. |  |
| :--- | :--- | :--- |
|  |  | 12345 |

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Comments and questions may be directed to E. F. Redish
Last modified March 2, 2001

## Appendix B: VASS

## VASS - P 204

Thank you for taking this survey that is part of a battery of instruments designed by Prof. Ibrahim A. Halloun in collaboration with a number of researchers in Lebanon and abroad. Each instrument is intended to identify factors that affect student understanding of particular aspects of science and that need to be accounted for in the design of instructional material.
For any information, please visit Prof. Halloun's website: www.inco.com.lb/halloun.
All data are confidential. Your identity will not be disclosed to any party.

1. By comparison to the rest of the class, I consider myself:
(a) weak in physics.
(b) excellent in physics.
2. What I actually learn in my current physics course is:
(a) good for my course exams.
(b) helpful in my everyday life.
3. My exam performance in my current physics course actually reflects how well I can:
(a) recall course materials the way they are presented in class.
(b) apply course materials in situations not discussed in class.
4. To do well in my current physics course, I actually need to go through the textbook or course materials and:
(a) find the important information and memorize it the way it is presented.
(b) reconstruct the material in my own way so that I can make sense of it.

The following twenty-three questions (5-27) are about physicists and their ways of doing physics. They are not about your physics courses. Please answer all these questions so as to reflect what you think physics is about as a science and this irrespective of how things are actually being done in your current courses.
5. Physics and chemistry are:
(a) related to each other by common principles.
(b) are separate and independent of each other.
6. When faced with a natural event that occurs for the first time in a given place, physicists:
(a) check whether this is a recurrence of a familiar event that took place elsewhere.
(b) look for aspects that distinguish this particular event from all other events.
7. Once they come up with new information, physicists:
(a) check whether it fits with the rest of their knowledge in physics.
(b) ascertain its merits independently of their knowledge in physics.
8. When they investigate a particular object in the natural world, physicists:
(a) look for all possible features that might be attributed to the object under investigation.
(b) concentrate on particular features that they consider relevant to the purpose of study.
9. In order to decide whether two different objects may behave the same way in the natural world, physicists check whether the two objects:
(a) are similar in all respects.
(b) are subject to similar conditions.
10. Physicists say that electrons and protons exist in an atom because:
(a) they have seen these particles in their actual form with some instruments.
(b) they have made observations that may be attributed to such particles.
11. Physicists say that the earth and the moon attract one another because:
(a) they have been able to detect and measure their mutual attraction with some instruments.
(b) the moon's revolution around the earth can be explained in terms of such attraction.
12. When they investigate a particular event in the natural world, physicists decide what data they need to collect:
(a) based on what they already know in physics.
(b) after observing the event in all possible details.
13. In order to decide whether two natural events can be investigated the same way, physicists first look whether the two events:
(a) involve similar objects.
(b) occur in accordance with the same physics principles.
14. Physicists working in one branch of physics, like mechanics or thermodynamics, investigate the natural world in ways that may be followed in:
(a) other branches of physics.
(b) other scientific disciplines, like chemistry or biology.
15. In order to figure out how things actually work in the natural world, physicists:
(a) survey aspects of this world that may be detected directly by our senses or through some instruments.
(b) imagine how things could possibly exist in ways that may not be humanly possible to detect.
16. Physicists' findings about the natural world are:
(a) dependent on current scientific knowledge.
(b) accidental, depending on physicists' luck.
17. When investigating a particular event in the natural world, physicists follow:
(a) one particular method that they consider most appropriate for the event under study.
(b) a variety of methods to see if they may come up with the same conclusion every time.
18. The same natural event may be investigated from different perspectives in accordance with:
(a) different principles coming from different branches in physics.
(b) different principles coming from different scientific disciplines.
19. Physicists use mathematics:
(a) to express their knowledge in meaningful ways.
(b) to get numerical answers to physics problems.
20. Scientific concepts of mass and electric charge are:
(a) inherent in the nature of physical objects and independent of how humans think.
(b) invented by physicists to represent properties that physical objects might possess.
21. Scientific concepts of force and energy are:
(a) inherent in the nature of physical objects and independent of how humans think.
(b) invented by physicists to represent properties that physical objects might possess.
22. Two different scientific concepts may correspond to the same physical object:
(a) in different respects.
(b) in the same respects.
23. Newton's laws of motion (like his second law often expressed in the form $\mathbf{F}=\mathbf{m a}$ ) apply to physical objects that may be located:
(a) anywhere in the universe.
(b) in specific places of the universe.
24. Physicists' current ideas about particles that make up the atom apply to physical objects that may be located:
(a) anywhere in the universe.
(b) in specific places of the universe.
25. A bit of information is considered scientific from physicists' perspective:
(a) when it has well-established merits regarding the natural world.
(b) when it is offered by a group of trustworthy physicists.
26. Ideas about the natural world that nowadays physicists have accepted and successfully used for a long time:
(a) may eventually be modified in some respects.
(b) will continue to be accepted in their present form in the future.
27. Physicists accept an idea about the natural world when the idea portrays this world:
(a) exactly the way it is.
(b) by approximation.

The following twelve questions (28-39) are about your physics courses. Please let your answers to all these questions reflect what you actually do in these courses, and how you actually feel about them.
28. Studying physics is for me:
(a) an enjoyable experience.
(b) a frustrating experience.
29. Learning physics requires:
(a) a serious effort.
(b) a special talent.
30. When I experience a difficulty while studying physics:
(a) I seek help, or give up trying.
(b) I try to figure it out on my own.
31. I go over the main body of a physics chapter:
(a) before the chapter is covered in class.
(b) after the chapter is covered in class.
32. I attempt to solve homework problems:
(a) before they are solved in class.
(b) after they are solved in class.
33. For me, discussing materials in my physics course with my classmates:
(a) is a waste of time.
(b) helps developing my reasoning skills.
34. For me, solving a physics problem more than one way:
(a) is a waste of time.
(b) helps developing my reasoning skills.
35. The first thing I do when solving a physics problem is:
(a) represent the situation with sketches and drawings.
(b) search for formulas that relate givens to unknowns.
36. After I have answered all questions in a homework physics problem:
(a) I stop working on the problem.
(b) I check my answers and the way I obtained them.
37. After the teacher solves a physics problem for which I got a wrong solution:
(a) I discard my solution and learn the one presented by the teacher.
(b) I try to figure out how the teacher's solution differs from mine.
38. After I succeed in solving a particular physics problem:
(a) I figure out under what conditions I can apply the same method to another problem.
(b) I memorize the method I followed in case I need it for solving a similar exam problem.
39. In order to solve a physics problem, I need to:
(a) have seen the solution to a similar problem before.
(b) know how to apply general problem solving techniques.

The following ten questions (40-49) are about the way you would like things to be done in your physics courses. Please let your answers to these questions reflect your own preferences or aspirations, irrespective of how things are actually being done in these courses.
40. I think that, when adequately presented, physics courses can be helpful to me:
(a) in my everyday life.
(b) if I were to become a physicist.
41. I would like my physics course to allow me relate physics:
(a) to the way I think about certain things in the natural world.
(b) to other sciences and their ways of dealing with the natural world.
42. I would like materials in my physics course to be covered in a way to help me:
(a) do well on physics exams.
(b) develop my reasoning skills.
43. I would like to study physics in order to satisfy:
(a) my own interests.
(b) what certain people expect of me.
44. I would like my understanding of physics courses to depend on:
(a) how much effort I put into studying.
(b) how well the teacher explains things in class.
45. I would like to learn about topics discussed in my physics course:
(a) from my physics textbook.
(b) from other sources.
46. In my physics course, I would like to:
(a) learn how physicists go about investigating the natural world.
(b) acquire information about certain objects and events in the natural world.
47. I would like my performance on physics exams to reflect how well I can:
(a) recall course materials the way they are presented in class.
(b) apply course materials in situations not discussed in class.
48. For any question asked in class, I would like my physics teacher to:
(a) provide the correct answer.
(b) show how we may get the answer.
49. When studying physics in a textbook or in course materials, I would like to:
(a) find the important information and memorize it the way it is presented.
(b) reconstruct the material in my own way so that I can make sense of it.
50. I answered the questions in this survey:
(a) to the best of my ability.
(b) without thinking seriously about them.

## Appendix C: EBAPS

## Part 1

## DIRECTIONS: For each of the following items, please read the statement, and indicate (on the scantron answer sheet) the answer that describes how strongly you agree or disagree.

A: Strongly disagree B: Somewhat disagree C: Neutral D: Somewhat agree E: Strongly agree

1. Tamara just read something in her science textbook that seems to disagree with her own experiences. But to learn science well, Tamara shouldn't think about her own experiences; she should just focus on what the book says.
2. When it comes to understanding physics or chemistry, remembering facts isn't very important.
3. Obviously, computer simulations can predict the behavior of physical objects like comets. But simulations can also help scientists estimate things involving the behavior of people, such as how many people will buy new television sets next year.
4. When it comes to science, most students either learn things quickly, or not at all.
5. If someone is having trouble in physics or chemistry class, studying in a better way can make a big difference.
6. When it comes to controversial topics such as which foods cause cancer, there's no way for scientists to evaluate which scientific studies are the best. Everything's up in the air!
7. A teacher once said, "I don't really understand something until I teach it." But actually, teaching doesn't help a teacher understand the material better; it just reminds her of how much she already knows.
8. Scientists should spend almost all their time gathering information. Worrying about theories can't really help us understand anything.
9. Someone who doesn't have high natural ability can still learn the material well even in a hard chemistry or physics class.
10. Often, a scientific principle or theory just doesn't make sense. In those cases, you have to accept it and move on, because not everything in science is supposed to make sense.
11. When handing in a physics or chemistry test, you can generally have a sense of how well you did even before talking about it with other students.
12. When learning science, people can understand the material better if they relate it to their own ideas.
13. If physics and chemistry teachers gave really clear lectures, with plenty of real-life examples and sample problems, then most good students could learn those subjects without doing lots of
sample questions and practice problems on their own.
14. Understanding science is really important for people who design rockets, but not important for politicians.
15. When solving problems, the key thing is knowing the methods for addressing each particular type of question. Understanding the "big ideas" might be helpful for specially-written problems, but not for most regular problems.
16. Given enough time, almost everybody could learn to think more scientifically, if they really wanted to.
17. To understand chemistry and physics, the formulas (equations) are really the main thing; the other material is mostly to help you decide which equations to use in which situations.

## Part 2

## DIRECTIONS: Multiple choice. On the answer sheet, fill in the answer that best fits your view.

18. If someone is trying to learn physics, is the following a good kind of question to think about?

Two students want to break a rope. Is it better for them to (1) grab opposite ends of the rope and pull (like in tug-of-war), or (2) tie one end of the rope to a wall and both pull on the other end together?
(a) Yes, definitely. It's one of the best kinds of questions to study.
(b) Yes, to some extent. But other kinds of questions are equally good.
(c) Yes, a little. This kind of question is helpful, but other kinds of questions are more helpful.
(d) Not really. This kind of question isn't that great for learning the main ideas.
(e) No, definitely not. This kind of question isn't helpful at all.
19. Scientists are having trouble predicting and explaining the behavior of thunder storms. This could be because thunder storms behave according to a very complicated or hard-to-apply set of rules. Or, that could be because some thunder storms don't behave consistently according to any set of rules, no matter how complicated and complete that set of rules is.
In general, why do scientists sometimes have trouble explaining things? Please read all options before choosing one.
(a) Although things behave in accordance with rules, those rules are often complicated, hard to apply, or not fully known.
(b) Some things just don't behave according to a consistent set of rules.
(c) Usually it's because the rules are complicated, hard to apply, or unknown; but sometimes it's because the thing doesn't follow rules.
(d) About half the time, it's because the rules are complicated, hard to apply, or unknown; and half the time, it's because the thing doesn't follow rules.
(e) Usually it's because the thing doesn't follow rules; but sometimes it's because the rules are
complicated, hard to apply, or unknown.
20. In physics and chemistry, how do the most important formulas relate to the most important concepts? Please read all choices before picking one.
(a) The major formulas summarize the main concepts; they're not really separate from the concepts. In addition, those formulas are helpful for solving problems.
(b) The major formulas are kind of "separate" from the main concepts, since concepts are ideas, not equations. Formulas are better characterized as problem-solving tools, without much conceptual meaning.
(c) Mostly (a), but a little (b).
(d) About half (a) and half (b).
(e) Mostly (b), but a little (a).
21. To be successful at most things in life...
(a) Hard work is much more important than inborn natural ability.
(b) Hard work is a little more important than natural ability.
(c) Natural ability and hard work are equally important.
(d) Natural ability is a little more important than hard work.
(e) Natural ability is much more important than hard work.
22. To be successful at science...
(a) Hard work is much more important than inborn natural ability.
(b) Hard work is a little more important than natural ability.
(c) Natural ability and hard work are equally important.
(d) Natural ability is a little more important than hard work.
(e) Natural ability is much more important than hard work.
23. Of the following test formats, which is best for measuring how well students understand the material in physics and chemistry? Please read each choice before picking one.
(a) A large collection of short-answer or multiple choice questions, each of which covers one specific fact or concept.
(b) A small number of longer questions and problems, each of which covers several facts and concepts.
(c) Compromise between (a) and (b), but leaning more towards (a).
(d) Compromise between (a) and (b), favoring both equally.
(e) Compromise between (a) and (b), but leaning more towards (b).

## Part 3

# DIRECTIONS: In each of the following items, you will read a short discussion between two students who disagree about some issue. Then you'll indicate whether you agree with one student or the other 

24. 

Brandon: A good science textbook should show how the material in one chapter relates to the material in other chapters. It shouldn't treat each topic as a separate "unit," because they're not really separate.
Jamal: But most of the time, each chapter is about a different topic, and those different topics don't always have much to do with each other. The textbook should keep everything separate, instead of blending it all together.

With whom do you agree? Read all the choices before circling one.
(a) I agree almost entirely with Brandon.
(b) Although I agree more with Brandon, I think Jamal makes some good points.
(c) I agree (or disagree) equally with Jamal and Brandon.
(d) Although I agree more with Jamal, I think Brandon makes some good points.
(e) I agree almost entirely with Jamal.

## 25.

Anna: I just read about Kay Kinoshita, the physicist. She sounds naturally brilliant.
Emily: Maybe she is. But when it comes to being good at science, hard work is more important than "natural ability." I bet Dr. Kinoshita does well because she has worked really hard.
Anna: Well, maybe she did. But let's face it, some people are just smarter at science than other people. Without natural ability, hard work won't get you anywhere in science!
(a) I agree almost entirely with Anna.
(b) Although I agree more with Anna, I think Emily makes some good points.
(c) I agree (or disagree) equally with Anna and Emily.
(d) Although I agree more with Emily, I think Anna makes some good points.
(e) I agree almost entirely with Emily.
26.

Justin: When I'm learning science concepts for a test, I like to put things in my own words, so that they make sense to me.
Dave: But putting things in your own words doesn't help you learn. The textbook was written by people who know science really well. You should learn things the way the textbook presents them.
(a) I agree almost entirely with Justin.
(b) Although I agree more with Justin, I think Dave makes some good points.
(c) I agree (or disagree) equally with Justin and Dave.
(d) Although I agree more with Dave, I think Justin makes some good points.
(e) I agree almost entirely with Dave.
27.

Julia: I like the way science explains things I see in the real world.
Carla: I know that's what we're "supposed" to think, and it's true for many things. But let's face it, the science that explains things we do in lab at school can't really explain earthquakes, for instance. Scientific laws work well in some situations but not in most situations.
Julia: I still think science applies to almost all real-world experiences. If we can't figure out how, it's because the stuff is very complicated, or because we don't know enough science yet.
(a) I agree almost entirely with Julia.
(b) I agree more with Julia, but I think Carla makes some good points.
(c) I agree (or disagree) equally with Carla and Julia.
(d) I agree more with Carla, but I think Julia makes some good points.
(e) I agree almost entirely with Carla.
28.

Leticia: Some scientists think the dinosaurs died out because of volcanic eruptions, and others think they died out because an asteroid hit the Earth. Why can't the scientists agree?
Nisha: Maybe the evidence supports both theories. There's often more than one way to interpret the facts. So we have to figure out what the facts mean.
Leticia: I'm not so sure. In stuff like personal relationships or poetry, things can be ambiguous. But in science, the facts speak for themselves.
(a) I agree almost entirely with Leticia.
(b) I agree more with Leticia, but I think Nisha makes some good points.
(c) I agree (or disagree) equally with Nisha and Leticia.
(d) I agree more with Nisha, but I think Leticia makes some good points.
(e) I agree almost entirely with Nisha.
29.

Jose: In my opinion, science is a little like fashion; something that's "in" one year can be "out" the next. Scientists regularly change their theories back and forth.
Miguel: I have a different opinion. Once experiments have been done and a theory has been made to explain those experiments, the matter is pretty much settled. There's little room for argument.
(a) I agree almost entirely with Jose.
(b) Although I agree more with Jose, I think Miguel makes some good points.
(c) I agree (or disagree) equally with Miguel and Jose.
(d) Although I agree more with Miguel, I think Jose makes some good points.
(e) I agree almost entirely with Miguel.
30.

Jessica and Mia are working on a homework assignment together...

Jessica: O.K., we just got problem \#1. I think we should go on to problem \#2.
Mia: No, wait. I think we should try to figure out why the thing takes so long to reach the ground.
Jessica: Mia, we know it's the right answer from the back of the book, so what are you worried about? If we didn't understand it, we wouldn't have gotten the right answer.
Mia: No, I think it's possible to get the right answer without really understanding what it means.
(a) I agree almost entirely with Jessica.
(b) I agree more with Jessica, but I think Mia makes some good points.
(c) I agree (or disagree) equally with Mia and Jessica.
(d) I agree more with Mia, but I think Jessica makes some good points.
(e) I agree almost entirely with Mia.

## Appendix D: SERMP

## Student Expectation of The Role of Mathematics in Physics

Course Code: $\square$
Group
$\square$

## Date:

Below are 30 statements which may describe your understanding of the Role of Mathematics in
Physics. You are requested to state your position regarding each statement by circling a number
From 1 to 5 where the numbers mean the following:

## 1: Strongly Disagree 2: Disagree 3: Neutral 4: Agree 5: Strongly Agree

- Your Participation is voluntary
- Your responses will be anonymous

| $\begin{aligned} & \hline \text { Item } \\ & \text { no } \end{aligned}$ | Item |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | I solve mathematical physics problems in order to learn physics. | 1 | 2 | 3 | 4 |  |  |
| 2 | Problem solving in physics means finding the right equation to use. | 1 | 2 | 3 | 4 | 5 | 5 |
| 3 | I understand the physical meaning of equations used in this course. | 1 | 2 | 3 | 4 | 5 | 5 |
| 4 | A necessary skill in this course is being able to memorize all the mathematical equations that I need to know. | 1 | 2 | 3 | 4 | 5 | 5 |
| 5 | Learning physics is a matter of acquiring knowledge that is specifically located in the laws and equations. | 1 | 2 | 3 | 4 | 5 | 5 |
| 6 | Physics laws relate to what I experience in real life. | 1 | 2 | 3 | 4 | 5 | 5 |
| 7 | I am able to solve a mathematical physics problem that I have never seen before. | 1 | 2 | 3 | 4 |  | 5 |
| 8 | I understand physics equations as relationship among variables. | 1 | 2 | 3 | 4 |  | 5 |
| 9 | Solving mathematical physics problems in the physics class is the same as doing so in the mathematics class | 1 | 2 | 3 | 4 |  | 5 |


| 10 | Physical relationships can be explained using mathematics. | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 11 | The most crucial thing in solving a physics problem is finding the right equation to use. | 1 | 2 | 3 | 4 | 5 |
| 12 | In solving a physics problem, I sometimes get a correct mathematical solution whose meaning I do not understand. | 1 | 2 | 3 | 4 | 5 |
| 13 | I take symbols in physical equations as representing numbers. | 1 | 2 | 3 | 4 | 5 |
| 14 | The use of mathematics in problem solving makes physics easier to understand. | 1 | 2 | 3 | 4 | 5 |
| 15 | Formulae describing physical relationship are "out there" to be discovered. | 1 | 2 | 3 | 4 | 5 |
| 16 | A mathematical solution to a physics problem must be meaningful to me. | 1 | 2 | 3 | 4 | 5 |
| 17 | It is necessary for lecturers to explicitly discuss with students, how mathematics is used in physics. | 1 | 2 | 3 | 4 | 5 |
| 18 | The first thing that I do when solving a physics problem is to search for formulae that relate givens to unknowns. | 1 | 2 | 3 | 4 | 5 |
| 19 | To be able to use an equation in a problem, I need to know what each term in the equation represents. | 1 | 2 | 3 | 4 | 5 |
| 20 | I would prefer to learn physics with no mathematics. | 1 | 2 | 3 | 4 | 5 |
| 21 | I learn physics in order to solve problems. | 1 | 2 | 3 | 4 | 5 |
| 22 | I spend a lot of time figuring out the physics derivations in the text. | 1 | 2 | 3 | 4 | 5 |
| 23 | There can be no physics without mathematics. | 1 | 2 | 3 | 4 | 5 |
| 24 | The main skill to learn out of this course is to solve physics problems. | 1 | 2 | 3 | 4 | 5 |
| 25 | I treat equations as representations of reality. | 1 | 2 | 3 | 4 | 5 |
| 26 | I always see symbols as representing physical measurements | 1 | 2 | 3 | 4 | 5 |
| 27 | The mathematics that I learned in the mathematics class is useful when solving physics problems | 1 | 2 | 3 | 4 | 5 |


| 28 | When I solve most physics problems, I think about the concepts <br> that underlie the problem. | 1 | 2 | 3 | 4 | 5 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 29 | If I do not remember a particular equation needed for a problem <br> in a test, there is nothing much I can do. | 1 | 2 | 3 | 4 | 5 |
| 30 | There should be more physics problem involving the use of <br> mathematics than those where students just explain. | 1 | 2 | 3 | 4 | 5 |

## Appendix E: CONSENT FORM

# UNIVERSITY OF SOUTH AFRICA <br> RESEARCH CONSENT FORM 

Researcher: Reuben D. Koontse<br>Tel: (267) 3655712<br>Cell: (267) 72907790<br>Private Bag 011 Gaborone, Botswana<br>Supervisor: Jeanne Kriek (Prof)<br>Tel: (0027) 0124298405<br>PO Box 392, Unisa 0003<br>Pretoria, South Africa

This is to verify that I $\qquad$ consent to voluntarily participate in a study that involves the role that mathematics plays in students understanding of physics.

I understand that:

- Participation is voluntary
- My responses will be treated confidentially
- I may withdraw from the research anytime without any negative consequences to my self
- Anonymity is insured
- Copies of my test, assignment and tutorial scripts may be used in the study
- I may be interviewed in the study


## Signed:

Date:

## Appendix F: UNISA Ethics Clearance

## INSTITUTE FOR SCIENCE AND TECHNOLOGY EDUCATION

31 January, 2011
Attention: Prof M Slabbert
Chairperson, URERC
REPORT OF ETHICAL CLEARANCE APPLICATIONS SUBMITTED TO THE ETHICS SUBCOMMITTEE OF THE INSTITUTE FOR SCIENCE AND TECHNOLOGY EDUCATION (ISTE)

OCTOBER - DECEMBER 2010 THROUGH TO JANUARY 2011

\begin{tabular}{|c|c|c|c|}
\hline APPLICANT \& DEPT./COURSE \& TITLE \& APPROVED/REJECTED <br>
\hline \& \& \& <br>
\hline Mr. Marumure, G. P \& Chemistry Education M.Sc. \& Problems and Prospects of Teaching Chemical Equilibrium at the FET Band \& Approved <br>
\hline Mr. Nhlanhla Lupahla \& Mathematics Edu M.Sc \& An Investigation into the Algebriac Problem solving Skills of Grade 12 Learners in Oshana Education Region \& Finally approved after resubmission <br>
\hline Mr. Bekele Gashe D. \& Physics Education

Ph.D \& | Diognosis of |
| :--- |
| Students` Alternative |
| Conceptions and Conceptual Change through Cognitive Perturbation and simulation in Undergraduate Electricity \& Magnetism in Ethiopia | \& Approved <br>

\hline Mr. Sam Kaheru \& Physics Education \& Exploring the Use of \& Approved <br>
\hline
\end{tabular}

|  |  |  |  |
| :---: | :---: | :---: | :---: |
|  |  |  |  |
|  |  |  |  |
| Mr. Reuben Koontse | Physics Education Ph.D | The Role of Mathematics in Students Understanding of Physics | Approved |
|  |  |  |  |
|  |  |  |  |

## GEOchonogor

Dr. C.E Ochonogor
Rep./Chair: Ethics Sub-Committee, ISTE.

# Appendix G: University of Botswana Research Permission letter 

Office of the Deputy Vice Chancellor (Academic Affairs)
Office of Research and Development

| Corner of Notwane | Pvt Bag 0070B | Tel: [267] 355 2900 |
| :--- | :--- | ---: |
| and Mobuto Road, | Gaborone, Botswana | Fax: [267] 3957573 |
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Ref: UBR/RES/IRB/1271
29 July 2011

Mr Reuben Koontse
P O Box 70961
UB Post Office
Gaborone

## RE : PERMISSION TO CONDUCT RESEARCH WITHIN UNIVERSITY OF BOTSWANA

Project Title : "The role of Mathematics in Students Understanding of Physics".
We are glad to advise that permission has been granted for you to conduct the above study within UB. You are further advised that your study has been granted exemption from research permit requirements since the study will be conducted within the confines of UB and is aimed at improving the teaching of Physics within UB. In conducting your study, you are however reminded to follow high ethical standards. Specifically, you will be expected to obtain informed consent from both lecturers and students and maintain confidentiality.

- APPROVAL NUMBER UB/IRB/1271

The above details should be used on all correspondence concerning this exemption.

- INVESTIGATORS Mr Reuben Koontse
- APPROVAL DATE -29 July 2011
- EXPIRATION DATE :This approval expires on 28 July 2012

After this date, this project may only continue upon renewal. For purposes of renewal, a progress report to ORD. The report should be submitted one month before the expiration date.

- REPORTING OF SERIOUS PROBLEMS: All serious problems having to do with safety or welfare of research volunteers as well as any serious problems impacting on study quality and progress (whether expected or unexpected) must be reported to ORD within 10 working days.
- MODIFICATIONS :Prior approval is required before implementing any significant changes to the Protocol.
- TERMINATION OF STUDY : On termination of this study, a report has to be submitted to ORD.
- QUESTIONS : Please contact ORD ext 2911 or e-mail on paul.ndebele@mopipi.ub.bw.
- Other:

Study has been exempted from the Govt Research permit requirement in terms of minute reference $\mathrm{E} / 1 / 20 / 2 \mathrm{X}$ (15) from MESD dated $24^{\text {th }}$ September 2010. You may accordingly proceed with your study.

Kind regards.


Paul Ndebele
For Director, Office of Research and Development


## Appendix H: Interviews

## $1^{\text {st }}$ Interview Sessions

## Group H

A discussion with group H about students general physics experience heralded the following excerpt:

R: Fizi, so let's hear, what's your side of the story, how are you liking the physics course?

H3: The lab side they have touched that, but it general....the tutorial is interesting.
$\mathbf{R}$ : They are interesting?
H3: Yaa. It's because all those things are maths related.
R: They are what?
H3: They are maths related.
R : Oh...they are mathematics related.
H3: Yaa.
R: Ok, so and you like mathematics?
H3: And I love mathematics.
R: So ... it means ... in the, in the lectures most of the time you do lot of mathematics.
H3: A lot.
$\mathbf{R}: \quad$ Alright, and in the tutorials?
H3: And in the tutorials.

H5: Ya, but with lectures, it's ok, right now it's not the same as last semester. Last semester from mechanics, then optics. But right now if you know potential difference you will still see it and apply it as you progress.

R: Eeh...
H5: Starting even with capacitors... but there is some correlations and not the same as last semester. Again first semester, eish it was a bit tougher than now, now as long as you can understand how the formula work like it will be easy for somebody to pass, it's not like last semester where even If you knew the equations, you may not be able to integrate it properly.

R: Ok, I have not heard Katy's story...
H6: Like they said lectures are fine.
R: Mmm.
H6: The thing is if you don't understand the concepts, you will have problems throughout. So for that part I think it is very important to understand the concepts.

R: Aha, I see. Alright let's talk about the tutorials; we have not talked about the tutorials. What's the story there, what's the story there? What do you like most about the tutorials? And what don't you like?

H4: Writing answers on the board, I don't like that.
R: You don't like that. What would you like?
H4: Just coming and explaining that we do that using this concept, that's what we want.
R: Isn't the answers all that you want?
No...!! (Three voices)
H3: We want the idea of how to solve the problem.
R: You want the idea?
H3: Yaa.
R: And you don't get the idea?
H3: We are...., but not in ... like a more efficient way. Because you know, a question like that will not appear the same way in a test. You have to know that if it is this way, we have to tackle it this way.

R: Alright, ok. Yes sir, you are quiet what have you got to say?
H2: Aah, I can only say what I don't like about the tutorials.
$\mathbf{R}: \quad$ Ok, what did you say about Tutorials?
H2: Tutorials, just the handouts, yaa, just handing out the papers.
R: Yaa.
H2: I would prefer if we did every question.
R: If you did every question?
H2: Yaa showing the thinking behind what every question actually wants.

R: Aha.
H2: Instead of just giving the, the paper, we are not sure where this thing actually comes from, or exactly what's going on?

R: Mmm. Let me ask you, when you do a tutorial question, you do it up to the end and then you get a correct answer, do you always understand the principles underlying that question, are you always in a situation to understand the principle underlying a question if you got it right?
[Students keep quiet]
$\mathbf{R}$ : That much is a given isn't it, is it? Or you don't get the question?
H3: I didn't get the question.
R: Yaa, if you solve a question, say a mathematical question and you get it correct in terms of the solution that you are looking for, umm, do you always at the same time understand the whole principle underlying that question or do you come upon a situation where you can solve a question correctly, mathematically and you do not necessarily understand the question.

H3: I think with this course it's kinda different because we are kind of relating different ... yes with physics it has its own concept like the theory part.

R: Aha.
H4: But then there is also the mathematical side. But then before you move to the mathematical side you should know the theory part as well.

R: Yes?
H4: The principles, understand everything ... because what really matters is the concept behind everything.
R: You think that's what matters.
H4: That's what matters.
$\mathbf{R}$ : So do you always get the concept?
H4: Because with mathematics we have calculators, we can always get answers right, but then what really matters is the principle.

R: $\quad O k$.
H4: Yes.
R: But do you always get the principle?
H4: I always, most of the time I get the principle.

R: Alright... mmm. Aha, Any you have been quiet, what are you thinking?
H6: For me, I sometimes... I can get an answer without knowing how.
R: Mam?
H6: Sometimes I get an answer without knowing how. It happens quite often.
$\mathbf{R}$ : Without knowing how you got it?
H6: Yes, sometimes without understanding I just do it and hei... with God's luck I get it.
R: Mmm...Ok, I think we have exhausted today's session. That's it. So we will stop here for
today.

## Group M

The first tutorial - group M students' interview was also along the same lines as that of group H. Students were allowed to share their general physics experience in as broadly as possible. After about $1 / 3$ of the way through the interview, it came to this:

R: Ok somebody else, what can you tell us, Mr. Fizi?
M2: I have a problem with the tutorials.
R: Problem, because Hmm...
M2: Because he comes with answers, he doesn't explain to us. He just writes the answers on the board and then he goes away, that's all. He just comes with answers and he pastes them on the board. And we don't understand how to... if you didn't do the tutorial it becomes a problem because you don't understand anything...you just see figures, numbers, equations, just puts them down and goes away.

R: Hmm...Ok... what would you prefer to be done Mr. Fizi?
M2: I would prefer not necessarily lecturing as such, but maybe explaining key concepts because I think a tutorial is a time where you kind of... where you explain your problems to your tutor, you share problems because that's the only time where you are free, not like lectures, the lecturers have to meet their... have to push the syllabus and stuff like that, so I think a little bit of explaining the problems, not just pasting them on the board.

S3: You see the tutorial is placed very nicely at the end of the week, on Friday afternoon after all lectures. But, but the problem is that...you know there are many rules and theorems in physics. So if we are taught that from Monday to Friday...

## R: Нтт...

S3: In our tutorial session, eeh...we should review those things. Like what we did from Monday up to Friday, not necessarily doing all the questions... we make take one or two from the tutorial script and concentrate on the concept that we learned.

R: Нтт...
S3: Because you find that those answers that are written on the board are. are meaningless to most of us. So we need only to review what we learnt.
$\mathbf{R}$ : So you want to concentrate on the principles and concepts on the tutorials.
S3: Yes.
R: Just to discuss that.
S3: Yes.
R: And not solve problems?
S3: No... Problems... not all problems in the tutorial.
R: Not all the problems.
S3: Yes...
R: So you are actually talking about a completely different structure to tutorials. You would prefer where...
(Student S1 interrupts)
S1: To understand the concepts.
S3: To understand the concepts.
$\mathbf{R}$ : The concepts are explained so that you can...
S3: Yes because again the tutorial answers are always posted on WebCt ${ }^{4}$.
R: Ooh... (The researcher was not aware of this)
S3: So if you need answers you can refer from WebCt.
$\mathbf{R}$ : So what is the difference between those answers that you get from WebCt?
S1: $\quad$ There is no difference (the girl interrupts).
$\mathbf{R} \quad$ And the tutorials?

[^5]S2: $\quad$ They are the same.
S3: Well, they are the same, because the questions are the same.
S1: So like Fizi was saying, the tutor should come explain key concepts, make us understand.

R: $\quad Н т т \ldots$
S1: Because we are going to find answers on WebCt.
R: Hmm Twizer ...what's your story. What do you like most about tutorials Mr. Twizer
S4: From my experience here with tutorials...I don't know about other classes from our class is generally the same problem which they just mentioned. Of which I feel that there is no much enough explaining of the key concepts.

R: Yeeh!
S4: Yeeh... they have already mentioned that we are just given the solutions. And there is no much explaining of the key concepts, of which is very vital. If you don't get something from the lecture, you are hoping to get it from the tutorials. And with our case that's not how it is.

R: Mmm.....now tell me from your experience of learning physics both from lectures and tutorials, when you solve physics problems, do you get an idea that the act of solving a physics problem and getting it correct, does it help you understand the physics concepts and principles you are talking about?

M3: I think actually getting a correct answer boosts your morale towards physics.
R: Yaa.
M3: Because it proves that what they said is, the principle what they said about it is right. It applies.
$\mathbf{R}$ : What they said?
M3: Like, if you take for example Coulomb's law.
R: Yaa.
M3: If you are somebody, you are just coming and being told about Coulomb's law. It quite confuses you the first time. But once you do the calculations and see, you will get it.

R: Alright, good....Fizo, does it help?
M2: Yes it does help, but sometimes ahh, I only use the equation and get the answer and say ahh here I don't understand. I just got the answer. I know how to find... I know
how to use the equation and find the answer. Not necessarily meaning I understand the concept.

R: Mmm.
M2: Because sometimes I get a question, ok fine, I look for the correct mmm... the right formula to use, I use that formula, I check the answer at the back of the book. Ok the answer is correct but not necessarily understanding the concept...so I do have a problem sometimes.

R: Your experience Mr. Roly?
M4: Yaa, most of them yaa, you feel that the answer is in line with the concept. But sometimes yaa you do feel that yaa here I just got the answer but you don't know what the meaning of the answer is.

R: Mmm.
M4: Like sometimes you get a negative answer sometimes you get a positive one. You don't really understand what the meaning of those answers is?

R: Ely, can you solve a problem that you have never seen before?
(Ely giggles)
R: Well like in the same field, like right now you are doing electricity and magnetism, right?

M1: Yes...
R: So if you get a question in electricity and magnetism that you have not seen before, can you solve it, or has it ever happened before?

M1: I just apply the equations.
(R laughs, Ely laughs too)
M1: I look at a question, look at what I am given in the question... yes look at what I am given ... this and this ... and try to relate it to an equation that I know.

R: So if you get a question in electricity and magnetism that you have not seen before, can you solve it, or has it ever happened before?

M1: I just apply the equations [blindly].

R: Ok...Mr. Fizi, can you learn physics without mathematics?
S2: Aah I don't think so.

R: It's impossible?
S2: It's impossible, it's very impossible. You need to apply maths in order to understand the physics.
$\mathbf{R}: \quad$ Tracy, do you share the same sentiments?
S5: I think that it is possible but there is a lot of maths.
$\mathbf{R}$ : There is a lot of maths. So you can learn physics without maths?
S5: Yes.
( R tries to explain some electricity and magnetism concept without maths. All students laugh and R laughs as well).

R: So, when you solve problem, a physics problem, for you to get it correctly what do you think is the most important thing to do?

S1: Read the question thoroughly, understand it. As they have been saying write down what you are given, and write down what you are looking for, that's basically it.

R: Yes.
S1: Yes.
S2: Looking for the correct formula.
R: But how do you know it's the correct formula.
S2: Obviously you have to understand.
R: You have to understand.
S2: Yes. .
R: Alright, now the mathematics does it simplify or makes physics easier, or more difficult.

S3: Well it makes it easier because mathematical illustrations they tend to make you understand or believe because they are proved.

R: They are proved.
S3: Yes, you know in physics there is a lot of proofing and you tend to get it more quickly when there is mathematics involved.

R: Yes, what's your take?
S1: Yaa... physics, maths, yaa when you are taught concepts and then you might not get, but then when you apply maths then... it makes you believe, then you understand.

R: Maths makes you believe?
S1: It complements.
( R laughs)
R: What do you say Mr. Twizer.
S4: I feel that maths is simpler than physics.
R: It's simpler than physics.
S4: Yaa of which the problem now is in most of the physics problems, you have to understand the physics part of the problem first, before you get to solve with maths. Of which I don't think...... ahh it makes me feel that it doesn't make any difference, with maths.

R: It doesn't make any difference.
S4: Yaa, because you have to go through the physics first before you go to the maths part of it which is the easy part?
$\mathbf{R}$ : The mathematical part is the easy part.
S4: Yaa.
S1: You know sometimes you can get the physics, your physics maybe right but your maths is wrong.

R: Ooh.... so either way.
S1: Yaa sometimes it's the physics and then the maths which is wrong.
R: Ok guys, I think our time is up...you have been very helpful. I got a lot of information from our discussion. So we can stop here. Thanks.
(R switches off the audio recorders and students prepare to leave for their next lesson)

## Second interview sessions

Two (2) weeks after the first interview, students have written the first test of the semester, Group H meet again with the researcher in the library seminar room. As the interviews progressed, the discussion was narrowed to students' work in tutorials and tests specifically in the topic of electricity, where students used a lot of mathematics in solving physics problems.

## Group H

R: What I want us to talk about today, ok you did you tutorial test two weeks ago and your test this week, no, right?

S1: Yes the test was this week.
R: So how were they?
S1: $\quad$ The test, I did not manage to finish.
R: Didn't manage to finish.
S1: Yes.
S2: Aah I think eeh the test was, I don't know whether they set properly or not, it was like everything in there I have never seen a question like it ever, everything was different. It was so different, like everything, everything was hard. From section A to section like every section.
$\mathbf{R}: \quad I s i t ?$
S2: Especially B1, I spent like 45 minutes trying to figure out stuff and then eish...
R: You are shaking your head, is it because...
S3: It's so true.
R: It's so true.
Yaa, yaa (at least three voices)
R: Is different from what, the tutorial and what you get from lectures or what?
S3: Ok the questions we could hear (the student here was speaking in vernacular, and the word for hear and understand can sometimes be used interchangeably) what they were saying but then ...how they were constructed and what they wanted did not make sense ...;because there was this question where they said we have a charge being placed such that we have an equilibrium system, but then the thing is the charge is placed between negative and positive charge, and when you put the particular charge being positive, it does not make sense because you are not going to have an equilibrium system. You can only put it on the other side. So if a question is like that, you don't know how to tackle it. Break the rules or.... just do it.

R: Yes... hmmm Mr. ..., how was the test, or tutorials test?
S4: But the test things aah, we don't know whether we have been taught or what.
(Others laugh...)
S4: Because tutorials and test are...
R: Tell me when you solve this problem; be it in a test or tutorial, can you solve a problem that you have not seen before, Slim?

S1: In Physics?

R: Yes in physics...can you solve a problem that you have not seen before?
S1: Umm.... It depends.
R: Yes...On what?
S1: Umm ... it depends on the structure, how that question is.
R: Umm, Pat.
S2: Umm... I don't know. Honestly I can't.
R: You can't?
S2: I need to have seen an example of the exact question.
R: Exact question.
S2: Yes, I need to have seen an example of it, or so that I apply it... but generally no.
S5: Yaa.
S6: Yaa same here...aah its quite difficult to solve because we expect like aah...an example of each type so that we know what to do... we know that if it is like this we do this.

R: Yes.
S6: If you have not seen it's going a bit a little hard to, unless maybe... it's your good day...you woke up on the good side of the bed (chuckles) or something.

S2: Without that no way!!
R: Yes!!
Students (S2, S5, S6) refute that they can solve a question that they have not seen before. Only student S1 says she may solve a physics question that she has never seen before. She however says "It depends".

R: Yes... Mr. Tony, when you solve a physics problem be it a tutorial, test, or just studying where do you normally star, what is the first thing that you do.

S7: The problem is that the questions that we do in tutorials, they are completely different from the ones in the test.

R: Yes...Brie, what's the starting point.
S3: I usually start reading what the question wants. Usually... maybe ...say it wants force, so I try to put down all the equations that I know, for force. But after seeing that test I don't think that is useful because I tried using this other equation, it refused, tried this one it refused,, there was nothing to do then ... eeh but it wasn't really bad, but eeh
some of the things were just too new. Because in class we do things that we understand honestly, but in the test, it's always a shock.
(Some of them laugh)
S2: It's a shock!!
R: It's a shock?
S2: Yes, sometimes. And even when I get a test paper I never look through it.
R: You never look through it.
S2: Yes at the beginning I just, because I don't want to think of a question that I saw, later on, at the beginning and I am not able to focus on the other ones, so I just tackle one question , next question...

R: When you go for a physics test, how much memorization do you do?

## A lot... a lot (at least 4 voices at the same time)!!

S1: Cramming!! I remember I walked in the test and I literally crammed a working of one question. (Some laugh, Glody shakes her head vehemently)

R: Glody, is that what you do, Glody?
S6: Aah, I don't do that.
R: You don't do that.
S6: Honestly I don't cram. I don't wanna lie, I don't cram.
R: $\quad O k$.
S6: $\quad$ The thing is I make sure that when I get the first formula, the basic one, the main one. I go along and link it with all the others that I know.

R: So you get the formula?
S6: I get the main formula and link it with the other ones that I know.
R: Formula you mean mathematical formula?
S6: Mathematical formula maybe force equal to mass over acceleration, then I link it with others that I know.

R: Yes.

S6: And also with that one, usually we are also given values written like this. The ones that we are supposed to know and use them in calculation, sometimes I just take the value that is given there with the units and try to derive a formula from that.

R: Yaa.
S6: So that's how I tackle everything.
R: So, Wada can you learn physics without mathematics. Do you think physics can be taught without mathematics?

S2: Yes I do but I don't think it will be as fun or it will be as interesting because answering questions, from senior school, answering theory questions proved to be more difficult than the mathematical part because the theory you have to read. Physics you know we don't usually re...ad physics. We just, I don't re...ad physics, I just find the question and see how they relate.

R: You solve problems.
S2: Yes, the structured ones I know it can be taught using the structured but, I think, a lot of people fail it. Again I think there are other chapters or parts of physics which is impossible to teach without the mathematics.

R: Ok, Slim do you think mathematics simplifies or makes physics more difficult.
S1: I think it simplifies it.
R: $\quad O k$.
S1: Yes, physics you have to apply and that application is...is related somehow to mathematics. It links mathematics with physics.

R: Ok, you have class at 12 is it .
S1: Sure.
R: Ok its 5 minutes to 12 so, I don't want to waste your time. So thanks a lot guys. Hope you benefiting something from this. I am benefiting a lot. I am sure the department will benefit a lot as well.

## Group M

Only three students came to the interview this time. Although the original number for interview groups was ten, from the previous interview these three students appeared to be among the most enthusiastic and with the benefit of hindsight proved to be quite open-
minded, free and informative. R had come with snippets of paper showing the formula $\mathrm{F}=\mathrm{k}_{\mathrm{e}}$ $\mathrm{q}_{1} \mathrm{q}_{2} / \mathrm{r}^{2}$. He gave them to participants and started like this:

R: So guys. now please know that... eeh I'm not interested in whether you get this wrong or right ok, neither is this a contest. I just want you to give me an understanding of how you see things ok, your understanding.

S1: $O k$
R: Now this particular thing I'm giving you appeared quite frequently in the last few weeks even in your test...so, this particular thing that im giving you in the first place, Mr. Lety what would you say, what is this?

S1: Yaa I would say this is the ... mmm...Coulomb's law.
R: It's Coulomb's law?
S1: Yes sir.
R: Ok, Bookie, what is that you have in front of you?
S2: This...yaa ... it's the force between two electric point .... two electric point charges.
R: It's the force between two electric point charges...
S2: Mmm.
R: Ok... Mmm. So Mr. Lety you wanted to say something...
S1: Something about this ... this equation
R: Yes, about that...it's an equation right?
S1: Yes...
R: Ok.
S1: So about this, I would rather .... yaa it contains a constant.
$\boldsymbol{R}$ : Нтт..
S1: The constant is .....Something eight point nine nine something, but we use nine point zero
times ten to the power nine.
$\boldsymbol{R}: ~ Н m т \ldots$
S1: So and two charges which act on one another at a certain distance which is $r$.
R: $O k$.

S1: Basically it is force, due eeh force between those charges, those point charges that are there.

R: $O k$.
S1: So it was derived by some... some scientists named Coulomb.
R: Ok, Bookie if we you asked to ... somebody say describe this, this equation in words, what would you say? Just do describe what it's all about.

S2: (She giggles) ...it's ....
$\boldsymbol{R}$ : Yes...
S2: In words, describing what I see...
$\boldsymbol{R}$ : Yes...
S2: You multiply the charges, then you divide it by the square of the distance between them, times the constant...

R: Yes.
S2: It gives you the force between two point charges.
R: Yaa ...Ok ...Mr. Fizi ...what is that in front of you?
S3: The electronic force ... the equation for electronic force, electric force.
R: Electric Force... Ok. (R giggles) so if somebody was to ask you, describe that equation in words, what would you say?

S3: Mmm...in words. $q_{1}$ and $q_{2}$ are different charges, and the $r$ represents the distance between the charges, that is squared. And $K_{e}$ is a constant, and I know its value is nine times ten to power nine if I'm not mistaken.

R: Alright...
S3: And the force is in Newton.

Appendix I: Instructor Solutions to Test Questions

Appendix I1: Instructor solution to question $1 \mathrm{~A}_{1}$
A. 1

Let $q_{1}$ and $q_{2}$ be hoo point charges. Fore between the charge, when kelt at a

$$
\begin{aligned}
& \text { distance (d) aport, } \\
& F=\frac{1}{4 \pi \pi_{0}} \frac{q_{1} q_{2}}{d^{2}} \quad[1] \\
& \begin{array}{l}
\text { Suspire the } 2 \text { rect behreen th hocharss becosis } \\
(F / 3) \text {, when the charges are kept al a distava (x) } \\
\text { [1] }
\end{array} \\
& \text { aport, Then } \\
& \frac{1}{4 \pi \varepsilon 0} \frac{q_{1} q_{2}}{x^{2}}=F / 3 \quad[1] \\
& \text { or } x=\sqrt{3} d \quad[1]
\end{aligned}
$$

## Appendix I2: Instructor solution to question $1 \mathrm{~B}_{2 \mathrm{a}}$

B2
(a)


Noting that $Q_{1}=Q_{2}$

$\left|E_{1}\right|=\left|E_{2}\right|=\frac{9 \times 10^{9} \times 5 \times 10^{-6}}{1.32}=34091 \mathrm{~N} / \mathrm{C}$
$E_{1 x}=-E_{2 x}=E_{1} \cos 30^{\circ} \approx 2 s \sin \mathrm{~N} / \mathrm{c}[2]$
$\Rightarrow E_{2 x}=-23524$
$E_{1 y}=E_{2 y}=E_{1} \sin 30^{\circ} \approx 17045 \sin$ [.2]
$\left.\begin{array}{l|c|c|c} & & \\ \hline x-\operatorname{Eomp} & 29524 & -29524 & 0 \\ \hline y-\text { comp } & 17045.5 & 17045.5 & 34091=E_{R x}\end{array}\right]$

$$
\begin{aligned}
E_{R} & =\left\{E_{R x}^{2}+E_{R y}^{2}\right\} \frac{1}{2} \\
& \left.=\left\{0^{2}+34091\right\}^{2}\right\} 1 / 2 \\
E_{R} & =34091 \mathrm{~N} / \mathrm{C}^{2} \quad \begin{array}{c}
\text { Resultant electric field } \\
\text { at } \\
E
\end{array}[2]
\end{aligned}
$$

Appendix I3: Instructor solution to question $2 \mathrm{~B}_{2 \mathrm{a}}$


## Appendix J: Students Use of Units, Variables and Constants

Appendix J1 Section A questions

| Student | Questions - Section A |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | A1 | A2 | A3 | A4 | A5 |
| V1 | - No units used <br> - Variables substituted <br> - Constant( $\mathrm{k}_{\mathrm{e}}$ ) substituted the $1^{\text {stline }}$ immediately after introduction of formula | - No units used <br> - Variables not substituted <br> - Constants not substituted | - Unit of distance(m) used in the last line, solution ( $8^{\text {th }}$ ) <br> - Variables $\left(q_{1}, q_{2}\right)$ substituted in $3^{\text {rd }}$ line <br> - Constant ( $\mathrm{k}_{\mathrm{e}}$ ) cancelled before being substituted | - Unit of potential difference ( V ) used in the last line ( $5^{\text {th }}$ ) <br> - Variables (m, Vi, q) substituted in penultimate line ( $4^{\text {th }}$ ) <br> - Question requires no constants | - Unit of capacitance (F) used in $8^{\text {th }}$ line. Unit of distance ( m ) used in last line ( $12^{\text {th }}$ ) <br> - Variables (U,V) substituted in $6{ }^{\text {th }}$ line. <br> - Constant $\mathrm{E}_{0}$ substituted in 12th line |
| V2 | - No units used <br> - Variables not substituted <br> - Constants never substituted | - Units of Force $(\mathrm{N})$ used in $3^{\text {rd }}$ line <br> - Variables (m,a) substituted in $1^{\text {st }}$ line immediately after introduction of formula ( $\mathrm{F}_{\mathrm{e}}=$ ma) <br> - Constant ( $\mathrm{k}_{\mathrm{e}}$ ) substituted immediately after the introduction of | - Units of distance (m)used in last line ( $12^{\text {th }}$ ) <br> - Variables $\left(q_{1}, q_{2}\right)$ substituted in $8^{\text {th }}$ line. <br> - Constant ( $\mathrm{k}_{\mathrm{e}}$ ) cancelled before being substituted | - Unit of energy (J) used in last line of first equation ( $6^{\text {th }}$ ). Unit of potential difference V used in last line of second equation (4 ${ }^{\text {th }}$ ) <br> - Variables ( $\mathrm{V}_{\mathrm{f}}$ and $\mathrm{V}_{\mathrm{i}}$ ) Substituted in $5^{\text {th }}$ line of first equation. Variables $\Delta U$ and $q_{0}$ substituted in $2^{\text {nd }}$ line of second equation <br> - Question requires no | - Not done |


|  |  | formula |  | constants |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| V3 | - No units used <br> - Variables not substituted <br> - Constants substituted | - Units of force (N)stated in first line <br> - Variables ( $\mathrm{m}_{\mathrm{e}}$ and g) substituted in the $1^{\text {st }}$ line immediately after introduction of formula. Variables ( $\mathrm{q}_{1}$, $\mathrm{q}_{2}$ ) were substituted in $2^{\text {nd }}$ line immediately after introduction of formula <br> - Constant ( $\mathrm{k}_{\mathrm{e}}$ ) substituted three lines after introduction of formula | - No units used <br> - Variables $\left(q_{1}, q_{2}\right)$ substituted in $6^{\text {th }}$ line after introduction of formula(penultimate) <br> - Constants (Ke) never substituted, cancelled in $2^{\text {nd }}$ line after introduction of formula. | - No units used <br> - No variables substituted <br> - Question requires no constants | - Units of Capacitance ( $F$ ) used in last line. Units of area $\left(\mathrm{m}^{2}\right)$ used in last line. <br> - Variables (U, V,C,d) substituted in $2^{\text {nd }}$ line after introduction of formulae. <br> - Constants $\left(\mathrm{K}, \mathrm{E}_{0}\right)$ substituted in $2^{\text {nd }}$ line after introduction of formula(penultimate) |
| V4 | - No units used <br> - Variables substituted <br> - Constants substituted | - Not done | - Units for charge ( $\mu \mathrm{C}$ ) used in first line <br> - variables $\left(q_{1}, q_{2}\right)$ substituted in the $2^{\text {nd }}$ line after introduction of formula <br> - Constant $\left(\mathrm{k}_{\mathrm{e}}\right)$ substituted in the $2^{\text {nd }}$ line after introduction of formula | - Not done | - No units used <br> - Variable (q) substituted in $1^{\text {st }}$ line immediately after introduction of formula <br> - No constants used |
| V5 | - No single units used <br> - Variables not substituted <br> - Constants substituted | - Units of force(N) used in the $1^{\text {st }}$ line immediately after introduction of formula <br> - Variable (F) substituted in $1^{\text {st }}$ | - Units of distance (m), used in last line ( $11^{\text {th }}$ ) <br> - Variables $\left(q_{1}, q_{2}\right)$ substituted in $3^{\text {rd }}$ line after introduction of formula <br> - Constant $\left(\mathrm{K}_{\mathrm{e}}\right)$ cancelled | - Units of potential difference ( V ) used in last line ( $8^{\text {th }}$ ) <br> - Variables ( $m, v_{i}, q$ ) substituted in $5^{\text {th }}$ line after introduction of formula | - No units used <br> - Variables (U,V) substituted in $2^{\text {nd }}$ line after introduction of formula <br> - Constants $\left(\mathrm{K}, \mathrm{E}_{0}\right)$ not substituted ( question |


|  |  | line, $\left(q_{1}, q_{2}\right)$ in $3^{\text {rd }}$ line after introduction of formula . <br> - Constant ( $\mathrm{K}_{\mathrm{e}}$ ) substituted in $3^{\text {rd }}$ line after introduction of formula | in $3^{\text {rd }}$ equation, the first time they were introduced | - Question requires no constants | not completed) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| M1 | - Units for distance(m) used in last line of solution <br> - Variables (F, q1,q2) substituted in $2^{\text {nd }}$ line after introduction of formula <br> - Constant ( $\mathrm{K}_{\mathrm{e}}$ ) substituted in $2^{\text {nd }}$ line after introduction of formula | - Units of $g\left(\mathrm{~m} / \mathrm{s}^{2}\right)$ used in the penultimate line <br> - Variables (F,q) substituted in the $3^{\text {rd }}$ line after introduction of formula <br> - Constant ( $\mathrm{K}_{\mathrm{e}}$ ) substituted in $3^{\text {rd }}$ line after introduction of formula | - Units for distance(m) used in last line of solution <br> - Variables (q1,q2) substituted in $2^{\text {nd }}$ line after introduction of formula <br> - Constant ( $\mathrm{k}_{\mathrm{e}}$ ) not substituted( cancelled out) | - Not done | - Units of potential difference ( V ) used in last line <br> - Variables (F, d) substituted in $1^{\text {st }}$ line after introduction of formula <br> - Constants $\left(\mathrm{K}, \mathrm{E}_{0}\right)$ not substituted |
| M2 | - Units for distance ( $m$ ) used in last line of solution <br> - Variables (F, q1,q2) substituted in $1^{\text {st }}$ line immediately after introduction of formula <br> - Constant ( $\mathrm{K}_{\mathrm{e}}$ ) substituted in $1^{\text {st }}$ line after introduction of formula | - Units for distance(m) used in last line of solution <br> - Variables (F, q1,q2) substituted in $1^{\text {st }}$ line immediately after introduction of formula <br> - Constant ( $\mathrm{K}_{\mathrm{e}}$ ) substituted in $1^{\text {st }}$ line after introduction of formula | - Units for distance(m) used in last line of solution <br> - Variables (q1,q2) substituted in $5^{\text {th }}$ line after introduction of formula <br> - Constant ( $\mathrm{k}_{\mathrm{e}}$ ) not substituted(cancelled out in $1^{\text {st }}$ line after introduction of formula) | - Units of potential difference (V) used in line of solution <br> - Variables ( $m_{e, ~}, \mathrm{q}_{\mathrm{v}}$ ) substituted in $5^{\text {th }}$ line after introduction of formula <br> - No constant used in solution | - Units (C,F, $\mathrm{m}^{2}$ )used in last line of each solution <br> - Variables ( $\Delta \mathrm{v}, \mathrm{d}, \mathrm{q}, \mathrm{u}, \mathrm{A}$ ) substituted in $1^{\text {st }}$ line after introduction of formulae <br> - Constants ( $k, \mathrm{E}_{0}$ ) substituted in $2^{\text {nd }}$ line after introduction of formula |


| M3 | - Units for distance(m) used in last line of solution <br> - No Variables substituted <br> - Constant ( $\mathrm{m}_{\mathrm{e},}$ a) substituted in $1^{\text {st }}$ line immediately after introduction of formula | - Units for distance(m) used in last line of solution <br> - Variables (F, q1,q2) substituted in $1^{\text {st }}$ line immediately after introduction of formula <br> - Constant ( $\mathrm{K}_{\mathrm{e}}$ ) substituted in $1^{\text {st }}$ line immediately after introduction of formula | - Not done | - Units of energy(J) used in last line of solution <br> - Variables (q,v) substituted in $1^{\text {st }}$ line immediately after introduction of formula <br> - No constant used | - Units for distance(m) used in last line of solution <br> - Variables (c,d) substituted in $2^{\text {nd }}$ line after introduction of formula <br> - Constants ( $\mathrm{k}, \mathrm{E}_{0}$ ) substituted in $2^{\text {nd }}$ line after introduction of formula |
| :---: | :---: | :---: | :---: | :---: | :---: |
| M4 | - No units used <br> - No variables substituted <br> - No constants substituted | - Units of force <br> $(\mathrm{N})$ used 4 lines before the last line. Units for distance(m) used in last line of solution <br> - Variables (F, q1,q2) substituted in $4^{\text {th }}$ line after introduction of formula <br> - Constant ( $\mathrm{K}_{\mathrm{e}}$ ) substituted in $4^{\text {th }}$ ${ }^{\text {t }}$ line after introduction of formula | - Units of charge $(\mu \mathrm{C})$ used in $4^{\text {th }}$ line after introduction of formula, seven lines before last line <br> - Variables ( $\mathrm{q} 1, \mathrm{q} 2$ ) substituted in $4^{\text {th }}$ line after introduction of formula <br> - Constant ( $\mathrm{k}_{\mathrm{e}}$ ) not substituted(cancelled out in $1^{\text {st }}$ line after introduction of formula) | - Units for potential difference(v) only used in last line of solution <br> - Variables (m,vi, q) substituted in $6^{\text {th }}$ ) line after introduction of formula <br> - No Constant used | - Units ( $\mathrm{C}, \mathrm{m}^{2}$ ) used in last line of each solution <br> - Variables (u,v) substituted in $1^{\text {st }}$ line immediately after introduction of formula. Variables (C,d, $\mathrm{E}_{0}$ ) substituted in $4^{\text {th }}$ line after introduction of formula <br> - Constant ( $\mathrm{E}_{0}$ ) substituted in $4^{\text {th }}$ line after introduction of formula |
| M5 | - Units for distance(m) only used in last line of solution | - Units ( $\mathrm{Nm}^{2} \mathrm{c}^{2}$, $\mathrm{kg}, \mathrm{m} / \mathrm{s}^{2}$ ) used in $1^{\text {st }}$ line | - Units for distance(m) used in last line of solution | - Units for energy (j) used in $6^{\text {th }}$ line after | - Units for area $\left(\mathrm{m}^{2}\right)$ used in last line of solution <br> - Variables (d,u,r) |


|  | - Variables not substituted(cancelled out) <br> - Constant not substituted(cancelled out) | immediately after introduction of formula, 2 lines before the last line <br> - Variables (F, q1,q2) substituted in $1^{\text {st }}$ line immediately after introduction of formula <br> - Constant ( $\mathrm{K}_{\mathrm{e}}$ ) substituted in $1^{\text {st }}$ line immediately after introduction of formula | - Variables (q1,q2) substituted in $6^{\text {th }}$ line after introduction of formula <br> - Constant ( $\mathrm{k}_{\mathrm{e}}$ ) not substituted(cancelled out immediately at introduction of formula) | introduction of formula, three lines before last line. Units of potential difference (V) used in last line. <br> - Variables (m,vi, vf) substituted in $1^{\text {st }}$ line after introduction of formula <br> - No constant used | substituted in the $1^{\text {st }}$ line immediately after introduction of formula <br> - Constants <br> ( $k, \mathrm{E}_{0}$ ) substituted in $1^{\text {st }}$ line immediately after introduction of formula |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |

Appendix J2: Section B questions

| Student | Questions - Section B |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | B1 | B2 | B3 | B4 | B5 |
| V1 | - Units(N) used in the last line of each solution for all the questions <br> - Variables $\left(q_{1}, q_{2}\right.$, $r_{2}$ ) substituted in the $2^{\text {nd }}$ line following introduction of the formula <br> - Constants $\left(\mathrm{K}_{\mathrm{e}}\right)$ substituted in the $2^{\text {nd }}$ line following introduction of the formula | - Units (N/c) used in the last line of each solution <br> - Variables $\left(\mathrm{q}_{1}, \mathrm{q}_{2}\right.$, $q_{3}, r$ ) substituted in the $1^{\text {st }}$ line immediately following the introduction of the formula <br> - Constant $\left(\mathrm{k}_{\mathrm{e}}\right)$ substituted in the $1^{\text {st }}$ line immediately following the introduction of the formula | - Not done | - Units (F, C, J, V/m, $\mu \mathrm{F}$ ) only used in the last line of each solution <br> - Variables (A, d, C, V, $\mathrm{E}, \mathrm{C}_{\text {eq }}$ ) substituted in the $1^{\text {st }}$ line immediately following introduction of formula. <br> - Constant $\left(\mathrm{E}_{0}\right)$ substituted in the $1^{\text {st }}$ line immediately following the introduction of the formula | $\begin{aligned} & \text { - Not } \\ & \text { done } \end{aligned}$ |
| V2 | - Units (C)only used with final answer <br> - Variables ( $\mathrm{q}_{1}, \mathrm{q}_{2}$ ) substituted in the $3^{\text {rd }}$ line following introduction of formula. Variable( $r$ )substituted in the $2^{\text {nd }}$ line following introduction of formula <br> - Constant ( $\mathrm{k}_{\mathrm{e}}$ ) substituted in the $3^{\text {rd }}$ line following | - Units (N/C) only used in last line of solution <br> - Variables (q, r) substituted in the $1^{\text {st }}$ line immediately following introduction of formula. <br> - Constant ( $\mathrm{k}_{\mathrm{e}}$ ) substituted $1^{\text {st }}$ line immediately following introduction of formula | - Not done | - Units (F, C, J,) used in the last line of each solution <br> - Variables (A, d, C, $\Delta V, Q$, , substituted in the $1^{\text {st }}$ line immediately following introduction of formula. <br> - Variables (A, d, C, $\Delta V, Q$, , substituted in the $1^{\text {st }}$ line immediately following introduction of | - Not done |


|  | introduction of formula. | - |  | formula. |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| V3 | - No units used <br> - Variables (q1 and q2) not substituted. Variable r substituted into the $1^{\text {st }}$ line after introduction of formula. <br> - Constant (K) substituted in the $3^{\text {rd }}$ line following introduction of formula | - No units used <br> - No variables substituted <br> - No constant substituted | - Not done | - Units (F, C, J,q) used in the last line of each solution <br> - Variables (A, d, C, $\Delta V, Q, q)$ substituted in the $1^{\text {st }} t$ line immediately following introduction of formula. <br> - Variables (A, d, C, $\Delta \mathrm{V}, \mathrm{Q}$ ) substituted in the $1^{\text {st }}$ line immediately following introduction of formula. | - Not done |
| V4 | - Units ( $\mu \mathrm{C}$ ) only used in the last line of the solution <br> - Variables (q1 , q2) not substituted <br> - Constant ( $\mathrm{k}_{\mathrm{e}}$ ) substituted in $3^{\text {rd }}$ line following introduction of formula | - No units used <br> - No variables substituted <br> - No constant substituted | - No units used <br> - Variables (Q, r, q, $\Delta \mathrm{E}$ ) substituted in the $1^{\text {st }}$ line immediately following introduction of formulae. <br> - Constant (k) substituted in the $1^{\text {st }}$ line immediately following introduction of formula | - Not done | - Not done |
| V5 | 1. Units of force (N) used in last line of each solution <br> - Variables $\left(q_{1}, q_{2}\right.$, | - Units (C/m2) used in the last line of each solution <br> - Variables $\left(q_{1}, q_{2}\right.$, | - No units used <br> - Variables (u, q) substituted in the $1^{\text {st }}$ line immediately | - Not done | - Not done |


|  | r) substituted in the $1^{\text {st }}$ line immediately following introduction of formula <br> - Constant ( $\mathrm{k}_{\mathrm{e}}$ ) substituted in the $1^{\text {st }}$ line immediately following introduction of formula | $r_{2}$ ) substituted in the $1^{\text {st }}$ line immediately following introduction of formula <br> - Constant ( $\mathrm{k}_{\mathrm{e}}$ ) substituted in the $1^{\text {st }}$ line immediately following introduction of formula | following introduction of formulae. <br> - Constant ( $\mathrm{k}_{\mathrm{e}}$ ) substituted in the $1^{\text {st }}$ line immediately following introduction of formula |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| M1 | - Units (N/m) used in the penultimate line <br> - Variables (q, r) substituted in the $1^{\text {st }}$ line immediately following the introduction of the formula <br> - Constant $\left(\mathrm{k}_{\mathrm{e}}\right)$ substituted in the $1^{\text {st }}$ line immediately following the introduction of the formula | - Units ( $\mathrm{m}, \mathrm{N}$, $N / C$ ) used in the $1^{\text {st }}$ line of solution <br> - Variables (q, r) substituted in the $1^{\text {st }}$ line immediately following the introduction of the formula <br> - Constant( $\mathrm{k}_{\mathrm{e}}$ ) substituted in the $1^{\text {st }}$ line immediately following the introduction of the formula | - Not done | - Units ( F,C, KJ, N/M used in last line of solution <br> - Variables (A, d, C, $\Delta \mathrm{V}, \mathrm{q}, \mathrm{r})$ substituted in the $1^{\text {st }}$ line immediately following the introduction of formulae <br> - Constant $\left(\mathrm{E}_{0}, \mathrm{k}_{\mathrm{e}}\right)$ substituted in the $1^{\text {st }}$ line immediately following the introduction of the formula | - Not done |
| M2 | - Units (C ) used in last line of the solution <br> - Variables $\left(\mathrm{q}_{1}, \mathrm{q}_{2}, \mathrm{r}\right)$ | - Units (N/m) used in last line of the solution <br> - Variables (q, r) | - Not done | - Units ( F,C, J, V/M, $\mu$ F) used in last line of the solution <br> - Variables (A, d, C, | - Not done |


|  | substituted in the $1^{\text {st }}$ line immediately following the introduction of formula <br> - Constant( $\mathrm{k}_{\mathrm{e}}$ ) substituted in the $1^{\text {st }}$ line immediately following the introduction of the formula | substituted in the $1^{\text {st }}$ line immediately following the introduction of formula <br> - Constant( $\mathrm{k}_{\mathrm{e}}$ ) substituted in the $1^{\text {st }}$ line immediately following the introduction of the formula |  | $\Delta \mathrm{V}, \mathrm{Q}, \mathrm{r})$ substituted in the $1^{\text {st }} t$ line immediately following the introduction of formulae <br> - Constant $\left(\mathrm{E}_{0}\right.$, ) substituted in the $1^{\text {st }}$ line immediately following the introduction of the formula |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| M3 | - Not units used <br> - Variables $\left(\mathrm{q}_{1}, \mathrm{q}_{2}, \mathrm{r}\right)$ substituted in the $1^{\text {st }}$ line immediately following the introduction of formula <br> - Constant $\left(\mathrm{k}_{\mathrm{e}}\right)$ substituted in the $1^{\text {st }}$ line immediately following the introduction of the formula | - Not units used <br> - Variables (q, r) substituted in the $1^{\text {st }}$ line immediately following the introduction of formula <br> - Constant ( $\mathrm{k}_{\mathrm{e}}$ ) substituted in the $1^{\text {st }}$ line immediately following the introduction of the formula | - Not done | - Units (C, J ) used in line of the solution <br> - Variables (A, d, C, V, $q, r$ ) substituted in the $1^{\text {st }}$ line immediately following the introduction of formulae <br> - Constant( $\left.\mathrm{E}_{0}, \mathrm{k}_{\mathrm{e}}\right)$ substituted in the $1^{\text {st }}$ line immediately following the introduction of the formula | - Not done |
| M4 | - Not done | - Units (m, N/c) used in last line of the solution <br> - Variables (q, r) substituted in the $1^{\text {st }}$ line immediately following the | - Units (v) used in last line of the solution <br> - Variables (Q, L ) substituted in the $1^{\text {st }}$ line immediately following the introduction of the formula | - Units ( $\mathrm{pF}, \mathrm{nC}, \mathrm{J}, \mathrm{V} / \mathrm{m}$, $\mu \mathrm{F}, \mathrm{V}$ ) used in line of the solution <br> - Variables (A, d, C, $\mathrm{V}, \mathrm{Q}, \mathrm{C}_{\text {eq }}$ ) substituted in the $1^{\text {st }}$ line immediately following | - Not done |


|  |  | introduction of formula <br> - Constant( $\mathrm{k}_{\mathrm{e}}$ ) substituted in the $1^{\text {st }}$ line immediately following the introduction of the formula | - Constant( $\mathrm{k}_{\mathrm{e}}$ ) substituted in the $1^{\text {st }}$ line immediately following the introduction of the formula | the introduction of formulae <br> - Constant $\left(\mathrm{E}_{0}\right)$ substituted in the $1^{\text {st }}$ line immediately following the introduction of the formula |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| M5 | - Not done | - Units (m, N/c) used in last line of the solution <br> - Variables (q, r) substituted in the $1^{\text {st }}$ line immediately following the introduction of formula <br> - Constant $\left(\mathrm{k}_{\mathrm{e}}\right)$ substituted in the $1^{\text {st }}$ line immediately following the introduction of the formula | - Units (V,J) used in last line of solution <br> - Variables ( Q, L )substituted in the $1^{\text {st }}$ line immediately following the introduction of the formula <br> - Constant $\left(\mathrm{k}_{\mathrm{e}}\right)$ substituted in the $1^{\text {st }}$ line immediately following the introduction of the formula | - Units ( $\mathrm{F}, \mathrm{nC}, \mathrm{J}, \mathrm{V} / \mathrm{m}$, $\mu F, C, V)$ used in line of the solution <br> - Variables (A, d, C, V, $Q, C_{\text {eq }}$ ) substituted in the $1^{\text {st }}$ line immediately following the introduction of formulae <br> - Constant $\left(\mathrm{E}_{0}\right.$, ) substituted in the $1^{\text {st }}$ line immediately following the introduction of the formula | - Not done |

## Appendix K: Students' Solutions to Questions

APPENDIX K1: Student V2 on question $1 \mathrm{~A}_{1}$


APPENDIX K2: Student $\mathrm{M}_{1}$ on Question $1 \mathrm{~A}_{1}$


APPENDIX K3: Student V3 on question 1B $\mathrm{Ba}_{2}$


APPENDIX K4: Student V4 on question $1 \mathrm{~B}_{2 \mathrm{a}}$


APPENDIX K4: Student V4 on question 1B $\mathrm{a}_{2 \mathrm{a}}$ (cont.)
BR.
(a)


$$
\vec{E}_{1}=k e \frac{Q_{1}}{r^{2}}
$$

$$
=25826.5 \mathrm{C} / \mathrm{m}^{2}
$$

$$
\begin{aligned}
& c^{2}=a^{2}+b^{2} \\
& 2^{2}=a^{2}+1^{2} \\
& 2 a^{2}=4-1 \\
& \sqrt{a^{2}}=\sqrt{3} \\
& a=\sqrt{3}
\end{aligned}
$$

$$
\begin{aligned}
=\frac{\left(9 \times 10^{9} \times 5 \times 10^{-6}\right)}{\left(1.32^{2}\right)} & \therefore \frac{\sqrt{3}}{2}
\end{aligned} \quad=0.86 \mathrm{~m}, ~ \approx 0.9 \mathrm{~m}
$$

$$
\begin{aligned}
& r^{2}=0.86^{2}+1^{2} \\
& r^{2}=1.32 \mathrm{~m}
\end{aligned}
$$

$$
\vec{E}_{2}=k_{e} \frac{Q_{2}}{r^{2}}
$$

X-Components

$$
\begin{aligned}
=\frac{\left(9 \times 10^{9} \times 5 \times 10^{-6}\right)}{\left(1.32^{2}\right)} \quad & E_{1 x}-E_{2 x}=E_{x} \\
& 25826.5 \operatorname{Cos} \theta-25826.5 \cos \theta=0
\end{aligned}
$$

$$
=25826.5 \mathrm{c} / \mathrm{m}^{2} \quad \therefore E_{x}=0
$$

$$
\begin{aligned}
& y \text {-components } \\
& E_{y}=E_{4 y}+E_{2 y} \\
& =25826.5 \sin \theta+25826.5 \sin \theta \\
& =2\left(25826,5 \operatorname{Sin} 60^{\circ}\right) \\
& E_{y}=44732.8 \mathrm{c} / \mathrm{m}^{2} \\
& \begin{aligned}
\text { Net electric field } & =\sqrt{E_{x}^{2}+E_{V}^{2}} \\
& =\sqrt{44732.8}
\end{aligned} \\
& \operatorname{Artan} \theta=\operatorname{artan}\left(\frac{4473}{0}\right. \\
& \text { erection }=\theta=90^{\circ}
\end{aligned}
$$

## APPENDIX K4: Student V4 on question $1 \mathrm{~B}_{2 \mathrm{a}}$ (cont.)




[^0]:    Copyright 2014 Turnitin. All rights reserved.

[^1]:    "The unipolar/sink model; the clashing currents model; the weakening current model; the shared current model (Osborne \& Freyberg, 1985; Koumaras et al., 1990; Driver et al., 1994; Borges \& Gilbert, 1999; Koltsakis \& Pierratos, 2006); the sequence model (Shipstone, 1984; Engelhardt \& Beichner, 2004); the local reasoning model (Cohen, Eylon \& Ganiel, 1983; Heller \& Finley, 1992); the short circuit model

[^2]:    ${ }^{1}$ Practically, the selection could only be done after the scripts were marked, whether a student got a question right or wrong was not a factor for consideration. Therefore the marking that appears on the students scanned scripts should be ignored as it did not inform any part of the analysis.

[^3]:    ${ }^{2}$ Epistemology is defined as a science that is concerned with knowledge acquisition and the act of knowing (Piaget, 1972). For students, learning is an epistemological practice. Epistemological frames therefore are what guides students to adopt particular learning cultures. These frames are important to unpack in this study, so as to put students' work when solving electricity problems into relevant context.

[^4]:    ${ }^{3}$ Fizo, Fizi are pseudonyms

[^5]:    ${ }^{4}$ Webct is the University e- learning site where teaching materials are placed for students to access.

