INSIGNIFICANT DIFFERENCES: THE PARADOX OF THE HEAP

by

WILLIAM EDWARD BRONNER

submitted in fulfillment of the requirements for the degree of

MASTER OF ARTS

in the subject

PHILOSOPHY

at the

UNIVERSITY OF SOUTH AFRICA

SUPERVISOR: EB RUTTKAMP
JOINT SUPERVISOR: SJ ODELL

MAY 2004

****

SUMMARY

Title of dissertation:

INSIGNIFICANT DIFFERENCES: THE PARADOX OF THE HEAP

Summary:

This study investigates six theoretical approaches offered as solutions to the paradox of the heap (sorites paradox), a logic puzzle dating back to the ancient Greek philosopher Eubulides. Those considered are: Incoherence Theory, Epistemic Theory, Supervaluation Theory, Many-Valued Logic, Fuzzy Logic, and Non-Classical Semantics. After critically examining all of these, it is concluded that none of the attempts to explain the sorites are fully adequate, and the paradox remains unresolved.

Key terms:

Eubulides, heap, paradox, sorites

CONTENTS

CHAPTER 1: INTRODUCTION

1-1) THE PARADOX

The paradoxical form of argumentation known as the <u>sorites</u>, or paradox of the heap, is an ancient logic puzzle dating back to the Greek philosopher Eubulides, a contemporary of Aristotle. The general form of the argument is as follows:

S[1]: One grain of sand is insufficient to constitute a heap.

S[2]: If one grain of sand is insufficient to constitute a heap, then two grains of sand are insufficient to constitute a heap.

S[3]: Therefore, two grains of sand are insufficient to constitute a heap.

S[4]: If two grains of sand are insufficient to constitute a heap, then three grains of sand are insufficient to constitute a heap.

S[5]: Therefore, three grains of sand are insufficient to constitute a heap.

.

.

S[2(10^{12})-1]: Therefore, a trillion grains of sand are insufficient to constitute a heap.   <1*1>

In more condensed form the paradox may be expressed simply as:

P[1]: One grain of sand is insufficient to constitute a heap.

P[2]**:** Adding a single grain of sand to a non-heap is not sufficient to

form a heap.

Therefore, no quantity of sand is sufficient to constitute a heap.

<1*2>

The problem here, of course, is how to account for such an implausible result given what looks like an otherwise compelling argument. For even though the conclusion appears to follow directly from obviously true premises, the idea that no quantity of sand is sufficient to constitute a heap is just too incredible to be regarded as correct. Consequently, it seems untenable either to accept or reject the inference. How, then, is it possible to resolve this dilemma? Many, if not all, logicians consider the sorites unsound, and its conclusion false. (In fact, the paradox is closely associated with the slippery slope fallacy in informal logic.) Nonetheless, there is little agreement as to where, exactly, the argument goes wrong. That the argument is flawed is more readily apparent than why it is.

1-2) EARLY HISTORICAL DEVELOPMENTS

Since its initial formulation over two millennia ago, the sorites paradox received rather limited attention until the period beginning at the end of the nineteenth century, when a renewed interest in the sorites finally began to emerge. Eventually, by the latter part of the twentieth century, interest in, and discussion of, this topic reached historic levels. Sustained attempts at solution have, by now, resulted in an appreciable body of literature on the topic. At present, the volume of material available to consult on the subjects of vagueness, in general, and the sorites paradox, in particular, is several times what it had been just a few decades ago.

Before focusing on these largely recent contributions to the theory of the paradox, however, it is worth conducting a brief survey of the main historical developments that led up to these ideas. Those interested in a more in-depth, detailed account are referred to the first three chapters in Timothy Williamson's book Vagueness (Williamson, 1994). These three introductory sections include discussions of (1) the early history of sorites paradoxes (the first sorites; Chrysippan silence; sorites arguments and Stoic logic; the sorites in later antiquity; the sorites after antiquity); (2) the ideal of precision (the emergence of vagueness; Frege, Pirece, and Russell); and (3) the rehabilitation of vagueness (vagueness and ordinary language; the Black-Hempel debate; family resemblances; open texture). What follows is a short summary of those historical events.

As already stated, formulation of the sorites paradox dates back to ancient Greece. (The term sorites, itself, is literally from the Greek term for heap.) Invention of the argument known as the 'paradox of the heap' is generally attributed to Eubulides of Miletus (also famous for several other paradoxes), although the historical record is too incomplete to guarantee that the sorites definitely originated with him. An earlier and somewhat similar puzzle (the Millet Seed) had been constructed by Zeno of Elea about a century before. The 'Bald Man' paradox, also ascribed to Eubulides, is essentially the same puzzle as the heap paradox, but considers how few hairs are required to make a bald man bald.

The sorites evidently served as a considerable source of division between two of the rival ancient Greek schools of philosophy: the Academy (the skeptics) and the Stoa (the Stoics). The paradox, having no readily apparent solution, was well suited to the skeptics of the Academy who, simply by inquiring, "Is one few? Are two few? Are three few? ..." made difficult any non-skeptical reply. Consequently, the sorites functioned as a convenient means to effectively undermine opposing philosophies. According to

Williamson (1994**:** 12-22), though, the Stoics hit upon a successful strategy for avoiding the intended skeptical implications of this line of questioning. Due in part, at least, to their acceptance of the Principle of Bivalence ('Every statement is either true or false'), the Stoics, it seems, were of necessity committed to the existence of a sharp cut-off between heaps and non-heaps. But this did not, in turn, require them to actually identify where that cut-off point would be located. Since, in general, assertions of fact are true or false independently of our knowing (or even our ability to know) whether they are actually true or false, mere lack of knowledge does not compel one to deny that there is some objective fact of the matter. Just as someone can know that the number of stars in the universe is either even or odd without knowing which, a person can, likewise, one might think, also be confident that a sharp boundary does exist between heaps and non-heaps without, thereby, also having the ability to locate that boundary. This, Williamson (1994, ch. 1) argues, is just what the Stoics came to conclude. Being ignorant as to the exact whereabouts of the division between heaps and non-heaps need not imply that a sharp cut-off does not, nonetheless, somewhere exist. There is not, therefore, necessarily any real inconsistency in holding both that a precise demarcation between heaps and non-heaps does exist, and, yet, that we might have no means of finding it. As early proponents of the view that vague concepts do, indeed, have precise, yet unknown boundaries, the Stoics succeeded in establishing one of the major theories regarding the nature of the sorites paradox.

Not until Gottlob Frege first began publishing on the subject of vagueness at the end of the nineteenth century (Williamson, 1994, sec. 2.2) was there again some major movement toward addressing the problems raised by the sorites. As Williamson (1994**:** 36) remarks**:**

"Only two traditions in the history of philosophy have found a serious

problem in sorites paradoxes. One culminated in Stoicism; the other is modern analytic philosophy. The reason has to do with the centrality of formal logic in both traditions. The conclusiveness of formal proof makes a sorites paradox hard to fudge. Its premises once granted, its disastrous conclusion seems inescapable.... [T]he rise of modern logic at the end of the nineteenth century caused sorites paradoxes to become again a topic of philosophical discussion after two thousand years".

Frege, followed by Charles Sanders Peirce and Bertrand Russell in the early twentieth century, all viewed vague language as an impediment to a fully functional logic, something that, if left unchecked, was quite destructive in its consequences. Their common response was to reject vagueness as, to some degree, illegitimate. Frege, in fact, considered it an absolute requirement that a concept have a sharp boundary since, otherwise, in his judgement, it was not a concept at all (Williamson, 1994, sec. 2.2). Thus, Frege espoused a radical wholesale rejection of vagueness, suggesting a prohibition on vague predicates was in order. In his view, sorites paradoxes are symptomatic of just what happens when vague predicates are permitted.

Peirce, in contrast, was less inclined to repudiate vague language entirely (Williamson, 1994, sec. 2.3). It was his belief that much of the time imprecise concepts in everyday discourse function quite adequately despite their ultimate lack of sharp boundaries. Inexact terms are typically sufficient for the task at hand, and it is rarely the case that highly precise terms are really needed. It is only when circumstances dictate otherwise, when additional precision is truly required that we are, in fact, compelled to sharpen our definitions. If not, no refinement to our language is necessary. One can draw the obvious conclusion that the sorites is, indeed, one of the exceptional cases. It is one instance where imprecise concepts fail us, and it demonstrates why

vague terms cannot be granted unrestricted use.

Despite these earlier works on the subject, it was not until Russell's 1923 paper on vagueness (Russell, 1923) that some significant effort aimed at resolving the sorites finally began to develop. In the years immediately following this influential paper, a few contributors made additions to what was still a rather limited body of literature on the topic until when, in the 1970's, discussion of the sorites reached a historically unparalleled level. It was during this period of dramatically increased publication in the 1970's and, indeed, the few decades that have since followed, that numerous suggested solutions to the sorites have been offered, and their validity subsequently debated and challenged. Russell, therefore, it seems, deserves some substantial degree of credit for reigniting interest in a subject that had been substantially dormant for a long length of time. As Keefe and Smith (1997: 1) point out regarding the historic place of Russell's writings on vagueness, "... although there was interest in the paradox in antiquity, there seems to have been relatively little further discussion of vagueness before Bertrand Russell's seminal paper [of 1923] ...". Williamson (1994: 70) adds to this:

> "Russell's paper of 1923 on vagueness raised many of the issues central to the subsequent debate. Its effects, however, were delayed. Nothing of significance, and little of insignificance, was published on vagueness between Russell's work and that of Max Black in 1937. The latter provoked several responses, and the topic has remained alive, if not always kicking, ever since".

Thus, even though Russell's writings may have seemed to have no immediate impact at the time, and his ideas on the sorites may have contributed only marginally in the way of ultimately resolving the paradox, Russell's historic contribution was, nonetheless, evidently of considerable significance in the long

term.

In theoretical terms, Russell's view of vagueness was a continuation of what had generally been propounded earlier by Frege and Peirce. Restricting the use of inexact terms is not only desirable, but in some cases absolutely necessary. In particular, Russell held that the Law of the Excluded Middle does not apply for vague expressions. That is, a disjunction is not true unless one of its disjuncts is true, and conversely, if a disjunction is true then so, too, must one of its disjuncts be true. But since vague expressions admit borderline cases, there will be instances in which neither disjunct can be established to be true, and, consequently, for which the Law of the Excluded Middle cannot be said to hold. If, for example, Smith were borderline tall, being neither clearly tall, nor clearly not-tall, then the assertion:

Smith is either tall or he is not.   <1*3>

would, according to Russell, not be a necessary truth. In fact, it would not be true, at all. But in classical logic, 'Q or not-Q' always comes out true. Accordingly, vague expressions are exceptional in that the rules of logic that apply elsewhere do not pertain to instances like this one. Vague terms are, indeed, somehow illegitimate in a way that precise terms are not, and cannot be. Moreover, Russell suggested no means by which one might reconcile vagueness with the laws of logic. He found no satisfactory way to allow for vagueness while still retaining logic, nor a way to somehow modify logic to accommodate vagueness that would be in any way legitimate or defensible. His answer to the sorites, therefore, was to disallow use of the logical form of the argument whenever vague predicates are involved. Even though the sorites might seem valid when imprecise terms are used because, unlike exact terms, there is no clear step from true premises to false ones, the overall argument, nonetheless, turns out to be invalid. This is simply one feature of vague

language that stems from its general failure to follow the most fundamental principles of classical logic.

In 1937 Max Black (1937) published a paper entitled *Vagueness: An Exercise in Logical Analysis*. Black suggests it is fundamentally mistaken to treat vague expressions, which are actually expressions of degree, as though they were all-or-nothing terms. Rather than inaccurately asserting, for example, that:

Smith is tall.   <1*4>

in cases where Smith is not clearly tall, but is merely borderline tall, the correct approach would be simply to state explicitly and directly the degree to which Smith is tall. Black went on to propose, moreover, that this assignment of degree be accomplished on a statistical basis. That is, the extent to which a vague predicate truly applies depends on what language users as a whole affirm as appropriate. Black's view of vagueness as a matter of degree (although not necessarily his statistical methodology) remains one of the main theoretical approaches to the sorites.

The main criticism of Black's proposal was provided by Carl Hempel a short time later in 1939 (Williamson, 1994, sec. 3.2). The major weakness in Black's analysis, it seems, is simply the fact that vague predicates are, in practice, consistently used in an all-or-nothing fashion, not as attributes present to only a partial degree. Thus, treating these terms in any other way appears dubious at the outset. Just in terms of our ordinary linguistic practice, itself, we should have some immediate concerns regarding the validity of Black's analysis and his suggestion that vague terms be treated somehow differently than exact ones. In particular, it is doubtful that vagueness really matters to logic, i.e., whether the validity of a specific logical form truly depends upon whether vague concepts are being used. If that were the case, one could

argue that a separate logic is needed for vague terms, one different from the classical logic used for exact terms. But that looks like an extreme measure, particularly given what seems like an indisputable observation with regard to how vague predicates operate. As Williamson (1994: 80) puts it:

> "'The sun is hot' designates the state of affairs that the sun is hot to no degree less than 1, and is therefore not an exact translation of any sentence which does designate that state of affairs to a degree less than 1.... Hempel denies that designation is gradable. There is no semantical concept of vagueness, so vagueness does not invalidate classical logic".

In Hempel's view, Black mistakenly included among the normative practices characteristic of vague language a property that is merely a descriptive (though authentic) feature of it, namely, partial application, a distinction subsequently noted by Sainsbury (1991: 177), as well. In point of fact, the full but inconsistent use of a predicate (a property distinctive of vague terms applied to borderline cases) is far different from the consistent but partial application proposed by Black. Thus, Black's degree theory does not follow in any obviously direct way from how language functions in unclear cases such as <1*4>. Moreover, the disagreement between Hempel and Black relates to yet another controversial but even broader issue regarding language: how use relates to meaning. Again quoting Williamson (1994: 83):

> "The debate raises wider issues about the relation between meaning and use. Black attempted to reduce semantic descriptions to descriptions of use. Semantic descriptions are normative; they classify uses as correct or incorrect. Black's descriptions of use were non-normative. In effect, he was trying to reduce the normative to the non-

normative. His attempt failed. Arguably, any such attempt must fail.
The attempt to connect meaning and use is hard to sustain unless use
is already described in normative terms, something later philosophers
of ordinary language were willing to do. Their notion of 'what we say'
was a notion of correct usage in our community, not a matter of
statistics. To invoke non-normative descriptions of use in analyzing
vagueness, as in statistical accounts of degree of truth, is to
misconceive the relation between meaning and use".

Thus, a difficulty raised by Black's proposed solution is whether it comports
with any valid theory of meaning. How that question gets answered
determines, in part, at least, how feasible his theory is, and how seriously we
should consider it.

The Black-Hempel debate brings us up to almost the midpoint of the
twentieth century. This is a convenient place to conclude this brief survey of the
historical developments surrounding the sorites, not because it is at this stage
complete, but because details of the still more substantial debate to come in
the latter half of the century are more easily discussed on a topical basis.
Nonetheless, in surveying the rather slow progress made up to here, three of
the six major theories regarding the sorites have already been introduced:
Epistemic Theory (the Stoics), Incoherence Theory (Frege, Peirce, and
Russell), and Fuzzy Logic (Black). These remain to the present day leading
candidates among proposed solutions to the paradox, each deserving serious
consideration. Before proceeding with a critical examination of these and the
three other main theoretical approaches to the paradox, however, a few
additional preliminary topics needed to complete the background on the sorites
have yet to be discussed.

1-3) PURSUING AN ANSWER

After some two thousand years of searching for a solution to the sorites, a variety of imaginative and intriguing attempts at resolving the paradox have been suggested. In examining and debating each of these, however, philosophers have, as yet, failed to arrive at any consensus regarding which, if any, proposed solution is the correct one. In fact, far from settling the issue, the collective effort seems to have only broadened the dispute while producing no clear-cut result. Instead, scholars continue to disagree whether any solution offered truly provides some genuine insight into how to resolve the paradox.

Given this lack of agreement, an accurate assessment of the current situation requires taking a closer look at each of the main theoretical accounts of the sorites, and why these have remained as problematic as they are. To accomplish this, six theories on the subject (one ancient, five relatively recent, and all, arguably, the most important and influential ones to date) will be examined in some detail. These are: Incoherence Theory, Epistemic Theory, Supervaluation Theory, Many-Valued Logic, Fuzzy Logic, and Non-Classical Semantics. As subsequent analysis will show, however, none of these explanations are entirely acceptable. At least some of these permit the sorites to be reconstructed in somewhat modified forms that resist solution, while others raise troubling logical questions of their own that are not easily answered. In fact, it is a principal aim of the present communication to show that all of these attempts are, in some way, seriously flawed and that they, subsequently, fail to adequately resolve the paradox. If this conclusion is, indeed, correct, then, as matters presently stand, the most serious attempts at solution remain unsatisfactory, and considerable difficulties still persist in providing a definitive answer to the sorites. This suggests that any final resolution of the paradox would involve either some substantive refinement to an already proposed solution, or, perhaps, just as likely, require an altogether different theoretical approach than has been offered up to now.

In view of the difficulties already alluded to, one might ask of what great

benefit it is to continue to seek a solution to this paradox. Why, indeed, bother to continue studying the sorites? To begin with, the sorites is an intriguing puzzle in itself. More so than not, published works on the sorites treat this subject as a matter worth pursuing in its own right, not merely as a secondary issue whose value depends fundamentally on how it relates to some larger issue in the field of logic or philosophy. The paradoxical nature of this problem is simply fascinating. And with no agreed upon solution, the prospect of finally resolving this ancient paradox provides a compelling reason to continue searching for a way to understand it.

In addition to being a perplexing argument, in itself, however, there are two additional aspects of this paradox that have potentially far reaching consequences. The first of these relates to the question of whether vagueness is a wholly undesirable aspect of language that ought be eliminated. Even though the most commonplace implementations might seem unproblematic, one group of philosophers, in particular (the Incoherence theorists), have suggested the sorites reveals an underlying inconsistency inherent in vague language, generally. If so, developing a genuinely well constructed form of language free of paradox might well require the complete eradication of vague terminology from the entire lexicon. But whether such a drastic step is really warranted can be decided only after obtaining some clear insight into the true nature of the paradox.

Secondly, the sorites raises some crucial questions about a number of principles widely (though not universally) accepted in the field of logic, principles that might otherwise prompt no real dispute. The paradox creates serious doubts regarding the validity of classical logic and semantics, and potentially makes difficult any generalized set of rules with respect to truth-functionality. (How borderline cases could possibly qualify as truth-functional and operate as such is unclear and problematic.) In particular, the sorites raises concerns regarding the validity of the Law of the Excluded Middle

('Either Q or not-Q') and the Law of Non-Contradiction ('Not (Q and not-Q)'), and even suggests some legitimate basis for adopting an Intuitionist model in logic (see section 6-4). In this regard, the sorites is not an isolated problem, but one whose resolution seriously impacts a number of other rather important questions in the field of logic. In fact, without some final resolution of this paradox, a coherent and comprehensive theory of logic may be unattainable.

At a minimum, then, the sorites commands our attention because it presents an obviously errant piece of logic having no obvious flaw in logic. This alone makes the problem difficult to dismiss without comment. In addition, as already mentioned, the answers to some basic questions only indirectly related to the sorites are, in fact, dependent on just how the sorites, itself, gets resolved. If any of these associated issues are of importance, then so, too, is arriving at a solution to the sorites. In any event, taking the paradox seriously means looking at precisely where the argument could possibly go wrong, and making an accurate assessment of what weaknesses exist in its construction. This, in turn, entails having to critically reexamine the paradox and past attempts to provide a solution to it. While this process, by itself, doesn't guarantee a result, the aim, at least, is to narrow the field of promising candidates to a minimum by identifying and eliminating faulty accounts of the sorites. This, in itself, is a positive step toward finding a solution.

1-4) THE TOLERANCE PRINCIPLE

If the sorites is written simply as:

S[1]: One grain of sand is insufficient to constitute a heap.
S[2]: Two grains of sand are insufficient to constitute a heap.
S[3]: Three grains of sand are insufficient to constitute a heap.

.

.

S[10^{12}]**:** A trillion grains of sand are insufficient to constitute a

heap.   <1*5>

it is implicit in this form that the statements listed are more than merely a string of independent, though ordered, assertions. What makes the sorites especially effective as an argument is that each assertion relates to its immediate predecessor and its immediate successor in a way that is relevant to the argument as a whole. The reason nine grains of sand are not sufficient to constitute a heap is not just that nine grains, in and of themselves, are too few in number to make up a heap (although this is a good part of why they are insufficient), but that, in addition, eight grains of sand are also considered too few to make a heap. In other words, if eight are too few in number, then so, too, must nine be too few in number. Moreover, since the same logic applies to any fewer number of grains, as well, then, by implication, part of the reason nine grains of sand are insufficient to constitute a heap is that seven grains are insufficient. In fact, inasmuch as this logic iterates throughout the entire chain, it is even true that one reason nine grains of sand are insufficient to constitute a heap is that a single grain of sand is insufficient to do so.

Whereas in the example just given in <1*5> this relationship between statements is merely implied, the sorites is often presented in a format that makes it explicit. For example, both versions of the sorites given in section 1-1 do this. In version <1*1>, for instance, the conditional statement**:**

S[2]**:** If one grain of sand is insufficient to constitute a heap, then two

grains of sand are insufficient to constitute a heap.

makes clear that statements S[1] and S[3] are interrelated, and for version <1*2> premise P[2] makes clear that this conditional would hold for any similar

-14-

pair of statements**:**

> P[2]**:** Adding a single grain of sand to a non-heap is not sufficient
> to form a heap.

P[2] is commonly referred to as the <u>Tolerance</u> <u>Principle</u>, the idea that no step in a sorites series is a jump that suddenly causes a property to apply where it had not done so before, or to cease applying where it had done so before. According to this principle, no two adjacent elements in a series like <1*5> are to be classified differently**:** every element in the sequence is to be placed in the same category as the preceding one.

One restriction on the Tolerance Principle, however, and really the only reason it is acceptable as a general rule, is that the change brought about by each step must be too insignificant, too minor to matter. Thus, the series**:**

> S[1]**:** A person 1000 millimeters in height is not tall.
> S[2]**:** A person 1001 millimeters in height is not tall.
> S[3]**:** A person 1002 millimeters in height is not tall.
>   .
>   .
> S[1001]**:** A person 2000 millimeters in height is not tall.

<div align="right"><1*6></div>

qualifies as a legitimate sorites-type argument, since the change at each stage is quite small. The place in the sequence where the first false statement occurs is unidentifiable. In contrast, the same final result arrived at in the following manner**:**

> S[1]**:** A person 1 meter in height is not tall.

S[2]**:**   A person 2 meters in height is not tall.   <1*7>

is not a true sorites argument. It is easy to see where this particular argument goes wrong, and to simply accept S[1] while rejecting S[2]. Here, unlike with <1*6>, there is no problem in identifying just which assertions are true and which are false, because, unlike with <1*6>, the change is not sufficiently gradual to create any borderline cases. The only cases to consider here are clear-cut ones.

The Tolerance Principle is relevant to any discussion of the sorites for three important reasons. First of all, it is characteristic of, and essential to any sorites-type argument.   Without this principle, there would be no logical connection between the elements of a sorites series, and the argument as a whole would be ineffectual. The sorites works (or at least seems to do so) only because the Tolerance Principle is an integral part of it.   Secondly, the Tolerance Principle is legitimate only insofar as each individual step in the progression results in a seemingly inconsequential change (even though, collectively, the steps add up to quite a substantial change). In order to assure both that the Tolerance Principle, itself, is plausible, and that there exist true borderline cases, the incremental differences must, of necessity, be small ones. Increments too large make the argument altogether unconvincing. Lastly, some theoreticians have argued that it is the Tolerance Principle, itself, that is the underlying flaw in the sorites. Although there is not general agreement as to exactly why this would be so, it is reasonable to consider the differing criticisms leveled against the Tolerance Principle, and ask whether any of these are truly merited.

1-5) HIGHER-ORDER VAGUENESS

One feature of vague language that distinguishes it from exact

terminology is the presence of borderline cases. That is, vague predicates, in addition to having clear-cut cases, the same as exact terms do, also always have instances where their application is uncertain. Thus, even though one grain of sand is clearly not a heap, and a trillion grains of sand clearly are, in some intermediate cases it will be altogether unclear whether or not the term 'heap' properly applies. In such <u>borderline</u> instances, no norm exists that would help decide whether the number of grains present is, or is not, sufficient to constitute a heap. As a result, borderline examples of heaps are characterized by inconsistent application of the predicate 'heap': the same particular number of grains sometimes will get classified as a heap, but at other times as a non-heap.

One interesting consequence of there being these borderline cases is that the transition from instances that are clearly heaps to those that are borderline is, itself, uncertain. In other words, the boundary between clear-cut cases and uncertain cases is indistinct in the same way that the division between heaps and non-heaps is. This means that the indeterminacy associated with vagueness does not mitigate or disappear as progressively finer distinctions between categories are considered. The blurry and indistinct boundaries that exist for the most basic categories (e.g., heap / non-heap) extend to every higher level of discrimination, making vagueness a property that is pervasive throughout. Thus, not only is the heap / non-heap boundary imprecise, so is the heap/borderline-heap boundary, the borderline-heap/borderline-borderline-heap boundary, etc. And given this, any endeavor to somehow eliminate the unwanted consequences of vagueness by finding where borderline cases begin and end promises to be unsuccessful.

That such a 'higher-order' vagueness does, indeed, genuinely exist is supported by the kind of thought-experiment one might actually design if one

were to try to pinpoint where the exact transition to borderline cases immediately begins. A sorites argument using a category for borderline-heaps can be constructed as follows:

    S[1]: One grain of sand does not constitute a borderline-heap.

    S[2]: Two grains of sand do not constitute a borderline-heap.

    S[3]: Three grains of sand do not constitute a borderline-heap.

    .

    .

    S[N]: N grains of sand do not constitute a borderline-heap.

    .

    .   <1*8>

Obviously, for some sufficiently large number of grains it eventually becomes false that N grains do not constitute a borderline-heap. Yet, if this progression is repeated multiple times, allowing for different judgements in each instance, the place where the first false assertion occurs cannot be expected to consistently remain the same. Instead, the location will vary across a limited range of values, no differently than in the somewhat simpler case already presented in <1*1>. But this means trying to fix a sharp boundary for borderline cases is no less problematic than attempting the same feat for any other boundary involving vague expressions. That kind of effort is bound to be unsuccessful.

    The obvious conclusion to take away from this is that vagueness, if it exists at all, implies an indefinite number of higher-order blurry boundaries. Since a progression similar to <1*5> can be offered up for any degree of differentiation, and since the number of subdivisions one might create is practically unlimited, no boundary for vague predicates, higher-order or not, can

be expected to be sharply defined. Such a conclusion is hard to avoid. Thus, vagueness implicitly requires the existence of higher-order vagueness, a result that any theory of vagueness must take into account.

1-6) RECIPROCAL LOGIC

Paradoxical arguments are ordinarily resolved in one of three ways. It could be argued that (1) one or more of the premises is false; (2) the logic is faulty (i.e., the conclusion is not a logical consequence of the premises); or (3) the conclusion is, in fact, true. Thus, absent any identifiable flaw in either the premises or logic (alternatives 1 and 2), the remaining option is to conclude that the argument is, indeed, sound, after all (alternative 3). In the case of the sorites, however, no one has seriously argued, or even suggested, that this is the case. Aside from the obvious difficulty in accepting the implication that no number of grains would ever be sufficient to constitute a heap, there is a second, quite compelling reason for concluding that the argument is unsound: rather than starting with the premise that one grain of sand is insufficient to constitute a heap, it is just as reasonable to begin with the premise that a very large number of grains of sand must be sufficient to constitute a heap. Thus, by reversing the direction of the argument, the sorites also takes the form:

P[1]: A trillion grains of sand are sufficient to constitute a heap.

P[2]: Removing a single grain of sand from a heap is not

     sufficient to form a non-heap.

Therefore, any quantity of sand is sufficient to constitute a heap.

<1*9>

Not only is this result again implausible, but it conflicts with the prior conclusion that no quantity of sand is sufficient to constitute a heap. If the initial sorites argument <1*2> and this reverse form are both correct, the two versions taken collectively would imply that any specified number of grains would be both sufficient <u>and</u> insufficient to constitute a heap. This, of course, makes no sense at all. If there are only two alternatives, heap or non-heap, only one categorization can be correct. In addition, whether any particular aggregate of sand is of sufficient size to constitute a heap could not very well depend simply on which direction the logic is applied. Even though vague terms like 'heap' permit discrepant classification in borderline cases, it must, at least, be <u>possible</u> to consistently apply such terminology. Yet, reciprocal forms of the sorites (pairs of arguments that proceed in opposite directions) guarantee inconsistent outcomes, even in non-borderline cases.

If the above reasoning is correct, it is unsurprising that discussions of the sorites have focused on <u>how</u> the argument is flawed, not on <u>whether</u> it is. Given that reciprocal forms of the paradox lead to inconsistent results, the option of accepting the conclusion appears ruled out from the start, and there seems little choice but to proceed on the basis that the argument is unsound. Just how it is so remains to be answered, a matter that will be the focus of the theoretical accounts to be considered in the sections that follow.

CHAPTER 2: INCOHERENCE THEORY

2-1) INTRODUCTION

It is at this juncture we turn to the first of several theoretical approaches to the sorites, each designed to provide a definitive solution to the paradox. We begin with Incoherence Theory, a view, as the name implies, that vague language is, in some pivotal sense, fundamentally incoherent. According to advocates of this position (Frege, Peirce, and Russell have already been cited as belonging to this group) it is sometimes, if not always, best to avoid use of vague terminology, and to limit or eliminate its application, as no legitimate form of logic can otherwise fully function properly. That is, unrestricted use of vague language invariably leads to inconsistent and incoherent results. The sorites paradox simply succeeds in making this fact strikingly obvious.

The sorites is a persuasive argument, in part, because it makes use (at least implicitly) of a standard form of inference known as <u>modus</u> <u>ponens</u>. The structure of modus ponens has the general form:

P[1]: If G, then H.
P[2]: G.

Therefore, H.   <2*1>

Assuming that the conditional premise P1 and the categorical premise P2 are both true, it follows that the conclusion must also be true. (Conversely, the conclusion is false only in cases where at least one premise is false.) Moreover, this logical relationship would be expected to hold for any argument of the same form, irrespective of the content of G and H. In the case of the sorites this would be:

P[1]**:** If N is a heap, then N+1 is a heap.

P[2]**:** N is a heap.

Therefore, N+1 is a heap.   <2*2>

This being so, it makes sense to think that the sorites is unsound not because of any fault in its logical construction, but because it contains one or more false premises. In other words, if the logical form is unassailable (and it seems to be), the only other alternative is to question the assertions that constitute the argument.

One particular group of philosophers (included in which would be Max Black (1937), James Cargile (1969), Michael Dummett (1975), Gottlob Frege, W.V. Quine (1981), Bertil Rolf (1982**;** 1984), Bertrand Russell (1923), Peter Unger (1979a**;** 1979b), and Samuel Wheeler (1975**;** 1979)) have interpreted this result to imply that vague concepts are inherently inconsistent and, therefore, incoherent. Accordingly, asserting that 'N is a large integer' or that 'Smith is tall', for instance, would be considered as nonsensical as talk of 'the largest number' or a 'cubic acre'.[1] The terms 'large' and 'tall' simply fail to designate any substantive attributes, as such, because they fail to draw any real distinctions. This same reasoning extends to many less obvious examples, such as persons (Unger, 1979a) and tables (Unger, 1979b), which have been held to not really exist, at least not as ordinarily conceived.

Thus, rather than developing a rationale to justify our practice of using inexact categories to draw fuzzy distinctions, Incoherence Theory simply rejects this form of language altogether. Those who hold to this view advocate completely eliminating vague discourse, and, in so doing, replacing it with exact terminology. Instead, for example, of stating that 'Smith is tall' one might assert 'Smith is greater than 200 centimeters in height' which is not to say that 'tall' means 'greater than 200 centimeters in height' but that distinctions should

always be based on a precise quantifier. Substitutions of this kind would be required for each vague term in the lexicon in order to guarantee a fully functional language and a consistent classification system.

The basic reasoning underlying this approach can be summarized in terms of four main points. (1) The logical structure of the paradox is perfectly legitimate. As previously noted, the sorites is merely a select instance of the more general logical form modus ponens and, as such, constitutes a valid form of inference. (2) A concept is vague only insofar as (i) there is no sharp division between categories and (ii) slight differences are always too insignificant to matter. In other words, the Tolerance Principle (the conditional premise in the sorites; see section 1-4) accurately reflects the meaning ascribed to the notion of vagueness. That N grains of sand are sufficient to constitute a heap if N+1 grains are is an essential feature of heaps as vague entities. Without this stipulation heaps would not very well be heaps (not in any ordinary sense of the word). (3) There exist clear exemplars that help define vague predicates. The shortest person in the world, for example, is neither tall nor borderline tall, and clearly is not. And the tallest person in the world is without dispute not short, nor borderline short. It would be impossible to think otherwise without misconstruing what these terms mean. (4) The sorites paradox has intolerable implications. (i) It cannot be correct that a single grain of sand is sufficient to constitute a heap nor that a trillion grains are insufficient to do so. The extreme consequences that follow from acceptance of the sorites argument are simply untenable. (ii) Moreover, vague terms are inherently inconsistent: any borderline heap will, by definition, sometimes be categorized as a heap, sometimes not as a heap. But this is not any more acceptable in the case of vague terms than it is for absolutely exact ones. Just as an integer cannot fairly be classified as both odd and even, it could not, either, one might suppose, unequivocally be designated as a large integer and not a large integer. But that is precisely what an inexact term like 'large'

entails.

These four claims lead to something of a dilemma. If assertions (2) and (3) are correct, the inference supported by the sorites (that a trillion grains of sand are insufficient to constitute a heap, for example) becomes inescapable. But to reject (2) or (3) means discounting what is arguably beyond dispute, namely, that the premises P1 and P2 are true, and are true by definition. This, according to Incoherence Theory, is precisely what cannot be allowed. In particular, the existence of heaps as heaps demands strict adherence to the Tolerance Principle, and to omit this requirement is only to distort the plain meaning of the word. Any such attempt at redefinition would have to be regarded as illegitimate.

The remaining option, then, is to conclude that vague predicates are intrinsically confused and internally inconsistent. Even though vague language may function adequately in routine discourse (or, at least, seem to), what the sorites paradox succeeds in doing is to show that vocabulary of this kind really is quite fundamentally flawed. The problem with vague terms is that each utilizes a fuzzy boundary that incorporates two incongruent elements: (1) drawing a distinction with (2) the blurring of distinctions. (The categorical premise establishes the distinction, the conditional premises blur it.) Combining these disparate constituents into a single concept cannot help but result in a logically incoherent idea. Accordingly, there is good reason to think that no boundary truly exists unless it is clearly demarcated. It might be said that by virtue of there being no point at which the addition of a single grain of sand definitely matters that there is no genuine difference between heaps and non-heaps (not in any rigorous sense). Consequently, when the boundaries drawn are indeterminate and indistinct, these fail to operate as real boundaries at all. That they apparently do function as substantive boundaries is simply mistaken, notwithstanding their, nonetheless, wide use in ordinary discourse.

2-2) THE NON-EXISTENCE OF ORDINARY THINGS

Perhaps the most notable, as well as intriguing, attempt to support this conclusion regarding the inadequacy of vague language is to be found in the writings of Peter Unger (1979a; 1979b). In a group of publications authored by Unger, the logical difficulties normally associated with the sorites argument when applied to <u>heaps</u> are maintained to apply with equal force to somewhat less obvious compositional forms, in particular to a substantial number of familiar, ordinary physical objects seldomly associated with the sorites paradox.

Consider this: An ordinary chair is a collection of an immense number of extremely small sized particles. In principle, any chair could be disassembled just one particle at a time. If, for example, only a single atom of a chair were removed would we not correctly consider the object that remained to still be a chair? If so, what if another atom were lost, and another, and yet another? Would any or all of these be regarded as chairs? Moreover, would these be regarded as the same chair as the original, or, at least, a chair identifiably related to the original? It is difficult to see how any of these questions could be given an affirmative response since, if allowed to proceed, the destructive process described would eventually leave behind just one atom from the total set of atoms that comprised the original, intact chair. At that point, no one could possibly conclude that the chair endures. It seems incomprehensible that what remains is still a chair when the incremental losses have removed all but a single atom.

Thought-experiments of this kind involving the progressive (and ultimately complete) deconstruction of an object are meant to lead us to a rather astounding conclusion, namely, that we cannot speak intelligibly of objects whose properties are not precisely delineated. Unger (1979a; 1979b) argues, on this basis, that ordinary objects such as tables, rocks, swizzle sticks, yo-yos, and even people do not really exist. In his article *I Do Not Exist*, Unger

(1979a**:** 236) says the following**:**

"I do not exist and neither do you.... For, as regards almost everything which is commonly alleged to exist, it may be argued ... that it in fact does not exist. There are, then, no tables or chairs, nor rocks or stones or ordinary stars. Neither are there any plants or animals. No finite persons or conscious beings exist, including myself Peter Unger**:** I do not exist.... Tables, as well as chairs, have often been believed to be paradigms of existing things or entities, but I shall argue that they do not exist at all. They are ... only fictions ...".

To make his point Unger utilizes a procedure he calls the 'sorites of decomposition', using even himself as a case study. For example, to disprove the existence of tables Unger (1979a**:** 237-238) provides this formulation**:**

"(1) There exists at least one table.
(2) For anything there may be, if it is a table, then it consists of many atoms, but only a finite number.
(3) For anything there may be, if it is a table (which consists of many atoms, but a finite number), then the net removal of one atom, or only a few, in a way which is most innocuous and favourable, will not mean the difference as to whether there is a table in the situation".

He adds**:**

"These three premisses, I take it, are inconsistent. The assessment of this inconsistency, I submit, leads one to reject, and to deny, the first

premiss, whatever one may subsequently think of the remaining two propositions (Unger 1979a**:** 237-238)".

If so, if we are led to conclude along with Unger that the first premise here is faulty, then it has to be true that there really are no tables, and by similar reasoning, no rocks, no people, nor anything that would be subject to the kind of decomposition Unger imagines possible. Our quite common belief that the world is populated with such ordinary objects is, on this account, a quite mistaken view of how things really are.

This conclusion needs a bit of clarification.   One person who has considered this particular argument in some detail is Mark Heller (1990**:** 68-69) who, in his chapter on the sorites paradox, interprets Unger's position simply as follows**:**

"It may be admitted that the words 'heap' and 'stone' and 'person' do not apply to anything because of their vagueness. Yet this fact should not lead us to doubt the existence of the things that we had thought we could refer to with the words 'heap' and 'stone' and 'person'. Strictly, then, there are no people, but this is only because none of the many things that there are can properly be called a person.... What the Sorites paradox demonstrates is that the world is not the way we think it is. The objects that really exist do not have many of the properties that tables, chairs, apes, and people purportedly have.... The paradox shows that nothing can have the properties we attribute to the objects of our standard ontology. In particular, nothing can have the kinds of persistence conditions and essential properties that we standardly attribute.   These properties are merely products of our way of conceptualizing the world. But it is just these properties that go into the identity criteria for an object**;** they are what make the object what it is.

The Sorites paradox shows us that the world is not as we think it to be and challenges us to give an account of just what the world is like".

Heller (1990: 108-109) continues:

"It might be suggested that the most I have shown is that there is nothing that has the persistence conditions and essential properties that we attribute to our standard objects. It does not follow from this that our standard objects do not exist. It may be that [a table] exists, but just does not have the properties that we typically attribute to it. Our being wrong about an object's properties is not itself a reason to suppose that the object in question does not exist.... I recognize that denying that there are tables sounds a little crazy ... The present suggestion gives me a way to avoid sounding so crazy.... the Sorites paradox does not provide an argument against tables and people, it just helps us to see what these objects must be like".

In a related article regarding the potentiality for there being 'vague objects' in the world, J. A. Burgess makes much the same point. Burgess (1990: 282) states:

"If we wish to maintain that the sense in which the world is precise is really a 'deep' sense, then we shall be committed to the view that, in the deep sense, hills, humans, tables and other vague objects do not exist.... [T]o say that ... there are no hills is to say no more than that hills are not precise ... ".

Samuel Wheeler (1975; 1979), another principal proponent of the Incoherence thesis, maintains the same position. In his article On That Which

Is Not, Wheeler (1979**:** 155) echoes Unger's view in that, "... very probably, none of the ordinary 'middle-sized' objects of the 'given' world exist" but adds the qualification, "In particular, there are no persons <u>as</u> <u>ordinarily</u> <u>conceived</u> ...". Wheeler (1979**:** 168) goes on to say, with regard to the sorites of decomposition, that "What this sorites establishes is that ordinary objects and persons <u>as</u> <u>we</u> <u>conceive</u> <u>them</u> do not exist". (Emphasis added.) The key provision included here by Wheeler is that 'as ordinarily conceived' objects like these do not exist. The point is, Unger's argument does not mean that the kind of objects specified above fail to exist just by virtue of their being composites of seemingly insignificant constituent parts (and, therefore, their potentially being subjected to incremental deconstruction). Rather, nothing corresponds to these objects <u>insofar</u> as they are imagined (understood) to persist after undergoing some measure of deconstruction, however insignificant. But in another sense just the opposite is also true. Tables, rocks, people, etc., do exist, but <u>only</u> <u>insofar</u> as they are conceived of as not undergoing any change in their constituent parts. It is because we are also capable of conceptualizing objects in this way, as fixed entities having an invariant set of parts, that we are entitled to claim that they do, in fact, exist.

The effect, then, of extending the sorites argument beyond heaps to a wide range of ordinary objects is twofold. First of all, it shows how expansive the consequences of the sorites really are simply in terms of the number of different entities to which the argument might apply. If heaps are problematic entities, so are a great many other objects in the world, and the difficulties associated with vagueness are, therefore, not confined to just the rather few classical examples of vagueness (such as heaps) ordinarily considered when discussing the paradox. Secondly, the 'Unger-Wheeler thesis' reinforces how seriously we ought take the implications of the paradox. If the apparent inconsistency in the premises is real, as is suggested by the decompostion paradox, no solution can be successful that remains tolerant of vague

descriptions. (This is Unger's point.) Thus, the only satisfactory way to block the paradox and ultimately resolve the issue is to make a broad dismissal of vague language, generally. Otherwise, we would be forced to accept the consequences of there being vague objects, that is, of entities that are continuously subject to gradual decomposition. And permitting that seems by no means a tenable option.

2-3) REFORMULATING THE PARADOX

If the line of reasoning presented thus far is correct, all the standard forms of the sorites paradox are somewhat misleading. A more accurate representation would have a construction similar to**:**

> P[1]**:** If one grain of sand is insufficient to constitute a heap,
>
> then a trillion grains of sand are insufficient to constitute a heap.
>
> P[2]**:** One grain of sand is insufficient to constitute a heap
>
> (but a trillion grains of sand are sufficient to constitute a heap).
>
> ---
>
> Therefore, a trillion grains of sand are insufficient to constitute a heap.
>
> <2*3>

This makes more obvious how problematic the conclusion really is. Even though one portion of the argument, namely**:**

> P[1]**:** If one grain of sand is insufficient to constitute a heap, then
>
> a trillion grains of sand are insufficient to constitute a heap.
>
> P[2]**:** One grain of sand is insufficient to constitute a heap.
>
> <2*4>

supports the conclusion that a trillion grains of sand are insufficient to constitute a heap, one could equally well argue from the second premise alone to the opposite conclusion**:**

> P[2]**:** One grain of sand is insufficient to constitute a heap (but a trillion grains of sand are sufficient to constitute a heap).
>
> --
>
> Therefore, a trillion grains of sand are sufficient to constitute a heap.

Indeed, when formulated in this way, the sorites invariably fails to provide any consistent result. As a result, whether N grains of sand are, or are not, sufficient to constitute a heap becomes undecidable, and any supposed division between heaps and non-heaps turns out to be illusory.

2-4) INCOHERENT OR UNRESOLVED?

One of the principal appeals of Incoherence Theory (perhaps the major one) is simply the absence of any convincing alternative for explaining the sorites paradox. Despite the fact that logicians readily concede the sorites represents a fallacious form of reasoning, no proposed solution, regardless of approach, has pointed out any specific, identifiable fault in the structure of the argument that is generally acknowledged as a clear and obvious error. As a result, there must be some degree of doubt regarding whether just the form of the argument, itself, not anything else, is what accounts for the unacceptable implications associated with the paradox. In fact, absent any evident defect in the logical structure of the argument, it must be considered at least as plausible, if not compelling, that the sorites reveals a basic residual flaw within vague language, generally. If no other more apparent source of error exists

than this, then the view that vague concepts are inherently incoherent becomes, by default, the most credible explanation available.

There are two obvious objections to this conclusion. A number of theories currently exist that claim to provide solutions to the sorites without abandoning vague language, itself. Thus, the paradox, though tricky, is thought to have some kind of determinate and, ultimately, discoverable flaw. That the nature of this is not thoroughly obvious and immediately recognizable is what accounts for there being a paradox in the first place. An argument having a concealed, but, nonetheless, genuine inconsistency is far different from one that is, in actuality, fundamentally sound. Before accepting Incoherence Theory as true, therefore, other seemingly less viable explanations ought also be considered and analyzed in some detail. Unless each of these alternative theories is found to be inadequate, Incoherence Theory cannot, itself, be acknowledged as a definitive solution to the paradox.

Secondly, even supposing all competing theories were invalid, this would not lead one necessarily to conclude that the only option is to endorse Incoherence Theory as correct. Indeed, just as there is something suspicious about the sorites paradox, itself, there is something equally unsettling about an approach that repudiates vague language as comprehensively as does Incoherence Theory. It is not too surprising, then, that even absent a clear weakness, this theory can still be rejected as "an affront to common sense" (Sorensen 1987: 774) and the non-existence of ordinary things dismissed as just one of the "bizarre solutions" the paradox engenders (Goldstein 1988: 447). The paradox aside, vague terminology at least seems to function quite well in ordinary discourse without generating any substantial logical difficulties. It is only in the context of the sorites that matters become so exceptionally troublesome. Hence, despite a notable lack of success in finding a readily persuasive solution to the paradox, it seems quite reasonable to continue focusing on finding some defect specific to the sorites argument, alone, rather

than opting for a broader solution whose impact would be much more far-reaching. Not having a simple way of resolving the paradox, then, does not mean acquiescing to characterizing vague language as incoherent.

The other option, then, is to reject every theory that claims some specific flaw in the sorites, without thereby accepting Incoherence Theory as acceptable. Even if no alternative theory has proven to be correct, this does not guarantee no future, but as yet unrecognized, solution to the paradox will be forthcoming. It simply is not feasible to rule out every potential solution that might show the sorites to be an invalid form of argument. As a result, it is quite possible to be skeptical of all past attempts to specifically locate and identify what might be wrong with the sorites, yet remain unconvinced that Incoherence Theory provides the right approach to explaining the paradox, either.

2-5) SORENSEN'S THREE OBJECTIONS

That having been said, one theoretician, at least, argues that Incoherence Theory is not a serious candidate for providing a viable solution to the paradox. Roy Sorensen (1987) contends that more than being simply reluctant to accept Incoherence Theory as truly adequate, we have compelling reasons for rejecting it as quite untenable. He contends that in addition to (1) being 'self-defeating and an affront to common sense', (2) Inconsistency Theory falsely predicts that vagueness is compositional and, moreover, (3) the theory violates the Principle of Charity.

Let's examine these in reverse order. The Principle of Charity states that in interpreting an expression that is ambiguous or in some way unclear, the expression should be understood in a way that makes it seem most rational. In applying this principle to Incoherence Theory Sorensen (1987: 774-775) comments that, "... inconsistency theory violates the weakest version of the principle of charity; avoid attributing obvious contradictions to speakers" to

which he adds, "For example, if someone says, '(1) That is a bank but it is not a bank,' he should not be interpreted as assenting to a contradiction. We should instead suppose that 'bank' is being used ambiguously or that there are unstated adjectives or that the sentence is in some other way incomplete or non-literal".

In cases where speakers make use of vague terminology, therefore, we should be disinclined to assert that this involves employing words in a manner that makes their application internally inconsistent. If we are entitled to suppose that speakers, under normal circumstances, consistently make rational use of language, then it only makes sense to conclude that references to 'heaps' or other vaguely-defined objects employ language in a non-contradictory way. But this is exactly opposite to how Incoherence Theory views the situation. As Sorensen (1987: 775) puts it:

"... the inconsistency theory of vagueness faces a fundamental evidential problem. To show that a term is vague they must show that competent speakers assent to some obvious inconsistencies when they use these terms. However, charity ensures that we are rarely if ever in a position to attribute such inconsistencies. It will almost always be better to infer an ambiguity or an elliptical usage. Thus, the methodology of interpretation thwarts attempts to show that predicates are vague in the way inconsistency theorists envision".

The view adopted by Incoherence theorists, then, is one taken in direct opposition to the Principle of Charity. By attributing to speakers who employ vague terminology an inconsistent and, therefore, flawed use of language, ordinary language users are, to some degree, made out to be inept in mastery of their own language.

The second criticism put forth by Sorensen concerns the rules for combining statements incorporating vague expressions. He states**:**

> "... inconsistency theory falsely predicts that vagueness is compositional.
> The predicates 'somewhat greater than 105' and 'somewhat less than
> 107' are each vague predicates. If vague predicates are inconsistent,
> the conjunction of these predicates should be inconsistent and hence
> vague. However, 'is somewhat greater than 105 and somewhat less
> than 107' is a precise predicate. Thus vagueness is not compositional,
> so the inconsistency theory is mistaken (Sorensen 1987**:** 774)".

Accordingly, there is a problem with an argument that contends vague expressions are inconsistent, and, yet, that these, nonetheless, also operate according to simple combinational rules. If Incoherence Theory is to avoid this difficulty, some reasonable account must be provided of how to resolve this issue.

Sorensen's remaining objection, that regarding the self-defeating nature of Incoherence Theory, stems from his observation that the distinction between vague and non-vague predicates can, itself, be vague. Sorensen (1985) illustrates this point by presenting a rather unusually constructed version of the sorites wherein the individual elements comprising the sorites series are, themselves, sorites-type arguments. The result is a kind of self-referential meta-sorites having the property of drawing imprecise distinctions with respect to imprecise distinctions.

Here is how this works. Define a sorites-N series as an argument of the form**:**

S[1]: 1 is a small positive integer and/or less than N.

S[2]: 2 is a small positive integer and/or less than N.

S[3]: 3 is a small positive integer and/or less than N.

.

.

where the parameter N is, itself, some integer. If N is extremely large, then the first false statement in the series would necessarily be S[N], since statement S[N-1] would be true simply by virtue of referring to an integer less than N, but statement S[N] would refer to an integer that is neither small nor less than N. Thus, the sorities-1,000,000,000,000, for example, being well past where N is still small, would have a clear, sharp transition point from true to false statements in going from S[999,999,999,999] to S[1,000,000,000,000].

By similar reasoning, a sorites-N series having an obviously small value for N would also have a precise transition point. This would, as before, coincide to the boundary between S[N-1] and S[N], only here it would reflect the step from the largest small integer less than N to the first small integer not less than N. Thus, the sorites-3, for example, in having an N value well below where N ceases being small, would have a sharp transition from true to false statements in going from S[2] to S[3].

The third and last category, then, consists of those intermediate cases where it is unclear whether or not the value of N is truly small. In such instances, it becomes problematic whether, for a given value of N, the corresponding series should be considered to have a sharp boundary or an indeterminate one. If N really is small, then a transition from true to false statements will occur precisely between S[N-1] and S[N]. Otherwise, if N is not small, the boundary will surely be an imprecise one: because 'small' is a vague concept to start with, the location of the border between true and false statements cannot be exact, either; that location becomes no easier to pinpoint

than the imprecise transition from 'small' to 'not-small'. Consequently, in borderline cases of N there is not only no way to resolve whether N really is small or not, there is, moreover, no way to determine whether or not the N-series has an abrupt, definite transition from true statements to false ones.

The remaining step in constructing the meta-sorites is to place the individual sorites-N series together in the following way:

S[1]*: Sorites-1 has a sharp boundary.

S[2]*: Sorites-2 has a sharp boundary.

.

.

S[10^{12}]*: Sorites-1,000,000,000,000 has a sharp boundary.

Each statement in this series states that the corresponding sorites-N series is not vague. But, as already argued, not only will not every constituent series from sorites-1 through sorites-1,000,000,000,000 have a sharp boundary, which particular sorites-N series is the first to lack a clear-cut boundary will be indeterminate. Consequently, which particular S[N]* statements in the meta-series are true, and which are false, is also uncertain. In simple terms, precisely which statements S[N]* are vague is, itself, vague.

Sorensen (1985: 135) goes on from this to conclude:

"The vagueness of 'vague' suggests that accounts of vagueness which describe vague predicates as incoherent are necessarily false. Such an account can be true only if the terms it *uses* to describe vague predicates are coherent. Since they use 'vague' in their thesis that all vague predicates are incoherent, their account can be true only if their account uses an incoherent term".

Thus, any defense of Incoherence Theory ends up advancing some type of argument whose linguistic form employs the very terminology that is claimed to be defective. But this, it seems, makes it impossible for Incoherence theorists to make their case without engaging in self-defeating logic.   To avoid becoming incoherent, itself, any argument in favor of Incoherence Theory would need to somehow be critical of vague language without incorporating vague terminology in its own formulation.   But it is rather doubtful that any such argument could be constructed in that manner and, consequently, that any version of Incoherence Theory could, itself, be coherent.

2-6) THREE REPLIES

In the famous opening line of <u>A</u> <u>Tale</u> <u>of</u> <u>Two</u> <u>Cities</u>, Charles Dickens (1980**:** 1) remarks**:**

"It was the best of times, it was the worst of times ..."   <2*5>

This statement might, if taken simply at face value, be mistaken for just a plain piece of nonsense. But we are not, in this instance, obligated to accept Dickens' assertion quite as literally as it is actually stated. We are, or, at least, ought to be, disposed to interpret this as a statement possessing some tacit stipulations not expressed by (yet, nonetheless, intended by) the author. It is simply left to the reader to make sense of this remark by supplementing it with the qualifiers needed to make it more
intelligible. Thus, one might reasonably suppose that Dickens' opening to his novel meant something closer to**:**

In some ways it was the best of times, in other ways, the worst.
                                                                                    <2*6>

than that <u>without</u> <u>any</u> <u>qualification</u> it was both the best and the worst of times. Some kind of reformulation of Dickens' original statement in this way would be the most natural reading of the text, as the alternative would be to judge, without having any real basis for so doing, that Dickens was somehow making a genuinely incoherent remark here. The inclination to not give very serious consideration to the possibility that Dickens was just being irrational in his thinking is exactly the approach that, in more general form, comprises a rational basis for the Principle of Charity.

There are, however, two good reasons for questioning applying this rather broad principle to a criticism of Incoherence Theory. It is not plausible, first of all, to think the Principle of Charity has unlimited scope. The most obvious way to interpret**:**

There is a largest prime number.   <2*7>

for example, is to construe it literally, notwithstanding that there is no largest prime and that a logical proof can be constructed to demonstrate that no largest prime does, in fact, exist. Unless it is reasonable, in some cases, at least, to think that an incoherent assertion is being advanced, there would be no room at all for making mistakes in logic. The effect would be to presume people are so rational that logical assertions made in error could never be interpreted as erroneous.

Secondly, unless a more plausible interpretation is available than the one being offered, it is rather difficult to maintain that a proposed interpretation is truly faulty. What is really needed to effectively counter the claim that a statement ought to be taken in a particular sense (one that makes it incoherent, for example) is at least one different interpretation that makes the statement more reasonable. That is why assertions like <2*6> do not have to be dismissed as nonsensical. But the point behind Incoherence Theory is that in

other cases, like <2*7>, there seem to exist no suitable alternatives to a straightforward literal interpretation. For that reason the most plausible interpretation of <2*7> is the most literal one, even though this acknowledges the assertion as inconsistent. By similar reasoning, it is likewise possible to argue that vague terminology ought also not be construed as coherent, either. Unless some other theory that gives a better account of vague language can be provided, there simply is not sufficient reason to think that the sorites represents a coherent situation (as with <2*6>) rather than an incoherent one (as with <2*7>). But the existence of such an alternative explanation is exactly what is missing when it comes to resolving the sorites. Thus, contrary to what one might otherwise think of it, the Principle of Charity does not help settle the question here of whether or not vague language should be regarded as incoherent.

The second criticism offered against Incoherence Theory, that regarding the compositional nature of vague statements, is also quite suspect. It should be rather apparent that conjunctions formed from individual conjuncts are not always truth-functional in a simple way. Thus, while**:**

Lincoln was tall.   <2*8>

and

Napoleon was short.   <2*9>

imply that**:**

Lincoln was tall and Napoleon was short.   <2*10>

it surely does not follow from**:**

The home team might win. <2*11>

and

The visiting team might win. <2*12>

that:

Both the home and the visiting teams might win. <2*13>

Clearly, true conjuncts do not invariably combine in the straightforward way that statements <2*8> and <2*9> obviously do, so as to always produce true conjunctions. This being so, it should come as no surprise that vague conjunctions also do not strictly adhere to simple combinational rules. Thus, not only do the two statements:

'Is somewhat greater than 105' is a vague assertion. <2*14>

and

'Is somewhat less than 107' is a vague assertion. <2*15>

not imply:

'Is somewhat greater than 105 and somewhat less than 107' is a vague assertion. <2*16>,

that this result does not follow should not strike us as particularly remarkable.

Because conjuncts that are logically related cannot be expected to consistently follow the same rules as those that are logically independent, it ought be far from surprising that <2*10> is an acceptable inference while at the same time <2*13> and <2*16> are unsatisfactory. Why, then, combinational rules would be considered singularly problematic just in the case of vague expressions is, itself, rather puzzling.   Without some preliminary reason to expect vague conjuncts like <2*14> and <2*15> to behave any differently than conjuncts like <2*11> and <2*12>, the 'compositional argument' affords no compelling reason to reject Incoherence Theory as inadequate. Quite the opposite appears true. Given that conjunctions such as <2*13> do not follow directly from conjuncts <2*11> and <2*12>, we would be surprised if similar examples involving vague predicates did not, likewise, also exist. That they do exist is neither surprising nor problematic.

The final objection to be addressed concerns whether a statement of the form**:**

Vague concepts are incoherent. <2*17>

is, in fact, inconsistent in a way that makes Incoherence Theory untenable. Perhaps the most straightforward way to respond to this argument is by considering a simple analogy. Even though the two statements**:**

Ghosts do not exist. <2*18>

and

Unicorns are white. <2*19>

have grammatical forms similar to**:**

Zebra are striped. <2*20>

<2*18> and <2*19> are, nonetheless, quite different logically from <2*20>. Statement <2*20> simply attributes the predicate 'striped' to the subject 'zebra'. But the predicate 'white' in statement <2*19> is ascribed to an imaginary subject (unicorns), not a real one. Unlike zebra, unicorns do not have material existence. To avoid any implication otherwise, <2*19> would be better written as:

If any unicorns were to exist, they would be white. <2*21>

This effectively blocks one from making the faulty inference:

Unicorns are white.
White is a color.
Anything that has color exists.

Therefore, unicorns exist. <2*22>

Similarly, <2*18> is better rendered as:

Nothing that exists has the properties of a ghost. <2*23>

Thus, rather than seeming to ascribe a predicate to a subject that is non-existent (as with <2*18>), <2*23> makes clear that nothing at all corresponds to the predicate used. Stated in this way, it is quite apparent that in the course of simply ascribing a predicate to a subject, there is no implication that the named subject is, in fact, alleged to actually exist.

This same argument applies with respect to incoherent concepts. For

example, rather than asserting**:**

The idea of a large even prime number is incoherent.   <2*24>

it is somewhat less confusing to say that**:**

No number is large, even and prime.   <2*25>

<2*25> makes it clear that the property of incoherence is not being attributed to a number that is otherwise large, even, and prime.   It is simply an acknowledgment that these other attributes do not, per se, pertain to anything at all.

These same observations regarding seemingly minor differences in form come into consideration when evaluating whether Incoherence Theory, itself, as expressed by statement <2*17>, fails to make a coherent assertion.   As already seen with example <2*22>, the basic difficulty with using <2*17> as the form to express the idea that vague language is incoherent is that it suggests a wrong conclusion.   The particular way <2*17> is constructed invites an argument such as**:**

Vague concepts are incoherent.
Incoherence is a property.
Anything that has properties must exist.

Therefore, vague concepts exist.   <2*26>

Of course, if vague concepts truly are incoherent (first premise), then they cannot exist, so the conclusion derived from assuming the first premise to be true leads to a contradiction.   If this argument were valid, therefore, something

would, indeed, be quite wrong with the premise that vague concepts are incoherent. Insofar as <2*17> correctly characterizes Incoherence Theory, then, there is arguably something quite wrong with considering that vague language might be incoherent.

The mistake made here, of course, is that <2*26> misinterprets what <2*17> actually means. The sort of confusion that seems to arise from the somewhat misleading grammatical form used in <2*17> can be avoided by restating <2*17> more precisely. All that Incoherence theorists really contend is**:**

      No concept is vague.   <2*27>

In other words, it is only well delineated concepts that truly function as concepts. So-called 'vague concepts' really are not concepts at all. But when expressed in this way, Incoherence Theory leads to no obviously inconsistent result and cannot, therefore, be summarily dismissed as self-refuting. That particular criticism is based on a faulty reading of <2*17> and of what it implies. Thus, Sorensen's argument for the 'vagueness of vague' is simply unconvincing.

CHAPTER 3: EPISTEMIC THEORY

3-1) INTRODUCTION

Most logicians take it as a given that the sort of fuzziness we associate with vague concepts like <u>heap</u> is simply an intrinsic feature of vague terminology that is in no way eliminable. Indeed, it is thought that the inherent lack of precision encountered in use of these terms is the very characteristic that makes vague terms vague, without which there would be no expressions of this type. Our inability to be entirely consistent in their application is merely a reflection of this fact, not an indicator that we fail to fully comprehend vague concepts. But one group of theorists in particular (the leading proponents being Roy Sorensen (1985; 1987; 1988a; 1988b; 1988c; 1989a; 1989b; 1991a; 1991b) and Timothy Williamson ((Williamson, 1992); (Williamson & Simons, 1992); (Williamson, 1994))), has taken issue with this standard view. Rather than allowing for the possibility of an inexact lexicon, per se, proponents of Epistemic Theory attribute any imprecision in language to a lack of knowledge on the part of the language users. Vagueness, it is argued, does not reside in terminology itself, but is indicative of an imperfect understanding of the concept being employed.

The basic rationale behind Epistemic Theory is simply this: the sorites paradox is valid as an argument. If we are forced to accept the premises as true we are, likewise, obligated to accept what, without a doubt, is an untenable conclusion. The only means to avoid this is to reject at least one of the premises of the argument. But since there are obvious examples of both heaps and non-heaps, it must be that the categorical premise (one grain of sand is insufficient to constitute a heap) is true while the conditional premise (if N grains are not sufficient to constitute a heap, then N+1 grains also are not) is false. And, for the conditional premise to be false there has to exist some

step in the sorites series where N+1 grains are sufficient to constitute a heap, even though N grains are not. Despite the fact that it might be unlocatable, such a place has to exist. Thus, contrary to our intuition, there are, in reality, no intrinsically fuzzy concepts. That it seems otherwise to us is what accounts for there being paradox at all.

This particular theory is supported, in part, by persuasive evidence that we can, indeed, apply precise terms in rather imprecise ways. A couple of simple examples help illustrated this point: (1) Suppose one were asked to gauge the number of items present in a large group without being permitted to count them. (The number of people in a crowd, or the number of blades of grass in a field would be examples of this sort.) It would be nearly impossible, by casual observation alone, to come up with exactly the correct result in a single try, although one might well come reasonably close to doing so. (2) If an attempt were made to ascertain the weight of an object only by lifting it and sensing how heavy it felt, again, one would expect some degree of accuracy in the result, but it would be surprising to not be off the mark to some extent. In either of these cases, the method used to derive the results guarantees the value obtained should be taken as no more than a mere approximation to the true value.

According to Epistemic Theory, these examples are practically identical to a sorites-type experiment. Whether someone is attempting to determine if a brick weighs 900,001 dynes or is, instead, trying to decide if a collection of sand constitutes a heap amounts to basically the same thing. Both are attempts to arrive at a true value using procedures that provide merely rough estimates. But in either case, the pattern is the same: repeating the test numerous times should result in a group of values clustered around the correct value. In fact, given enough tries, the experimental value ought eventually to match the correct result. The point is, the difficulty in obtaining an exact result need not lead to the conclusion (the false conclusion, that is) that such an exact

result does, in fact, not exist.

What a typical formulation of the sorites means to Epistemic Theory, then, is something quite different from its usual interpretation. Since the paradox is seen as akin to estimating the number of people in a crowded stadium, the sorites argument is thought to be analogous to proposing**:**

S[1]**:** The number of people in the stadium is greater than zero.

S[2]**:** The number of people in the stadium is greater than one.

.

.   <3*1>

which, quite obviously, is false at some stage. Similarly, the sorites would be interpreted to say**:**

S[1]**:** N+1 grains of sand is greater than one grain of sand.

S[2]**:** N+1 grains of sand is greater than two grains of sand.

.

.

S[N]**:** N+1 grains of sand is greater than N grains of sand.

S[N+1]**:** N+1 grains of sand is greater than N+1 grains of sand.

<3*2>

which, again, is clearly incorrect. Not only do these comparisons eventually result in a false statement, this occurs at some definite location (at S[N+1]) in the sorites sequence, before which every statement is true and after which every statement is false. Hence, it is apparent both that there is a distinct error in the argument and where the source of error occurs.

At this stage in the discussion, however, how reasonable it is to think the

same logical construction holds for heaps as it does for crowds has not been answered, and the parallel need not be accepted as very convincing. Unless something more can be said that would aid in settling the matter one way or the other, it is difficult to reach any firm conclusion regarding the validity of Epistemic Theory. In the next four sections, therefore, several key issues relating to Epistemic Theory will be examined, ones that will be used to help evaluate this theory. This analysis will show a number of logical problems presently exist that make it difficult to consider Epistemic Theory a viable approach to solving the paradox.

3-2) THE METHODOLOGY PROBLEM

The two cases just considered, of the crowd and the heap, clearly differ in one important respect. We know how to determine the number of people in a crowd (by counting them), but do not know how to determine what minimum size is needed to create a heap. Thus, for crowds a means exists by which to determine if an estimated size is correct: it is simply compared to the actual count. But for heaps no such methodology exists. In fact, one cannot even imagine a procedure that would lead to acquiring that sort of definite information.

This methodological problem poses some difficulty for Epistemic Theory. Our inability to recognize a fixed transition point from non-heaps to heaps, by itself, raises doubt that such an entity genuinely does exist. Accordingly, the absence of even some hypothetical method for identifying where this transition occurs makes the theory all the more suspect. Proponents of Epistemic Theory, therefore, might be expected to either (1) suggest how the dividing line between heaps and non-heaps could be arrived at or (2) provide a rational basis why that information is unobtainable. To date, however, no theorist has specified a methodology that could satisfy (1), nor provided a detailed

explanation that would accomplish (2). Not only is a method lacking that could reliably locate the point of division between heaps and non-heaps, it is not clear why not.

In response to this criticism, it might be argued that more is being asked of Epistemic Theory than is really required. First of all, it need not be conceded that some procedure capable of identifying a precise boundary has even to exist (let alone be formulated in detail) in order that a sharp division for a vague predicate actually be present. (That would be to confuse a question of fact with a method for arriving at the facts.) Secondly, it is not apparent to what degree our inability to locate that boundary is, or must be, explicable (assuming the boundary does, indeed, exist). Insofar as something can be true without it being apparent that it is true, one would not necessarily think it a requirement to furnish an explanation of that sort (it being self-evident that truths are not always recognizable). Perhaps, then, we are entitled to infer the existence of precise categories for seemingly vague concepts without, at the same time, having any idea why their boundaries are unidentifiable by us. If so, it would be unwarranted to expect Epistemic Theory to account for this lack of knowledge in a more expanded way.

The question remains unanswered, then, whether Epistemic Theory is truly deficient for neglecting to provide a methodology capable of locating what are thought to be fixed boundaries, as well as for failing to fully explain this obvious omission. While one might well want more than this from Epistemic Theory, the theory in its current form either succeeds or fails as one that is to this extent incomplete. As will be seen next, however, the methodology problem is not the only reason to question the soundness of Epistemic Theory.

3-3) UNRELIABLE ESTIMATES: DO VAGUE TERMS MEAN WHAT
         WE THINK THEY MEAN?

The methodological problem of having no means for ascertaining the 'true' extension of vague terms like <u>heap</u> has associated with it an added difficulty. Because we do not know precisely at what point addition of a single grain results in a non-heap becoming a heap, we also never know to what extent our intuition as to where this transition point occurs is inaccurate. This being so, the question arises as to whether vague terms, as normally employed, provide even reasonable approximations to 'correct' use. As pointed out by Neil Cooper (1995**:** 257)**:**

> "... the very idea of a precise but unknowable cut-off point between the thin and the non-thin (e.g., as Sorensen and Williamson have urged), would mean that each one of us could be unknowably wrong about classifying somebody as 'thin' or whatever".

Whereas we ordinarily think of standard usage as being indicative of what is correct use, Epistemic Theory suggests that this could be far from true. Not only would it be possible to make minor mistakes in judgement regarding the scope of vague terms, Epistemic Theory provides no reason why, in principle, these estimates need be anywhere close to accurate. Thus, a sorites series such as**:**

S[1]**:** A person 1 day old is not an adult.

.

.

S[18,262]**:** A person 18,262 days old is not an adult.

S[18,263]**:** A person 18,263 days old is an adult.   <3*3>

would, in theory, be defensible, this despite the fact that a person 18,262 days (i.e., fifty years) old would never, by ordinary standards, be considered young

enough not to be an adult. Yet, if we are permitted to dismiss our intuitions as fundamentally unreliable, this result hardly seems avoidable.

One curious implication, then, of Epistemic Theory is the possibility that vague terms have meanings far different from those normally associated with them. Because this theory provides no reliable link between a word's 'true' extension and its standard use (most attempts to fix a boundary will be incorrect), how to even approximate 'correct' use becomes problematic. Hence, not only does Epistemic Theory incorporate a typical skepticism regarding borderline cases, but even what seem like unquestionable exemplars of heaps (a collection of a trillion grains of sand, for example) have to be distrusted. If such extreme instances are also in doubt as potentially not heaps at all, one must wonder whether vague language is at all meaningful given this account.

3-4) THE AMBIGUITY PROBLEM

In addition, if, as Epistemic Theory argues, every vague term (e.g., 'adult') has a corresponding precise equivalent (e.g., 'at least 6001 days old'), then it would be reasonable to suppose that one could construct a language having a vague term for each precise one. Thus, not only would there be a vague term '6001 days-inexact' corresponding to the precise '6001 days-exact', but also a term '6000 days-inexact' corresponding to '6000 days-exact', etc. However, given the fact that every vague term is not totally precise in its actual application but does, instead, cover a range of values, means that any closely spaced pair (or group) of terms would have substantial overlap. For instance, if another word is similar in meaning to the word 'adult' but refers to someone at least 6000 (not 6001) days old, the word 'adult' would, by virtue of being imprecise, sometimes (though mistakenly, according to Epistemic Theory) be used to refer to a person who is merely 6000 days of age.

Once this feature of vagueness is acknowledged, however, an immediate difficulty presents itself. Since it is true both that (1) 'false' cutoffs are indistinguishable from 'true' cutoffs (because the true ones are unlocatable) and (2) similar terms overlap in their actual use, it must be that the true boundary for any particular vague concept will always match a false boundary for other similar concepts. This, in turn, means that misuse of one term is indistinguishable from correct use of a similar term. Accordingly, there exist no adequate criteria for differentiating whether:

A person 6000 days old is an adult.   <3*4>

should be interpreted as saying:

A person 6000 days old is at least 6001 days old.   <3*5>

which is clearly false, or, instead, that:

A person 6000 days old is at least 6000 days old.   <3*6>

which is clearly true. The point is that without first establishing that the word 'adult' is not being used ambiguously to refer sometimes to a person 6000 days old but at other times to a person 6001 days old, it becomes impossible to draw a distinction between proper use and improper use. Without there being some independent means for ascertaining where the 'true' boundary for 'adult' would genuinely be located, this kind of ambiguity simply cannot be ruled out.

Epistemic Theory is mistaken, then, in claiming that the sorites is resolved merely by the act of sharpening our vague concepts. Even if we were to presume it possible to entirely eliminate all imprecision connected with

vague terminology on an individual basis, it does not follow that the difficulties associated with the sorites paradox would be eliminated or even substantially altered for vague terms collectively. If vague concepts had boundaries which were not only precise but also unknowable, this would make closely related concepts ('6000 days-inexact'/'6001 days-inexact') inherently indistinguishable in cases like <3*4>. So, in trying to eliminate vagueness in this way, the imprecision characteristic of vague terminology just reemerges in a form that is no less systemic and no less problematic. As Cooper (1995: 257) notes, in such circumstances, "... an unknowable cutoff point would be of no practical or theoretical use". Rather than establishing that vague concepts can have clearly demarcated, fixed boundaries, Epistemic Theory, in supposing that there can be sharp boundaries, actually leads to the conclusion that no particular sharp boundary can be exclusively associated with a particular vague term. But if that is true, if there exists no <u>particular</u> boundary, and if closely spaced sharp boundaries are not clearly distinguishable from one another, then really no definite boundary has been established at all.

3-5) RECONSTRUCTING THE PARADOX: HOW THINGS SEEM

There is one last point of criticism with regard to Epistemic Theory. To be effective as an argument, Epistemic Theory relies on the ability to draw a distinction between where we merely think a boundary exists and where it truly does exist. It must be potentially true, at least, that some attempts to set a fixed boundary for vague terms like 'heap' are mistaken. In this way it becomes meaningful to differentiate proper application of a vague concept from inappropriate use of it. But it would seem rather easy to construct a sorites-type argument that does not allow for such a distinction and, therefore, could not possibly be resolved by Epistemic Theory. Here is such a modified version of the sorites that resists solution:

S[1]: One grain of sand would not be considered sufficient to constitute a heap.

S[2]: If one grain of sand would not be considered sufficient to constitute a heap, then two grains of sand would not be considered sufficient.

S[3]: Therefore, two grains of sand would not be considered sufficient to constitute a heap.

S[4]: If two grains of sand would not be considered sufficient to constitute a heap, then three grains of sand would not be considered sufficient.

.

.

etcetera   <3*7>

This, of course, leads to the undesirable conclusion that no quantity is sufficient to be considered a heap. But here it is not possible to argue that there is a precise, but unknown cutoff. Since it seems we cannot be mistaken concerning our own judgements (that, at least, is highly doubtful), it seems neither can we be mistaken regarding where we consider the transition from non-heaps to heaps to occur. For a sorites argument of this type, then, where the 'actual' cutoff is located becomes identical to where one believes it to be located. But this is exactly what prevents Epistemic Theory from being a plausible solution here. By default, the 'correct' or 'true' boundary is necessarily located wherever it is presumed to be located.

Furthermore, because this modified sorites employs the same logical form as any other sorites argument, other variants of the paradox should be no more amenable to solution than it is. Accordingly, unless this modified form is somehow exceptional, Epistemic Theory evidently fails to account for any version of the sorites, and the underlying idea behind this theory (that vague

terms really do have a fixed, determinate boundary) does not succeed in providing a convincing basis for resolving of the paradox.

CHAPTER 4: SUPERVALUATION THEORY

4-1) INTRODUCTION

By repeatedly judging whether a collection of N grains of sand does, or does not, constitute a heap, three outcomes are possible: (1) N is sufficiently large to be always considered a heap; (2) N is too small to ever be categorized as a heap; or (3) Classification of N is inconsistent: it is sometimes classified as a heap, but sometimes as a non-heap. As a result, as N varies across a wide range of values (from zero to infinity, for example), three separate regions, or zones, are formed. The center area (where the heap / non-heap categories overlap) is characterized by inconsistent use of the term 'heap', whereas the two extremes are regions where sizes are judged consistently (here there is total agreement as to whether N makes up a heap).

Supervaluation Theory attempts to explain the sorites paradox in terms of this structure. (For further exposition of this approach, readers are directed toward the works of Michael Dummett (1975), Kit Fine(1975), Bas van Fraassen(1966), and Hans Kamp (1981).) By starting with an accurate description of how vague terms actually operate in practice, it should be possible (or so one might think) to work back toward what this indicates about the structure of the argument, itself. Still, what consistency of classification reveals about the underlying logic of the sorites is open to some interpretation, and the same descriptive approach might be used to support considerably different explanations. In fact, at least two distinct ideas (those of localized and global supertruth)[2] stem from this one approach. Broadly speaking, both of these comprise theories of Supervaluation (or, alternatively, different facets of the same theory), although the term is sometimes applied with only one of these two senses in mind. To adequately survey Supervaluation Theory as a whole, each of these two variants needs to be evaluated separately.

Before examining this topic more closely, however, two additional features of Supervaluation should be mentioned. Supervaluationism arguments characteristically (1) reject the conditional/hypothetical premise (the Tolerance Principle) while also (2) adhering to classical logic.   In so doing, these proposed solutions provide a straightforward resolution of the paradox (the argument is valid but unsound), without the need to introduce modifications to standard logic.   Supervaluationism thus avoids two of the main criticisms associated with several other theories.   In particular, it (1) concedes (as one might hope) that the conclusion of the sorites does, indeed, follow from the premises given, without (2) adopting an unconventional (and, therefore, dubious) form of logic.   Not surprisingly, the resulting theory has decided advantages over some competing theories.

4-2) LOCALIZED SUPERTRUTH

Consider the statement**:**

     Roses are red.   <4*1>

What conditions make this assertion true?   Since <4*1> is, in fact, ordinarily construed as equivalent to the categorical statement**:**

     All roses are red.   <4*2>

this is true just in case all roses are red, but is false otherwise**:** A single counter-instance (a yellow rose, for example) is sufficient to render the declaration false.   It should be obvious that, on this account, statements like <4*1> cannot tolerate any exceptions and remain true.   By analogy, it could be argued that the statement**:**

N grains of sand are sufficient to constitute a heap.   <4*3>

is true only if it is always the case that N grains of sand are sufficient to constitute a heap. Thus, for <4*3> to be true it must be consistently true (i.e., supertrue).

If this idea of supertruth is applied individually to each value of N, the result is a localized theory of truth (localized in that every value of N is evaluated independently). Accordingly, any interpretation of the sorites based on localized supertruth (1) requires that only consistently true statements count as true, while also (2) permitting neighboring/adjoining positions (N+1 grains of sand vs. N grains) to hold different truth values. In effect, localized supertruth asserts the sorites argument should read as follows**:**

S[1]**:** A trillion-and-one grains of sand are always sufficient to
            constitute a heap.

S[2]**:** A trillion grains of sand are always sufficient to constitute a
            heap.

.

.

S[10^9+1]**:** One grain of sand is always sufficient to constitute a
            heap.   <4*4>

Like standard versions of the sorites, the conclusion is problematic. But unlike standard versions, the progression from the initial statement S[1] to the final statement S[10^9+1] is obviously invalid. Well before the last statement is reached, a prior statement is eventually encountered that is false, because at some point there is a transition from consistently true statements to inconsistent ones. This first inconsistent statement lies at the boundary between 'true' (that

is, supertrue) assertions and 'false' ones, i.e., at the border between regions of consistent and inconsistent use. And since these regions (in contrast to the constituent truth values (true / false) that comprise them) cannot overlap, the regional boundaries (unlike the boundaries between individual constituents comprising the series) must have sharp, not fuzzy, borders. The abrupt transition from supertruth to 'partial' truth guarantees such a sharp partition will always exist. Thus, contrary to what one might first think, there is a sense in which sharp boundaries for vague terms do, in fact, exist. It follows from this, according to Supervaluationism proponents, that the conditional premise (which is taken to imply the absence of any distinct boundaries) is erroneous. There is always a transition point in every sorites series where the Tolerance Principle fails. Hence, the sorites is an unsound argument, and its conclusion false.

4-3) LOCALIZED SUPERTRUTH: TRUTH CONDITIONS

Does this theory of localized supertruth provide sufficient grounds for rejecting the Tolerance Principle and, in turn, for rejecting the conclusion of the sorites argument, as well? To help answer this, consider again the truth conditions for the statement:

Roses are red.   <4*5>

How would the truthfulness of this ordinarily be assessed in a situation where some roses were definitely red, but all others merely borderline-red (reddish-violet, for example)? In a case like that the ability to assign a definite truth value to <4*5> would hinge on whether an unambiguous color assignment were always possible. If the hue of any rose were neither clearly red nor clearly not-red, it would then be indeterminate whether all roses are red. In those

circumstances the most that could be said is that it is uncertain whether roses are red and, moreover, it would be unresolvable whether they are.

But this conclusion is quite different from what would be decided on the basis of localized supertruth theory. Because borderline-red roses would sometimes be classified as not-red, these would serve as counterexamples that render <4*5> false. So, by this standard, statement <4*5> would be not just indeterminate, it would be clearly false. Thus, strict adherence to localized supertruth leads to the curious conclusion that what is indeterminate (and, therefore, possibly true, one would think) by normal standards should be taken as simply false. Yet, this is exactly opposite of what 'indeterminate' actually means.

Another problem arises in borderline cases when dual predicates are involved. Suppose, for example, that all roses consisted of a single reddish-violet rose, neither clearly red, nor clearly violet. Not only would this rose be sometimes categorized as not-red (i.e., violet), it would, likewise, at other times be categorized as not-violet (i.e., red). Thus:

Roses are violet. <4*6>

would, like <4*5>, fail to be consistently true. Yet, if both <4*5> and <4*6> are counted as false, the only two realistic options here (red or violet) for classifying borderline cases are eliminated. One would, then, apparently be committed to the proposition:

It is not true that roses are red and not true that roses are violet.

<4*7>

even though it seems equally obvious that:

Reddish-violet roses must be classified as red or violet.   <4*8>

Supervaluation Theory provides no simple answer as to whether these two assertions can be reconciled and, if so, how. In fact, the very procedure used to decide that <4*5> and <4*6> are false supports the accuracy of statement <4*8>: to conclude that a reddish-violet rose is neither red nor violet entails first judging it to be both red and violet.

Both these difficulties associated with borderline cases suggest that the analogy drawn between generalized truths such as <4*2> and sorites-type premises like <4*3> cannot be suitable across an entire sorites series (S[1] through $S[2(10^{12})-1]$, for example). The explanation for this lies with the criteria relevant to assigning higher-order truth values. In order to be supertrue, the claim:

Roses are red.   <4*5>

must meet two conditions. Every rose must be categorized as red (the consistency requirement), and, in addition, every rose must actually be red (the truth requirement). But it is difficult to see how localized supertruth ensures both these standards are met. In reformulating the sorites argument from:

S[1]: A trillion grains of sand are sufficient to constitute a heap.

  .

  .

$S[10^{12}-N]$: N+1 grains of sand are sufficient to constitute a heap.

$S[10^{12}+1-N]$: N grains of sand may or may not be sufficient to
                constitute a heap.

  .

.   <4*9>

to:

       *S[1]: A trillion grains of sand are sufficient to constitute a heap.

       .

       .

       *S[10^{12}-N]: N+1 grains of sand are sufficient to constitute a heap.

       *S[10^{12}+1-N]: N grains of sand are not sufficient to constitute a

                  heap.

       .

       .   <4*10>

localized supertruth alters some original (lower-order) truth values and assigns new, derivative (higher-order) ones. The indefiniteness of statement S[10^{12}+1-N] (due to conflicting judgements over whether N grains can constitute a heap) is replaced by a single definite determination *S[10^{12}+1-N] that N grains cannot constitute a heap. But this means at least some initial lower-order judgements (e.g., those which assert that N grains <u>are</u> sufficient to constitute a heap) must have been made in error.

       What localized supertruth leads to, then, is a situation where preliminary lower-order truth values, because they are subject to revision, come to be viewed as unreliable. If so, this leaves unanswered how to establish which classifications are <u>really</u> consistently true, not just apparently so. Localized supertruth implies that not only are truth values in the borderline (overlap) region undependable (that these are not consistently true is obvious), but any consistent evaluations are also questionable. Unless the lower-order truth values are, themselves, trustworthy, there is no reason to conclude that even a

consistently held judgement must be consistently right: there is no distinguishing consistent truth from mere consistent classification. For this reason, neither the location of, nor the existence of, a boundary between supertrue (meaning both consistent and true) statements and non-supertrue statements can be established. If the underlying truth values are doubtful, no finite number of grains is guaranteed to be large enough to constitute a heap, nor small enough to be a non-heap. Even clear-cut cases seemingly beyond dispute (a single grain of sand, or a trillion grains of sand) cannot be relied upon.

Localized supertruth fails, then, to establish that the division between consistent and inconsistent use matches the division between heaps and non-heaps. There is never an identifiable pair within a composite sorites series (that is, within an aggregate of sorites series considered collectively) that could serve to invalidate the Tolerance Principle and show that a single grain is enough to make a difference. But without this, localized supertruth provides no simple resolution of the paradox.

4-4) GLOBAL SUPERTRUTH AND THE FORCED MARCH

The second principal theoretical feature of the Supervaluation approach is its reliance on the so-called forced march as a model for interpreting the sorites. Quite simply, this views the incremental progression of the logical form:

S[1]: One grain of sand is insufficient to constitute a heap.

S[2]: Two grains of sand are insufficient to constitute a heap.

S[3]: Three grains of sand are insufficient to constitute a heap.

.

.   <4*11>

as indicative of the actual process by which heaps are distinguished from non-heaps. In other words, the meaning of the term <u>heap</u> is established by how one would, in practice, proceed in classifying ever larger (or ever smaller) collections of sand. The inevitable pattern that results from applying this kind of ordered procession is not to expand the set of non-heaps indefinitely, but to reach an abrupt break at some point in the sequence. For any sorites series there will always exist some adjacent pair where N grains of sand are not considered a heap, but N+1 grains are, where the continuity suddenly terminates**:**

S[1]**:** One grain of sand is insufficient to constitute a heap.

S[2]**:** Two grains of sand are insufficient to constitute a heap.

.

.

S[N]**:** N grains of sand are insufficient to constitute a heap.

S[N+1]**:** N+1 grains of sand are sufficient to constitute a heap.

<div align="right"><4*12></div>

However, in repeating this procedure over and over again, the place where addition of just a single grain is decisive will undoubtedly vary. N and N+1 will not be at the same fixed position in the series each and every time. As a result, forced marches <u>as</u> <u>a</u> <u>group</u> have no unique breakpoint, but rather are comprised of multiple breakpoints (at different locations) extending across a limited range of values. Thus, the aggregate collection of all forced marches, unlike each individual forced march, has no specific transition point that sharply differentiates heaps from non-heaps.

Viewed in this manner, the kind of unbroken chain of inference that

characterizes the sorites and justifies its conclusion cannot be truly representative of how vague terms actually function in practice. The continuity of the argument is invariably interrupted by a sudden switch in classification, one that makes use of the same category throughout an entire sorites series unrealistic. Because every forced march is marked by such a discontinuity, it never becomes reasonable to accept the logical form of the sorites argument nor its conclusion as correct.

One strength of this particular approach to resolving the sorites is that it manages to remove an element of imprecision associated with the paradox without going to the other extreme (as does Epistemic Theory) of eliminating vagueness completely. In fact, this global form of Supervaluation Theory succeeds in this by interpreting the sorites as simply a specific type of compositional paradox, i.e., one where the individual members of a set have a relevant property different than the set

as a whole. In the case of heaps, since every individual forced march departs from the Tolerance Principle:

If N grains of sand are insufficient to constitute a heap,

then N+1 grains of sand are also insufficient to constitute a heap.

<4*13>

somewhere along the way, it becomes correct to say, paradoxically, both that all forced marches (as individual members of the set) violate the Tolerance Principle and also that all marches (collectively, as the set itself) adhere to the principle (since no common value of N applies in each case). Thus, Supervaluation Theory succeeds in resolving the issue by neither discounting the Tolerance Principle entirely, nor by embracing it categorically. Insofar as the Tolerance Principle is interpreted as a rule of inference, it (and, by implication, modus ponens) is rejected, even though the same principle

correctly and accurately depicts the boundary between heaps and non-heaps as blurred (because of the multiplicity of locations where a march might suddenly reverse course). Thus, how the Tolerance Principle functions logically must be kept altogether distinct from how it operates descriptively.

4-5) REDUCTIONISM AND FUNCTIONAL EQUIVALENCE

The degree to which Supervaluation Theory is persuasive and convincing as a solution to the sorites paradox depends, in large part, on the extent to which the forced march provides a suitable model for interpreting the problem. But it must be remembered that there exist other means of generating a sorites series in addition to the forced march. One might, for example, proceed by selecting a random number from 1 to 1,000,000,000,000 and then judging whether that number of grains of sand would be sufficient to constitute a heap. In repeating this experiment (a random shot experiment) enough times to obtain multiple results for each random number, the outcome should look as though it had been produced by multiple runs of a forced march. Indeed, it is because of this sort of similarity that the forced march has the potential to serve as a comprehensive response to the paradox. Not just the random shot, but any method, in general, capable of producing a sorites series will generate results that in the long run are indistinguishable from an aggregate of forced marches.

This means, in effect, every sorites-type experiment is functionally a kind of forced march and, regardless of how a sorites series is arrived at experimentally, the sorites as an argument can invariably be viewed as though it were constructed from a set of forced marches. But since no true forced march continues on without at some stage violating the Tolerance Principle, by extension no other means of generating a sorites series should do so either. Accordingly, Supervaluationism singles out the Tolerance Principle as the one

flawed piece of logic that renders the entire argument illegitimate.

The most immediate difficulty raised by this whole line of reasoning is, however, whether there is any real justification for giving such preference to the forced march as a model for understanding the sorites. The basis for holding this view to begin with (depending on how one interprets Supervaluationism as a theory) is that (1) the only legitimate procedural method for generating a sorites series is the forced march; or (2) every procedure is reducible to a forced march; or (3) every procedure violates the Tolerance Principle (the forced march most obviously of all). But each of these is problematic in itself.

First of all, in practice several procedures other than the forced march (the 'random shot' being one such example) have been used as methods for analyzing the sorites. It is simply empirically false to insist that the forced march is (and, therefore, should be) the only means relied upon in deciding how to sort vague entities into separate categories (heap / non-heap). Alternatives that are equally important to our understanding the paradox are well known. Moreover, since each different procedure generates a pattern of results consistent with vague terminology (there is always an overlap region bounded by regions of consistent use) there is no intrinsic reason to favor the forced march over other models. These considerations make assertion (1) unsupportable.

The basis for assertion (2) has already been discussed at some length: it is because alternative methods (such as the random shot) yield long term results which are similar to the forced march that there is an argument to be made for these alternatives to be viewed as reducible to the forced march. But before conceding this point in its entirety, there are two additional issues to be addressed. The first of these concerns the uniqueness of claim (2). If assertion (2) is understood to mean not just that every procedure is reducible to a forced march, but only to a forced march, then this claim is untenable. Because each form of generating a series is functionally equivalent (in that the

long run results should look much the same), each method is reducible to any other. But if so, this means the very same reductionist argument that supports the forced march as a model for the sorites could just as well be used to infer that some other approach (such as the random shot method) serve, instead, as the standard model for interpreting the paradox. Consequently, assertion (2) is acceptable only if it is acknowledged that there exist alternative reductionist models for depicting a sorites series, not only the forced march.

The second ambiguity involving assertion (2) concerns the extent to which a reductionist approach is feasible in the context of the sorites. Assertion (2) may be interpreted as saying:

> Insofar as a procedure that generates a sorites series can be legitimately treated as though it were a forced march, that procedure can be thought of as though it violated the tolerance principle.
> <4*14>

But this makes sense only if the following corollary is also imposed:

> It is not necessarily the case that because a procedure that generates a sorites series can be legitimately treated as though it were a forced march that that procedure actually does violate the tolerance principle.
> <4*15>

Unless this corollary is added, <4*14> might be interpreted to mean just the opposite: that the ability to think of, e.g., a random shot experiment as though it were a forced march means it literally has all the relevant properties of a forced march. This, however, presses the analogy too far. It should be quite obvious that at least <u>mechanistically</u> the random shot proceeds in a fashion which is nothing like a forced march, notwithstanding that the pattern of results

might be identical. Accordingly, assertion (2) ought to be accepted only in the limited sense imposed by corollary <4*15>, but rejected otherwise.

The further point to be made, however, is that aside from what are close variants of the forced march, no procedure gives any independent reason to dismiss the Tolerance Principle as defective. If the Tolerance Principle is false expressly because a systematic march invariably reaches a breakpoint, then absent any breakpoint there is no actual breach of this principle. Thus, given that procedures like the random shot experiment lack a true breakpoint, assertion (3) cannot very well be correct (at least, not in other than a rather incidental sense). Yet, unless (3) is, indeed, correct, the whole rationale behind adopting the forced march as an explanatory model in the first place is brought into question. Without this provision, Supervaluation Theory would be limited to only those sorites series which actually derived from a forced march experiment.

4-6) INDISTINGUISHABLE PAIRS

Before leaving the topic of Supervaluationism, one last issue needs to be examined. Imagine there exist 1,000,001 tiles, each painted a different color. The first tile is colored with a pure blue pigment, the second with a color mixture that is 1 part green and 999,999 parts blue pigment, the third tile with a 2:999,998 green/blue mixture, etc. A sorites-type experiment might then be conducted as follows: A test subject is shown tiles #1 and #2 simultaneously and asked to classify each one as blue or green. The same procedure is repeated with tiles #2 and #3, #3 and #4, and so on until all the tiles have been examined and classified.

The pattern that invariably emerges from this kind of pairwise forced march has these features: (1) Adjacent tiles, when examined together, are always classified alike; (2) Like a standard forced march experiment, there

always comes an abrupt breakpoint where the classification suddenly changes (in the present case from blue to green); (3) One of the tiles will be classified both as blue and as green within the same run (i.e., within a single march). Characteristics (1) and (3) not only differentiate this kind of forced march from its standard version, but create a situation that is especially troubling for Supervaluation Theory. To understand why, we need a closer look at the details relating to this kind of sorites experiment.

The colored tile version of the sorites just described is a special instance of the pairwise forced march for the following reason: The gradations in color between adjacent pairs of tile are not only minor (which is typical of adjacent elements in a sorites series, anyway), but also considerably too slight to be distinguished by a subject who has the opportunity to make a direct visual comparison between the tiles. As a consequence, even someone aware that no two tiles are exactly the same color would be at a loss in trying to place members of the series in their correct order (more blue to less blue). When considered in a pairwise fashion, therefore, neighboring elements in a sorites series which are impossible to discriminate on a visual basis end up being categorized in the same way. In the present example, for instance, adjacent tiles are always designated as <blue/blue> or <green/green>, but never <blue/green>. By design, this type of sorites experiment makes it unreasonable to do otherwise.

Yet, as with any forced march the classification must change someplace in the progression. Since there is no <blue/green> pair where this transition occurs, what happens instead is that a <green/green> pair is designated as immediately following a <blue/blue> pair. And because every mixed color tile is evaluated not once, but twice, one of these tiles will be judged as blue initially, and as green subsequently. The result looks like this:

S[1]: Tile #1 and tile #2 are blue.

S[2]: Tile #2 and tile #3 are blue.

.

.

S[N]: Tile #N and tile #N+1 are blue.

S[N+1]: Tile #N+1 and tile #N+2 are green.

.

.   <4*16>

Thus, in contrast to all preceding members in the series (those consistently assessed as blue), tile # N+1 is evaluated in an inconsistent manner, as both blue and, later, as green (although at slightly different points in the sequence).

This result implies something critical for Supervaluation Theory. The effect of having at least one tile classified as both blue and green is to allow the Tolerance Principle to apply throughout a whole sorites series without interruption. One of the main arguments in favor of Supervaluation Theory, it must be remembered, is that the conditional statement:

If N+1 grains are sufficient to constitute a heap, then so are N grains.
<4*17>

is breached whenever applied to any individual forced march. But that presumes a certain consistency in classification that does not obtain in the colored tiles experiment. There is simply no place anywhere in the colored tiles march where one or the other of the following is not true:

If tile #N is blue, then tile #N+1 is blue.

If tile #N is green, then tile #N+1 is green.   <4*18>

Hence, even though, like the standard forced march, there exists an abrupt break where the classification suddenly changes (here, between S[N] and S[N+1]), nonetheless, every step along this pairwise march still conforms to <4*17>. But if <4*17> is, indeed, never violated, the Tolerance Principle cannot be the source of the paradox in this case. Consequently, the colored tiles experiment presents an awkward problem for Supervaluation Theory to handle. Without some reason to dismiss either the experiment or the implications of it, the pairwise forced march leads to the conclusion that Supervaluationism cannot be correct as a general theory of the sorites.

CHAPTER 5: MANY-VALUED LOGIC AND FUZZY LOGIC

5-1) MANY-VALUED LOGIC: INTRODUCTION

Two reasons borderline cases are so problematic is that (1) standard usage provides limited and, indeed, inadequate guidance in deciding how to categorize difficult cases of this sort and (2) repeated attempts to arrive at a conclusive result inevitably result in inconsistent classification. In fact, decisions regarding how to classify obviously borderline cases can seem quite arbitrary, lacking in any criteria that would serve to justify classification one way rather than another. Given a blue-green tile, for example, that appears as much blue as green, how would it be possible to either support one color category as correct (e.g., blue), or contest the other (green) as incorrect? Neither choice of color would be a reliable indication of the tile truly being that hue more than the other, nor would it be a good indicator of how the tile might be categorized if the classification experiment were repeated.

However, it does seem the difficulty in dealing with this sort of borderline case might be remedied by a simple procedure. Rather than employing a blue/green-only classification scheme to cases like this, a more complex grouping is possible. If a separate category were added specifically to account for bluish-green colored items, then the blue-green tile in question would be easily categorized (in this instance, as bluish-green) when evaluated using an expanded <blue/bluish-green/green> color classification system. By introducing a finer distinction than originally adopted, the colored tile would fit neatly in one particular color category (bluish-green). This would eliminate the questionable and inconsistent classification of the tile as either simply blue or green.

In the case of the sorites, where classically there exist just two groupings (heaps and non-heaps), the same procedure is possible. Collections not large enough to be clearly heaps nor small enough to clearly be non-heaps could be counted separately as borderline cases of heaps if another, third category were created for just that purpose. By designating indeterminate cases of heaps as borderline cases, the indecision connected with classifying those as simply heaps or non-heaps would be avoided. Thus, the most difficult and problematic cases could be eliminated just by changing the classification system to include an additional category to account for these.

This attempt to improve upon two-valued logic is the underlying idea behind many-valued logic. If increasing the number of categories (or truth values) from two to three (or some still larger, yet finite, number) succeeded, this would provide a ready-made solution to the sorites. Since it is only the indeterminate cases, not the clear-cut ones, that are difficult to classify, providing a means for grouping these together would effectively eliminate all the problem cases. And if that were true, it would be possible to have a sorites series that is non-paradoxical.

Noteworthy contributors to the development of such many-valued logics have included Jan Lukasiewicz (1987), Kenton Machina (1972; 1976), Michael Tye (1989; 1990; 1994), and Peter Simons (1992; 1994-1995) (a proponent of four-valued logic).

5-2) TRUTH-TABLES

There exist two principal difficulties in developing a viable many-valued logic. The first of these relates to how one might successfully construct a truth-functional calculus similar to one that exists for standard two-valued classical propositional logic, one whose truth-table values make intuitive sense. If one starts with the standard truth-values of 'true' and 'false' and adds to these a

third truth-value such as 'indefinite', the question arises as to how the basic logical operators such as 'and', 'or', 'negation', and 'implication' are to be interpreted. That is, what truth values are to be assigned to propositions that incorporate a new truth-value such as 'indefinite', values that are subsequently placed in the corresponding truth-table?

There are two noteworthy results relating to the various attempts by theoreticians to build many-valued truth tables for propositional logic. The first of these is the decided lack of agreement among those who have done so. George Lakoff (1973**:** 503) in his 1973 article, for instance, displays the truth table constructed by Bochvar for his three-valued logic with the truth table constructed by Lukasiewicz for his three-valued logic. A comparison of the tables shows different values in some cases for the simple connectives of conjunction and disjunction when indefinite truth-values are involved. Similarly, an inspection of the three-valued truth-tables created by Hallden (Williamson 1994**:** 104), Korner (Williamson 1994**:** 109), and Tye (1990**:** 544) display important differences in many instances. These inconsistencies highlight some of the difficulty one is bound to encounter whenever trying to introduce a supplemental truth value in addition to 'true' and 'false'.

The other notable aspect of working toward a truth-functional many-valued logic is the often counterintuitive results that ensue. Williamson (1994**:** 105) points out, for example, that many classical forms of inference, such as modus ponens, are invalid following Hallden's three-valued logic. He remarks that**:**

"The argument from 'Jack is not a philosopher' to 'Jack is not a bald philosopher' has a true premise and a 'meaningless' conclusion if Jack is a non-philosopher on the borderline of baldness. This result is hard to accept (Williamson 1994**:** 107)".

Similarly, Williamson (1994: 110) notes that in Korner's three-valued logic ~(p&~p) comes out 'neutral' if p is 'neutral'. Thus, the Law of Non-Contradiction fails, a result that is also quite disturbing. Michael Tye (1990: 544-545), himself a proponent of three-valued logic, attempts to defend some of the odd results of his own proposed logic as follows:

> "It may perhaps be charged that the proposed truth-tables yield some implausible truth-value assignments in connection with certain compound sentences having indefinite components. In particular, if A is indefinite, then A -> A is indefinite as is A^~A. I concede that the tables would certainly be mistaken, if they permitted A -> A to be false and A^~A to be true. But they do no such thing. A -> A is a quasi-tautology and A^~A is a quasi-contradiction. So, while the former statement cannot be false and the latter cannot be true, both can be indefinite".

But whether explanations of this sort salvage many-valued logics from their often strange implications is far from clear and far from a settled issue. Those who are not proponents of many-valued logics to begin with are apt to find these kinds of interpretations unconvincing.

5-3) BORDERLINE-BORDERLINE CASES

Despite the difficulties just mentioned regarding the construction of truth-tables for many-valued logics, truth-functionality, it turns out, is not the most troubling aspect of this approach. A second, still more problematic difficulty relates to the property of higher-order vagueness (see section 1-5). While increasing the number of categories (or truth values) might be an effective strategy for handling some intermediate cases, such as a colored tile that is midway between pure blue and pure green, it, nonetheless, seems seriously

inadequate for dealing with all instances of borderline behavior. A three-valued logic that might be sufficient for a rather limited and select group of cases, for example:

     S[1]: Pure blue is blue.

     S[2]: Pure green is not-blue.

     S[3]: An equal mixture of blue and green is neither-blue-nor-green.

<div align="right"><5*1></div>

would not be generally adequate across an entire sorites series. In the case of heaps, for instance, adding another truth value to the two-valued logic normally employed would result in something like the following:

     S[10^6+1]: A trillion-and-one grains of sand are clearly sufficient to constitute a heap.

     S[10^6]: A trillion grains of sand are clearly sufficient to constitute a heap.

     .

     .

     S[N+1]: N+1 grains of sand are clearly sufficient to constitute a heap.

     S[N]: N grains of sand are not clearly sufficient to constitute a heap.

     .

     .   <5*2>

or equivalently as:

S[10^6+1]: A trillion-and-one grains of sand are sufficient to

                constitute a heap.

S[10^6]: A trillion grains of sand are sufficient to constitute a heap.

  .

  .

S[N+1]: N+1 grains of sand are sufficient to constitute a heap.

S[N]: N grains of sand are sufficient to constitute a borderline-

     heap.

  .

  .   <5*3>

The problem here, as usual, is the sharp cutoff between groupings part way through the sequence. Since, to be truly vague, a concept must not only have no clear boundary between its extension and its non-extension, it must also have no precise boundary between its clear application and uncertain application, then the border between N+1 and N grains must, itself, be inexact. Although there can be clear cases of both heaps and borderline-heaps, at some point this distinction (the higher-order boundary), itself, becomes indeterminate, too. Thus, introducing an additional category to account for borderline-heaps does nothing overall to circumvent the fundamental problem that arises from vague terms always having indistinct boundaries. Creating a separate category just for borderline-heaps has the unfortunate effect of generating borderline cases of borderline cases. This consequence has been widely appreciated, and is the main reason few theorists consider many-valued logic a plausible approach for dealing with vagueness. Williamson (1994: 111) puts it this way:

"The objection to two-valued logic was the supposed impossibility of

classifying all vague propositions as true or false**:** but the phenomenon of second-order vagueness makes it equally hard to classify all vague propositions as true, false or neither. As grain is piled upon grain, we cannot identify a precise point at which 'That is a heap' switches from false to true. We are equally unable to identify two precise points, one for a switch from false to neutral, the other for a switch from neutral to true".

It should be obvious that the end result of increasing the number of truth values is invariably to increase the number of imprecise boundaries, not eliminate them. Even though clear-cut examples of borderline-heaps are easily classified if there is a separate category for borderline-heaps, to deal with unclear cases (the borderline-(borderline-heaps)) would require yet another such subdivision. The problem is, no matter how many partitions are created, and how fine the distinctions are drawn, for any finite number of categories, the number needed will always exceed the number present. Borderline-(borderline-cases) would need to have a category for borderline-(borderline-(borderline-cases)) which, in turn, would need a category for borderline-(borderline-(borderline-(borderline-cases))), etc. It is simply not possible to ever have enough divisions to adequately manage the new borderline cases that arise at each new level. Consequently, the many-valued approach ultimately fails both to eliminate indeterminate cases, and to provide a plausible solution to the paradox. The problem with vagueness, as it turns out, is not simply a matter of having too few categories to accommodate the range of cases present.

One final observation worth mentioning before leaving the topic of many-valued logic concerns the effect of employing those many-logics having a very large number of truth values. As was already noted, increasing the number of categories used for classification also increases the number of boundaries

where borderline cases can arise. A simple two-valued logic, for example, has just one region where this occurs, but a three-valued logic will have two such regions, a four-valued logic will have three such regions, and so on. The problem is, as the number of truth values becomes quite large, so, too, the total number of borderline regions will, itself, also become quite large. In fact, the number of truth values could increase to the point where it approaches or even exceeds the total number of items in a sorites series. In that case, the crowding together of categories used to make classifications would turn every item in the series into a borderline case. Since, however, the idea of applying many-valued logic to the sorites is to remove borderline cases, not create more of them, the strategy of continually increasing the number of categories in an attempt to manage difficult cases is actually counterproductive. When fine distinctions must be drawn across a long series, expanding the number of truth values not only does not work in eliminating borderline cases, in the extreme it leads to nothing but borderline cases.

5-4) FUZZY LOGIC: INTRODUCTION

One particularly interesting modification and extension of many-valued logic involves expanding the classification system for vague terms to one based on an infinite, rather than a finite, number of values. Whereas multi-valued systems of logic sort items into a fixed, but limited, number of categories, so-called fuzzy logic, in contrast, uses a continuous range of values to rank items and assign a quantitative rating to each. The fundamental advantage of this approach, if successful, would be to replace the abrupt discontinuity present in any multi-valued system with a transition region having smooth, gradual progression. In so doing, this method avoids (or, at least, attempts to) the main problem associated with a sorites series, namely, an unavoidably sharp transition from non-heaps to heaps. To illustrate this point,

compare a forced march sorites using a two-valued system of logic**:**

      S[1]**:** One grain of sand is insufficient to constitute a heap.

      S[2]**:** Two grains of sand are insufficient to constitute a heap.

       .

       .

      S[N]**:** N grains of sand are insufficient to constitute a heap.

      S[N+1]**:** N+1 grains of sand are sufficient to constitute a heap.

       .

       .   <5*4>

with the same series using fuzzy logic**:**

      S[0]**:** Zero grains of sand are sufficient to constitute a heap.

                                                 [[0.000]]

      S[1]**:** One grain of sand is sufficient to constitute a heap.

                                               [[0.001]]

       .

       .

      S[10^9]**:** A trillion grains of sand are sufficient to constitute a heap.

      [[1.000]]

                                               <5*5>

The first of these, <5*4>, somewhere has a sudden transition from non-heaps to heaps. But the series based on fuzzy logic simply assigns a value in the range [[0-1]] to each positive assertion without attempting to provide any clear line of demarcation between heaps and non-heaps. Any entirely unwarranted

statement is simply given the value 0, and any definitely trustworthy statement the value 1. As a result, fuzzy logic grants no sudden starting point for heaps (nor for non-heaps). Instead, there is a gradual increase in the extent to which collections of sand are deemed to be heaps. By entirely eliminating any classical type boundary in this manner, fuzzy logic tries to also eliminate the typical difficulties in logic associated with the sorites.

5-5) DEGREES OF TRUTH

In 1965 L.A. Zadeh published an influential paper on the notion of 'fuzzy sets' in his article entitled, appropriately enough, *Fuzzy Sets*. In this publication Zadeh (1965**:** 338) sets forth the basic principles of fuzzy set theory, included in which is the following definition**:**

> "A fuzzy set is a class of objects with a continuum of grades of membership. Such a set is characterized by a membership (characteristic) function which assigns to each object a grade of membership ranging between zero and one".

The unusual notion introduced here (one that contrasts with the classical notion of sets) is that membership in a set need not be an all-or-nothing affair in which membership necessarily implies full membership. Instead, there is an allowance for partial membership, and this can occur to varying degrees across a complete range of values (from total membership to none at all, and anywhere in between). That is how the idea relates simply to sets, themselves. Zadeh (1965**:** 339) goes on to add, however**:**

> "If the values of $f_A(x)$ are interpreted as truth values, the latter case corresponds to a multivalued logic with a continuum of truth values in the

interval [0,1]".

In other words, the concept is applicable not only to set membership, but can be extended to logic, itself, by introducing and allowing for the idea of 'partial truth'. Truth, like set membership, is thought of, in this way, as a property that can arise to varying degrees. It is this extension to logic that was soon recognized by a group of theoreticians (included in which would be J. A. Goguen (1968-1969), John King (1979), David Sanford (1974; 1975a; 1975b; 1976; 1979), Graham Priest (1979; 2000), and Dorothy Edgington (1992)) as a possible means for handling the troublesome difficulties set forth by the sorites paradox. The result has been a number of focused attempts to provide a solution by offering an explanation based on the idea of fuzzy logic developed from the notion of fuzzy sets.

To better understand the theory behind this idea of 'fuzzy logic' we turn to a simple example. In Fuzzy Set Theory, a bluish-green hue would be considered part member in the set 'green', while also part member in the distinct set 'blue'. Furthermore, a bluish-green hue twice as green as blue would be assigned a 'green' value of 2/3 and a 'blue' value of 1/3, signifying the color's membership in the set 'green' is twice as great as its membership in the set 'blue'. This kind of example is easily handled by Fuzzy Set Theory, and the meaning behind the assigned values is straightforward enough. The question that arises, however, is how this idea that originated in set theory translates to the altogether different context of a sorites series where the meanings of the terms 'partial membership' and 'degree of membership' are not so readily apparent, and might be open to differing interpretations. In fact, the even broader question is whether attempting to apply Zadeh's theory to the specific situation presented by the sorites is at all feasible, or whether this simply overextends Zadeh's original concept regarding sets.

On one view, for fuzzy logic to be relevant to the problem of the heap,

the notion of 'degree of membership' should translate to 'degree of truth'. In other words, saying:

One grain of sand is a heap to degree 0.001. <5*6>

which is equivalent to:

One grain of sand is sufficient to constitute a heap. [[0.001]]

<5*7>

means exactly:

It is true to degree 0.001 that one grain of sand is sufficient to constitute a heap. <5*8>

One reason this <u>Degree</u> <u>of</u> <u>Truth</u> <u>Theory</u> makes sense is we expect the categorical premise of a standard sorites argument ('Zero grains are insufficient to constitute a heap') to be entirely true, but the conclusion ('A trillion grains are insufficient to constitute a heap') to be totally false. Interpreting 'degree' as 'degree of truth', in these circumstances, preserves our fundamental intuitions about how obviously true (or false) we find such extreme examples.

Secondly, fuzzy logic as a Degree of Truth Theory provides a plausible rationale for affirming the Tolerance Principle (see section 1-4) for each individual step in a sorites series, without, however, affirming the conclusion of the argument. According to Degree of Truth Theory, even though the Tolerance Principle is, strictly speaking, not altogether correct, it is, nonetheless, nearly so. That conclusion is based on the following reasoning: If we hold that a conditional is true when it is truth preserving, and it is truth preserving when an

antecedent and its consequent have the same truth values ([[0.42]] and [[0.42]], for example), then a conditional is true when an antecedent and its consequent have the same truth values. But if truth comes in degrees, then a conditional is true just to the extent that an antecedent and its consequent have <u>similar</u> truth values ([[0.42]] and [[0.43]], for example).   In the case of the Tolerance Principle, the statement**:**

> If N grains of sand are insufficient to constitute a heap, then N+1 grains are insufficient to do so.   <5*9>

is nearly truth preserving in that the assertions**:**

> N grains of sand are insufficient to constitute a heap.   <5*10>

> N+1 grains of sand are insufficient to constitute a heap.   <5*11>

are true to nearly the same degree as one another.   Furthermore, the conditional <5*9> is invariably close to true (that is, for each step throughout the entire series) since the difference in degree of truth between adjacent elements will consistently remain small regardless of the size of N. And because every sorites-type argument is similarly characterized by small stepwise changes, every sorites conditional (like <5*9>) will be nearly true throughout.

At the same time, however, there is a cumulative effect from repeated application of the Tolerance Principle. While conditional <5*9> is nearly truth preserving for any pair of adjacent elements in a sorites series, its failure to be entirely truth preserving means that multiple applications of <5*9> reduce the overall effect. For example, the conditional**:**

> If 0 grains of sand are insufficient to constitute a heap, then 20 grains

are insufficient to do so.   <5*12>

(which reflects the combined result of 20 applications of the Tolerance Principle) is far less truth preserving than any single step in the sequence would be. Since every step along the way results in some loss, the total combined effect can be quite substantial. For example, when applied across an entire sorites series, such as**:**

> If 0 grains of sand are insufficient to constitute a heap, then
> 1,000,000,000,000 grains are insufficient to do so.   <5*13>

the loss is so great as to not be truth preserving at all. Thus, the Tolerance Principal fails to preserve truth across a sorites series collectively even though it is substantially truth preserving with each individual step in the series. This explains how it is that the Tolerance Principle is, in one sense, truth preserving (it is largely so for each small step) and yet, in another sense, is not (cumulatively). Degree of Truth Theory provides a tenable account of why we are inclined to accept the conditional premise of the sorites, yet also reject the conclusion that would otherwise seem to follow from it.

5-6) TRUTH-FUNCTIONALITY

In the view of some, at least, the success of fuzzy logic, both practically as well as theoretically, hinges on whether it is possible to develop an adequate formal calculus, one comparable, in kind, to that which already exits for classical propositional logic. In fact, to no small extent, the debate between advocates of fuzzy logic and those with rival views has centered around whether attempts to formulate such a calculus have been at all successful. It is none too surprising, therefore, that proponents have often seen it as their

primary task to work out the details of a formal logic, and that critics have directed a substantial effort towards identifying the problems in doing so.

The difficulties in developing an adequate calculus for fuzzy logic can be better understood by considering just two of the common connectives in classical propositional logic. The rules for using the disjunctive 'or' and the conjunctive 'and' are straightforward: compound statements employing 'or' are true if-and-only-if at least one disjunct is true; compound statements employing 'and' are true if-and-only-if all conjuncts are true. Because the truth of any such compound statement is governed by the truth values of its constituent parts in this manner, and in a way that is rule governed, the logic and its calculus are be said to be 'truth-functional'. Unlike classical logic, however, fuzzy logic permits conjunctions, disjunctions, and their constituent conjuncts to take on intermediate values that indicate partial truth. As a result, it is far from obvious how to properly evaluate compound statements within the framework of fuzzy logic. Not only does the assignment of truth values to compound statements, itself, become problematic, it is, in addition, questionable whether fuzzy logic can, like classical logic, be rendered truth-functional. (And even supposing it cannot be, what, then, are we to conclude? If fuzzy logic turns out to not be truth-functional, does this automatically eliminate it as a possible solution to the sorites, or might it somehow still be an acceptable way of resolving the paradox?)

The basic criticism with regard to truth-functionality has been noted by a number of critics. To begin with, Timothy Williamson (1994: 114) notes:

"Continuum-valued logic assumes more than a continuum of degrees of truth. It makes the further assumption that the main sentence functors satisfy generalizations of truth-functionality to those degrees. When the application of such functors builds a complex sentence out of simpler ones, the degree of truth of the former is held to be determined as a

function of the degrees of truth of the latter".

But, then, how to accomplish this creates some notable difficulties. Sorensen (1988c**:** 58) raises the following issue**:**

> "One cost of this solution is the revision to logic. Intuitively, 'This collection of sand is either a heap or not a heap' is a tautology and so should have a degree of truth equal to 1. But given that the collection of sand is a borderline heap, 'This is a heap' and 'This is not a heap' will have degrees of truth equal to less than 1. The standard many-valued rule for determining the truth value of a disjunction is to assign the disjunction the higher of the truth values assigned to its disjuncts. Accordingly, the truth value of 'This is a heap or not a heap' will be less than 1. So unless the standard rule is replaced, many-valued theorists must either follow Sanford and deny the truth functionality of the logical connectives (to preserve classical theorems) or follow Machina and deny the classical theorems (to preserve truth functionality)".

Indeed, the difficulty presented here has also been recognized by the proponents of fuzzy logic, as well. Dorothy Edgington (1992**:** 200-201) remarks**:**

> "How do degrees of truth work? The widely held view is that we generalize from a two-valued truth-table, retaining the idea that the value of a complex proposition is a function of the values of its parts.... The favorite proposal for conjunction and disjunction is**:**
> > (a) $v(A\&B) = \text{Min}[v(A),v(B)]$
> > > $v(A\lor B) = \text{Max}[v(A),v(B)]$.
> > An alternative proposal is**:**

(b) v(A&B) = v(A) · v(B)

v(AVB) = v(A) + v(B) - v(A) · v(B)

... I think these proposals are wrong. First we can rule out the alternative proposal (b) for conjunction and disjunction by letting A and B be synonymous: let A be 'x is small' and B be 'x is little'. Then A&B, AVB, A and B should have the same value....

Second, although proposal (a) works in the above case , it is wrong, I think, when A and B are independent. Let the objects x, y, z be balls of various colours and sizes. Suppose:

v(x is red) = 1, v(x is small) = ½

v(y is red) = ½, v(y is small) = ½

v(z is red) = ½, v(z is small) = 0.

Intuitively, 'x is red and small' has a higher degree of truth tha[n] 'y is red and small'; and 'y is red or small' has a higher degree of truth than 'z is red or small'; yet all of these are ½ true, on proposal (a)".

Thus, this standard approach to fuzzy logic seems quite inadequate. To retain fuzzy logic as a viable form of logic, therefore, Edgington (1992: 201) proposes that, rather than adhering to the standard view, we treat conjunctions in fuzzy logic in a manner following probability theory:

"We can do better with degrees of truth by giving them a probabilistic structure. p(A&B) and p (AVB) are not determined by (but are constrained by) p(A) and p(B); p(A&B) = p(A) · p(B|A)".

Consequently, the truth values of the composite conjunctions do seem to make sense in light of the values assigned to the individual conjuncts, and the standard criticism of fuzzy logic in terms of truth-functionality is no longer problematic. This is one potential way to handle the issue.

A second approach suggested by Kenton Machina and David Sanford, though, is simply to challenge the supposed requirement that fuzzy logic must be truth-functional.[3]   Although it has often been taken for granted that fuzzy logic, like classical logic, should be truth-functional, is this necessarily a requisite feature? Machina (1976**:** 56) states**:**

> "We may now go on to ask whether our desired logical system is entirely truth-functional with regard to its sentential connectives....
>
> It does not seem entirely clear at the outset whether, say, a formula of the form 'p&q' should always find its value from the values of 'p' and 'q', even when 'p' or 'q' have intermediate values. For what do we say about an expression of the form 'p&~p' when 'p' is half true? After all, both conjuncts are true to a degree. Does that mean that contradictions can be 'half' true, as wholes? If we take the truth-functional approach, it seems we shall be committed to the partial truth of some contradictions. On the other hand, if we were willing to give up truth-functionality, we could insist that in the case of a contradiction, even though both conjuncts are true to a degree, nevertheless the whole formula is completely false, because the conjuncts are not logically independent of one another. One might be reminded here of probability theory, in which the conjunction of two events, A and B, each having probability greater than 0 and less than 1, may nevertheless have probability 0 when A and B are mutually exclusive events.
>
> Similar questions may be raised with respect to the law of the excluded middle, 'pV~p', when neither 'p' nor '~p' is completely true or completely false. In such a case, the truth-functional approach demands that 'pV~p' be treated just like any other formula of the form 'pVq'. So it would seem that the most natural sort of truth-functional definitions of the connectives 'V' and '&' in our multi-valued logic are likely to result

in the loss of both the law of noncontradiction and the law of the excluded middle".

Sanford (1976: 202-203) makes a similar point:

"   [pVq] = max ([p],[q])

        [p&q] = min ([p],[q])

Other writers, myself included, find some of the consequences of these definitions unacceptable. When [p] = 0.5, for example, [p&~p] = 0.5....

    Conjunction, therefore, is not to be defined truth-functionally. Logical relations between conjuncts must be taken into account when assigning values to a conjunction, although when there are no relevant logical relations, the value of a conjunction is determined by the value of the conjuncts.   Disjunctions   and   conditionals   must   receive   similar treatment....   Kamp and Fine both argue against using any truth-functional many-value semantics to deal with the logic of vagueness, and I agree completely with their conclusion. But their arguments are arguments merely against truth-functionality. Neither Kamp nor Fine gives any argument against the Principle of Infinite-Value Semantics, a principle which is logically independent of the Principle of Truth-Functionality. They each ignore the possibility of treating a logic of vagueness in an infinite-value semantics in which some connectives are not defined truth-functionally".

Thus, even though fuzzy logic might not be truth-functional in the way classical logic is (or in any other way, for that matter), it is arguably true that it could, nonetheless, still work as a solution to the sorites. In view of this, the unsettled debate over truth-functionality between proponents and critics of fuzzy logic could be largely irrelevant. If Machina and Sandford are not mistaken, resolving

this particular issue might have little to do with whether fuzzy logic is capable of providing a viable solution to the sorites paradox.

5-7) PARTIAL TRUTH

A second, and perhaps the most foundational objection to a theory of vagueness based on 'degrees of truth' is the criticism that the very notion of partial truth is, itself, unsatisfactory, that the theory of truth advanced by fuzzy logic in support of this idea is inherently faulty. As noted previously, the ideas introduced by Zadeh involve an important distinction between fuzzy logic (with its degrees of truth), and the somewhat different idea of fuzzy sets. While these ideas are related, they are not identical, and the inference from one to the other is merely conjectural:

> "Zadeh's original paper concerns fuzzy sets, not fuzzy logic. It is nevertheless natural to suppose that if membership of the set of heaps is a matter of degree, then so too is the truth of 'That is a heap'.... (Williamson 1994: 122)".

This raises the obvious question of whether there is any real justification for making the logical leap from attributes of sets to those of truth, itself. Sainsbury (1986: 98-99) remarks:

> "The following philosophical issue remains: all that the comparisons uncontroversially entitle the theorist to is degrees of the property the predicate expresses: degrees of redness or degrees of adulthood. To make good degree theoretic semantics, its central concept, degrees of truth, has to be justified. Accepting that the data demand a recognition of degrees of redness does not, in and of itself, require accepting that

they demand recognition of degrees of truth. There is no obvious path from the fact that things can vary in how red they are to the claim that sentences can vary in how true they are....

Let's suppose that the degree of truth theorist uses 1 to represent complete or maximal truth, 0 to represent complete or maximal falsity, and the numbers in between to represent the intermediate cases characteristic of vagueness. Thus 'a is bald' will be accorded degree 1 if a's scalp is hairless, degree 0 if a has a full head of hair, and some number in between if a is neither definitely bald nor definitely not bald. The question is: why should we think of these numbers as representing degrees of truth, rather than degrees of baldness"?

The basic objection, then, is that regardless of the fact that some attributes can and do exist to varying degrees, it need not follow that assertions employing these attributes are, themselves, capable of being true to varying degrees.

One reason for thinking that Sainsbury's skepticism here is justified is that, as ordinary conceived, truth is an iterative property. If one were to assert:

It will rain tomorrow.   <5*14>

it follows that one is also asserting that it is true that it will rain tomorrow, that it is, likewise, true that it is true that it will rain tomorrow, that it is true that it is true that it is true that it will rain tomorrow, etc. If one of these assertions is made, then implicitly all of them are. Thus, if fuzzy logic gives a realistic account of truth, it should allow statements like these to be iterative in this same way.

Additionally, besides the fact that this iterative property holds for full truth values in standard usage, there is another reason why we should expect iteration to also be an attribute of partial truth. To begin with, the underlying

idea behind degrees of truth is that a characteristic being present to a degree is equivalent to it being true to that same degree that the property is present. One would say of a hue that is 80 percent blue, for example, both that it is true that it is 80 percent blue or, equivalently, that it is 80 percent true that it is blue. In general, fuzzy logic holds that being true to a degree is the same as having a characteristic to that degree. But since truth is, itself, a characteristic, it, too, ought to be subject to the same principle. There should be such a thing as being true to a degree, itself, being true to a degree. Thus, truth becomes self-referential and iterative. Moreover, a statement such as:

    This hue is blue.   <5*15>

is actually just another way of saying:

    It is true that this hue is blue.   <5*16>

Thus, the assertion that it is 80 percent true that (this hue is blue) is really to say that it is 80 percent true that (it is true that this hue is blue), i.e., it is 80 percent true that (it is 100 percent true that this hue is blue). At its simplest, most basic level fuzzy logic is already self-referential when it ascribes a partial-truth value, and this occurs in a way that makes it an iterative function.

      That conclusion, however, leads to serious difficulty: Is it proper, then, to say of statements that are partially true that they, too, are partially true? There is a dilemma in trying to answer this. If partial-truths are ruled as legitimate only when they, themselves, are wholly true or wholly not-true, there ought be some valid theoretical basis for such a rule. Not only does fuzzy logic not provide that, the fundamental idea that partial-truths correspond to partial characteristics goes against this. Statements like this, then, that attribute partial truth to partial truths ought be allowed. But permitting such statements

does not seem a viable option. What sense can be made of the assertion:

It is eighty percent true that it is ninety percent true that this hue is blue.
<5*17>?

The idea that degree of truth relates to degree of set membership suggests that it is correct to assert a hue is ninety percent blue only if the degree of membership in the set blue is exactly ninety percent. But then, in cases where the membership is different from ninety percent, one ought say:

It is zero percent true that it is ninety percent true that this hue is blue.
<5*18>

(which contradicts <5*17>). For the same reason, <5*17> is not simply equivalent to stating that the hue is seventy-two percent blue. That also makes no sense in terms of set membership. The only way <5*17> could be true is if it were possible for both the embedded clause 'is ninety percent true' and the entire clause 'is eighty percent true that it is ninety percent true' to both be true, simultaneously. But that, of course, makes no sense, either.[4]

The ultimate result, then, is that fuzzy logic either retains the property of iterative truth at the risk of becoming incoherent, or prohibits statements that assert partial truths are, themselves, partially true. In the latter instance, however, fuzzy logic eliminates what is arguably an essential attribute of truth, namely, its ability to iterate. If so, fuzzy logic appears to survive only at the cost of altering our normal understanding of what it means for something to be true. This, in turn, suggests that what is decreasing across a sorites series is not, as fuzzy logic claims, truth, itself, but merely the very property we ordinarily consider as relevant to the series. What fuzzy logic seems not to do is apply a concept of truth that captures its genuine meaning in any normal sense of the

term.

5-8) PRECISION

  The third main difficulty relating to a theory based on fuzzy logic is the question of precision. There are two possible ways one might try to construct (or interpret) an infinite-value logic[5] and apply it to the sorites. One approach is to view the truth values specified as absolutely precise, rather than mere estimates. Thus, a designated value of $[[0.22]]$ would not allow that a slightly different value of, say, $[[0.23]]$ might also (or instead) be correct. That would imply the assigned value had some margin of error. The trouble with this particular idea is that it appears to introduce an artificial degree of exactness into concepts that are inherently inexact. If one adopted the standard, for example, that what constitutes a heap is a collection of grains of sand of sufficient number to always be considered a heap (i.e., a collection assigned a value of $[[1.00]]$), then there would be a clear numerical demarcation between heaps and non-heaps. The only question would be where to find this dividing line between consistent and inconsistent classification in terms of the actual number of grains of sand. Similarly, if the standard for being a 'heap' were that of being more of a heap than a non-heap, there would, again, be a clear line of demarcation, in that case set at a value of $[[0.50]]$. But either way, it would, in principle, be possible, as well as perfectly legitimate, given this interpretation of fuzzy logic, to have a sharp division between heaps and non-heaps. If one supposed that precise values could, in fact, exist, this would allow for criteria to be set that would define sharp boundaries such as these. Yet, this is just what cannot be allowed of any legitimate theory of vagueness. To accurately reflect what it means to be vague, terms like 'heap' cannot be allowed to be made overly precise, and any well reasoned theory of the sorites ought reflect this. Whatever solution is proposed for this paradox, it must be one in which

vague concepts remain imprecise and retain their vague character.

The alternative view is that fuzzy logic operates by ascribing merely approximate truth values, not precise ones. Thus, assigning a numerical value of [[1.00]], for example, to a particular number of grains of sand cannot be taken as guaranteeing that this amount is definitely a heap, nor can a value of [[0.51]] be deemed as assuring that the quantity present is more a heap than not a heap. These values are themselves inexact, not because they are mere estimates of more precise values, but because, for vague terminology, truly precise values do not actually exist. As a result, fuzzy logic need not result in imprecise terms being made into precise terms.

The negative consequence of this, however, is that the sorites still retains its paradoxical nature. Since the designated truth values are, themselves, to be regarded as imprecise, it remains possible to construct a sorites-type paradox as follows:

S[0]: Zero grains of sand are sufficient to constitute a heap.

[[0.50]]

S[1]: One grain of sand is sufficient to constitute a heap.

[[0.50]]

.

.

S[N]: N grains of sand are sufficient to constitute a heap.

[[0.50]]

S[N+1]: N+1 grains of sand are sufficient to constitute a heap.

[[0.50]]

S[N+2]: N+2 grains of sand are sufficient to constitute a heap.

[[0.50]]

.

.

S[109]): A trillion grains of sand are sufficient to constitute a heap. [[

0

.

5

0

]]

<5*19>

where the same truth value of [[0.50]] has been assigned to each statement in the series. Obviously, the initial group of assertions beginning with S[0], as well as the last group of assertions ending with $S[10^9]$, would not have values anywhere near [[0.50]], a number that reflects a borderline case. Assigning the value of [[0.50]] to any of these would plainly be inaccurate.   But it is, also, apparent, given the imprecision inherent in assigning values to borderline cases, that there would somewhere intermediate in the sequence be a range of statements, not just a single one, that might legitimately have a truth value of [[0.50]], and the location of these would, itself, be indefinite. The problem is, this gives rise to the same difficulty characteristic of any sorites series, namely, that what is true of one statement is also true of the next. In this case, if a truth value of [[0.50]] were valid for the claim that N grains of sand are sufficient to constitute a heap, then the same value ought to apply to N+1 grains, as well. But this makes it unavoidable that each subsequent statement again be assigned the value of [[0.50]], a result that is quite obviously false, most notably as the series approaches one trillion grains of sand. Thus, the idea of an infinite-valued logic based on inexact values, just like the alternative based on absolutely exact values, provides no real solution to the sorites.

CHAPTER 6: NON-CLASSICAL SEMANTICS

6-1) INTRODUCTION

There is an old adage that even a stopped clock is right twice a day. Although this might seem obviously true for just about any time of day (a clock displaying, say 9:34, would plainly match the actual time at both 9:34 A.M. and 9:34 P.M.), it is debatable whether this would also hold for a clock reading 12:00. Because 12:00 midnight divides one day from the next but is neither the last moment of one day nor the first moment of the following day, it is unclear whether 12:00 midnight should count as one of two instances when the clock time matches the actual time. Since midnight does not correspond to any day in particular, it is arguably the case that only once each day (at 12:00 noon) would a clock fixed at 12:00 agree with the true time: The match that occurs at 12:00 midnight cannot be assigned to a specific day, and, therefore, it cannot be included in the count for any day. Thus, one might conclude that a clock stopped at 12:00 is right just once a day, not twice.

On the other hand, that line of reasoning might not seem wholly convincing. An opposing view is that during any 24 hour period that does not begin and end at 12:00, there will always be two instances where the clock display of 12:00 corresponds to the actual time of day. It is really only by adopting and following the convention of dividing days exactly at midnight (or, alternatively, at noon) that the question even arises as to whether a clock fixed at 12:00 (unlike all other times) would be correct twice a day. Without such an arbitrary rule, 12:00 would be as unexceptional as any other time. Consequently, there is no truly compelling reason why 12:00 midnight could not, in principle, be assigned to the day prior to, or the day following midnight, even though, in practice, it ordinarily isn't. Yet, if either of these assignments were

made (and regardless of which one were made), the result would be that 12:00 would occur twice during every day. Hence, a stopped clock could be deemed always right twice a day, even if it does read 12:00.

Or consider the following situation: A person while driving his automobile in the state of Colorado tosses a candy wrapper out the car window near the border with Arizona, Utah, and New Mexico. The momentum of the throw carries the wrapper into Arizona where it bounces up off a rock and subsequently lands in Utah. No sooner does it come to rest there than a gust of wind blows the wrapper across another state line into New Mexico where it becomes firmly embedded in a cactus. Supposing that each state has a law prohibiting littering (but also that no law is sufficiently detailed to clarify whether the action described did, strictly speaking, constitute an act of littering within its own borders), the question arises as to whether the driver is, indeed, guilty of littering, and, if so, where? It seems that the driver did, in fact, litter, just as surely as had every phase of the event occurred entirely within the bounds of a single jurisdiction and had no border been crossed at any time. The action of the driver is not materially changed by this minor difference. Hence, one might suppose he must have broken the law somewhere. At the same time, however, it is difficult to make a compelling individual case that he littered in Colorado, or, separately, that he littered in Arizona, or in Utah, or in New Mexico. Not having met both criteria of what ordinarily constitutes littering (tossing litter and landing it) in any one particular place, it is uncertain how he could be in violation of any specific law. Moreover, if either criterion were a non-essential aspect of littering, if just one were sufficient to constitute a violation, the driver might be prosecuted multiple times in various states for the same single act. That does not seem like the right outcome either.

In both these examples there are two competing arguments, neither of which is without warrant, but neither of which is clearly decisive, either. It is open to debate how cases like these are to be resolved and whether more

general assertions (e.g., the driver littered somewhere) have truth values even when more specific related assertions (e.g., the driver littered in Colorado) might not. In fact, there exist two diametrically opposed theories on the matter, and finding the right solution to the sorites paradox may well depend on which of these is correct.

6-2) TRUTH-FUNCTIONALITY

According to classical logic, compound statements employing the simple connectives 'and' or 'or' are truth-functional: the truth of a compound statement is entirely dependent on the truths of its individual components. Thus, the truth value of the conjunction:

Socrates was Athenian and Socrates was not color-blind.

<6*1>

is, on this account, contingent upon the truth or falsity of each of the component assertions:

Socrates was Athenian. <6*2>

and

Socrates was not color-blind. <6*3>

Accordingly, unless it is possible to establish the truth values of these constituent parts, the truth value of the conjunction cannot, itself, be settled: If Socrates was, indeed, Athenian, but it is indeterminate whether Socrates was ever color-blind, it must also be indeterminate that Socrates was both Athenian

and not color-blind. That that conclusion follows from those premises seems quite straightforward.

Not every statement constructed from indeterminate propositions, however, follows the same rules of inference. In contrast to statement <6*1>, some conjunctions that combine indeterminate assertions seem to allow for definite truth values. For example, even though it might be doubtful that:

Socrates was never color-blind.   <6*4>

and also doubtful that:

Socrates was color-blind.   <6*5>

it is difficult to see how the conjunction:

Socrates both was color-blind and was never color-blind.

<6*6>

could possibly be true, or how the disjunction:

Socrates either was color-blind or was never color-blind.

<6*7>

could be other than true. <6*6> conjoins two mutually exclusive situations, while <6*7> conjoins two jointly exhaustive ones. Thus, without knowing whether Socrates was or was not ever color-blind, it would seem, nevertheless, possible to surmise that statement <6*6> is, in fact, false while statement <6*7> is true. On this non-classical view of semantics, compound statements can, at times, be true (or false) without their constituent parts being true (or false). Thus, what

counts as being truth-functional for one semantic form can be considerably different from what counts as truth-functional for another.

There is a close parallel between the type of conjunctions just examined and certain of compound statements based on vague concepts. For instance, while it might be unclear whether a particular blue-green colored tile should be classified as green or as blue, it does, nonetheless, seem obvious that:

This tile is green or it is blue.   <6*8>

would apply to any bluish-green tile, including one that is borderline between green and blue. Again, according to non-classical semantic theory, this assertion is true for a borderline blue-green tile even though neither:

This tile is green.   <6*9>

nor

This tile is blue.   <6*10>

themselves, have any obvious truth value in such cases. What non-classical semantics permits, in instances like this, is for a disjunctive statement like <6*8> to be assigned a truth value without similarly assigning any such values to the corresponding disjuncts (<6*9> and <6*10>). This, in turn, suggests a means for recasting the sorites in a form that provides a semantic solution to the paradox.

6-3) THE SEMANTIC FORMULATION

Non-classical semantic theory is relevant to the sorites argument in two

different ways. First of all, it says something about individual borderline cases. Even though neither**:**

N grains of sand are sufficient to constitute a heap.   <6*11>

nor

N grains of sand are insufficient to constitute a heap.   <6*12>

can be assigned any clear truth value if N is borderline case, Non-Classical Semantic Theory tells us that the disjunction**:**

Either N grains of sand are sufficient to constitute a heap or are insufficient to constitute a heap.   <6*13>

is, nonetheless, a true assertion. Thus, the sorities can be represented as**:**

S[1]**:** One grain of sand is insufficient to constitute a heap.
S[2]**:** Two grains of sand are insufficient to constitute a heap.
  .
  .
S[N]**:** N grains of sand are sufficient to constitute a heap or are insufficient to constitute a heap.
S[N+1]**:** N+1 grains of sand are sufficient to constitute a heap or are insufficient to constitute a heap.
  .
  .
S[10^{12}]**:** A trillion grains of sand are sufficient to constitute a heap.

<6*14>

since each borderline case (where classification as a heap is uncertain) is capable of being expressed as a disjunction.

What follows from this and what, in fact, is key in resolving the paradox, is how Non-Classical Semantic Theory applies to these disjunctive borderline cases taken collectively. Because a sorites series lacks a sharp cutoff, the borderline cases cover a range of values and are, therefore, themselves, expressible in the form of a collective disjunction:

Either

(N-1 grains of sand are insufficient to constitute a heap

and N grains are sufficient to constitute a heap.)

or (N grains of sand are insufficient to constitute a heap

and N+1 grains are sufficient to constitute a heap.)

or (N+1 grains of sand are insufficient to constitute a heap

and N+2 grains are sufficient to constitute a heap.)

.

.

etcetera   <6*15>

Here, a single disjunction substitutes for the series of individual disjunctions employed in version <6*14>. But since the disjuncts that comprise this list are, again, both mutually exclusive and jointly exhaustive, Non-Classical Semantic Theory holds that the entire disjunction <6*15> is true as a generalization, even though the same cannot be said of any of the constituent disjuncts.

This interpretation of the paradox provides reasonable theoretical grounds for rejecting the logical form employed by the sorites. Remember that for the conclusion of the sorites to follow, the Tolerance Principle (i.e., the

conditional premise) must hold throughout. But if the Tolerance Principle is expressed as**:**

It is not the case that

    (N-1 grains of sand are insufficient to constitute a heap

     and N grains are sufficient to constitute a heap.)

or (N grains of sand are insufficient to constitute a heap

     and N+1 grains are sufficient to constitute a heap.)

or (N+1 grains of sand are insufficient to constitute a heap

     and N+2 grains are sufficient to constitute a heap.)

   .

   .

etcetera   <6*16>

then the Tolerance Principle is simply a denial of <6*15>. Consequently, to reject the conditional premise of the sorites, it is not required that a specific number of grains N exist for which**:**

N grains of sand are insufficient to constitute a heap,

but N+1 grains are sufficient to constitute a heap.   <6*17>

Statement <6*17> need not be true in order for <6*16> to be false. It is sufficient simply that the disjunction <6*15>, which encompasses all borderline cases, be correct. If Non-Classical Semantic Theory is right in this regard, if <6*15> can be true and <6*16> false without <6*17> being true (the reason being that here the truth-functionality is non-classical), then the Tolerance Principle cannot be accepted as valid. In that case, the sorites fails as an argument, and its conclusion is unwarranted.

6-4) TWO THEORETICAL FOUNDATIONS

There have been two distinct attempts made to furnish a sound theoretical basis for the semantic theory just presented.[6] One of these, Supervaluationism, grounds semantic theory in its concept of supertruth (a form that, in this context, might be termed extended supertruth). The idea is simply this**:** A blue-green tile that is borderline in color will sometimes be classified as blue, sometimes as green. But regardless of which way it is classified, it will never be categorized as anything else other than blue or green. Any way of drawing the boundary between these two colors will result in only one of these two outcomes. Thus, the disjunction**:**

This tile is green or it is blue.   <6*8>

will be not just true, it will be invariably true (i.e., supertrue). And since the Supervaluationist standard for truth is supertruth, <6*8> is assessed as a fundamentally true statement. On the other hand, the two component disjuncts**:**

This tile is green.   <6*9>

This tile is blue.   <6*10>

like all borderline cases, are not consistently true and, consequently, are not judged to be fundamentally true. Thus, it turns out that some disjunctions (like<6*8>) are true even when the constituent disjuncts are not. But this conclusion is precisely the same idea posited by non-classical semantics in statements <6*14> and <6*15>, the two forms of the sorites used to provide a semantic solution to the paradox. Thus, it is possible to interpret Supervaluation Theory as a variant of non-classical semantics, one that not only

acknowledges disjunctions like <6*8>, <6*14>, and <6*15> as true, but also attempts to provide a theoretical justification for accepting them as true.

The second approach that has been used in conjunction with non-classical semantics is <u>Intuitionist</u> <u>logic</u>, a form of non-classical logic. A number of theoreticians (including Hilary Putnam (1983; 1985) and Peter Mott (1994)) have developed a theory of the sorites relying on this particular form of logic. In order to understand a solution based on Intuitionist logic, it is first important to recognize that there exists among logicians a fundamental divide on a basic issue in logic. One group, the epistemological realists, hold the view that propositions are true or false irrespective of our ability to find out which. Propositions have truth-values without regard to the possibility of our finding them out, and there can, in principle, be truths that we have no way of knowing (Read 1995: 203). In the opposing camp, the <u>Intuitionists</u> maintain the opposite view. The truth of a claim requires a demonstration, a proof that it is true. For example, 'A or not-A' is interpreted as meaning 'Either A is provable or not-A is provable'. Consequently, one may assert the conjunction 'A or not-A' only when in a position to assert or deny A, itself (Read 1995: 219).

What matters with regard to interpreting the sorites is that Intuitionists differ with realists regarding two fundamental principles in logic (not that these are the only two, however). Most notably, Intuitionist logic (1) omits the Law of the Excluded Middle:

Either (P is Q) or not-(P is Q).   <6*18>

as a theorem, and (2) rejects the inference from the universally quantified:

Not (all G's are H).   <6*19>

to the existentially quantified:

There is some G that is not H.   <6*20>

If correct, these two elements of Intuitionist Theory indicate something of importance about how we ought to interpret the sorites. Because <6<7*18> is not accepted as a theorem[7], Intuitionist logic (in contrast to classical logic) does not require the disjunction**:**

Either (N is a heap) or not-(N is a heap).   <6*21>

to be true. In fact, for borderline cases, <6*21> does not have any definite truth value at all.   This notwithstanding, since <6*19> can be interpreted as a negation of the Tolerance Principle, and <6*20> as affirming that there somewhere exists a number of grains N corresponding to an exact cutoff between heaps and non-heaps, Intuitionist logic does, in fact, agree with Supervaluationism in one essential way**:** rejecting the Tolerance Principle does not necessitate committing to the existence of a sharp cutoff. By affirming <6*19> without also affirming <6*20>, Intuitionism, despite embracing a non-standard form of logic, adopts the same semantic approach implicit in the related Supervaluationist solution to the paradox. In this way, it affords an alternative theoretical basis for understanding the sorites in terms of non-classical semantics.

6-5) ARE VAGUE TERMS TRUTH-FUNCTIONAL OR NOT ?

The introductory remarks presented in section 6-2 relating to truth-functionality offered two competing views on the subject. The question is, does Non-Classical Semantic Theory provide a better account than does classical theory concerning (1) the general issue of conjunctive statements and (2) the form of logic underlying the sorites argument? The most direct challenge that

could be raised regarding non-classical theory is whether statements like <6*7> and <6*8> really can be assigned definite truth values. If we return to the case of bluish-green tiles, for example, a strict adherent to classical semantics might simply reject statement <6*8>**:**

This tile is green or it is blue.   <6*8>

on the grounds that for a tile characterized by the statements**:**

It is indeterminate whether this tile is green.   <6*22>
It is indeterminate whether this tile is blue.   <6*23>

it must be the case that**:**

It is indeterminate whether this tile is green or blue.   <6*24>

The rationale behind this conclusion is that <6*24> follows directly (either in some intuitive sense, or just by the standard rules of classical semantics) from <6*22> and <6*23>, two assertions which, themselves, hardly seem disputable. Yet, since statement <6*24> conflicts with statement <6*8>, <6*8> cannot be valid if <6*24> is. While <6*8> certainly is true for any tile that is truly either green or blue, the false step in non-classical semantics (if there is one) is in treating borderline cases similarly to clear-cut ones. It is reasonable to question how these indeterminate cases (ones that receive inconsistent classification) are better characterized by statement <6*8> than <6*24>.

Is it possible to decide which of these two positions is correct, and which is mistaken? Since <6*8> is taken simply as self-evident, not as a derived truth, there does not seem to be any substantive rebuttal to an opposing semantic theory that just outright rejects <6*8> in deference to

<6*24>. In fact, it is hard even to imagine a methodology that could possibly resolve a dispute of this kind. If not, however, then in the final analysis how one assesses non-classical semantics may ultimately depend on nothing more than whether <6*8> is accepted as fundamentally correct, or <6*24> is. In that case, one might question whether the debate is really resolvable, at all.

The second difficulty facing the non-classical approach is the problem of indistinguishable pairs, an issue already raised in relation to Supervaluation Theory in section 4-6. The semantic formulation of the paradox given by collective disjunction <6*15> presumes (implicitly, at least) that pairwise inspection of adjacent elements in a sorites series will not result in consistent categorization of both elements in a pair throughout the whole series. That is, somewhere (although no exact location can be specified), there must be a <heap / non-heap> pair in the sequence. Otherwise, there would be no accounting for the transition from non-heaps to heaps as the number of grains increases. The problem is, this supposition violates the very notion of indistinguishable pairs, namely, that some property changes are not just too insignificant to matter, but also too small to be even noticeable. When examined pairwise, elements in a series (e.g., colored tiles) that are indistinguishable are invariably classified the same as each other (<blue/blue> or <green/green>, but never <blue/green>), meaning the Tolerance Principle <6*16> is not anywhere violated, even throughout an entire sorites series. But if this is the case, the semantic solution to the paradox, which ultimately is to reject the Tolerance Principle, cannot be entirely correct. A sorites series comprised of indistinguishable elements will always operate functionally in a manner consistent with the Tolerance Principle being true. Thus, non-classical semantics fails to provide a complete solution that adequately addresses all variants of the sorites. At best, this theory offers a partial, not a total, explanation of the paradox.

CHAPTER 7: CONCLUSION

This study has critically examined six of the major theoretical approaches to the sorites paradox. It is concluded that none of the theories considered here provide a fully adequate analysis of this argument, nor are any successful in furnishing a convincing solution to it. All of these accounts have fundamental problems that make their explanations of the sorites less than satisfactory; ultimately they leave unanswered exactly how to effectively resolve the paradox. We close this study, therefore, with a brief survey of the major difficulties associated with each proposed solution to the sorites.

Epistemic Theory makes the argument that an exact cutoff exists for every sorites series even though (1) no criteria exist that would establish where the correct boundary lies, or that (2) the criteria that do exist for establishing a boundary are simply unknown (perhaps unknowable) to us. There are two principal reasons for criticizing this view of the sorites. The first, and most common objection is that no boundary can be claimed to truly exist unless some set of available criteria also exist that would establish where that boundary is located (the methodology problem). If either (1) there are no relevant criteria, or alternatively (2) the criteria that do exist are unknown to us or inaccessible to us, this is really no different than the absence of a boundary. What Epistemic Theory ultimately argues for is a theoretical boundary that is functionally indistinguishable from the absence of a boundary.

The second main difficulty is that Epistemic Theory provides no means for differentiating vague terms that are nearly identical in meaning (the ambiguity problem). If we were to assume that precise, sharp boundaries did, in fact, exist for every vague term (yet for which we were unaware), closely related terms (those with nearly identical boundaries) would still, nonetheless, often be used interchangeably. As a result, if the sort of boundary envisioned by Epistemic theorists did exist, this would make no conceivable practical

difference to how vague language actually does operate. (The reconstructed paradox in section 3-5 illustrates this point.) Again, what is being offered by Epistemic theorists is merely a theoretical distinction that makes no practical difference. Yet, unless there is some concrete, tangible difference somewhere, it is difficult to see how an Epistemic-type boundary could matter at all, let alone in a way that is helpful in resolving the sorites.

Supervaluation Theory provides a reductionist approach to the paradox, one that considers the structure of the sorites argument as fundamentally reducible to a stepwise procession along a progressively changing series. Accordingly, there will always occur an abrupt breakpoint somewhere in the sequence, a sudden transition that interrupts the string of inferences well short of the final conclusion that ends the argument. Thus, the sorites argument is viewed as obviously flawed, and we are entitled to reject the logical form of the sorites, as well as its conclusion.

There are two basic problems with this proposed solution. Most fundamentally, this theory provides no convincing basis for employing a forced march model as an exclusive view of the sorites. There are any number of other ways to generate a sorites series, but none of these are considered as also relevant to how one should view the logic underlying the paradox. The pairwise forced march, in particular, shows that Supervaluationism does not provide an adequate generalized account suited to every type of sorites series. The particular reductionist model underlying Supervaluation Theory is simply too narrow to provide a comprehensive solution capable of dealing with the sorites argument in a general way.

The second problem relates to the notion of supertruth. One variant of Supervaluation Theory is based on a localized supertruth account of truth, i.e., one for which any statement in a sorites series is held to be true only if it is consistently held as true. But this theory of truth is just difficult to accept as correct with respect to borderline cases. If there is a dispute as to whether N

grains of sand are sufficient to constitute a heap (because the number N would sometimes, but not always, be considered sufficient), this means something far different from saying that N grains are, indeed, insufficient. That would turn what is essentially doubtful (whether N grains are sufficient) into something clear-cut. Nonetheless, that is precisely what localized supertruth argues, and what makes this particular interpretation of Supervaluationism additionally untenable as an account of the sorites.

Solutions to the sorites based on many-valued logics are attempts to avert the paradoxical implications of the sorites by substituting logics having three or more truth values in place of the standard two-valued logic normally employed by logicians. This strategy seeks to resolve two difficult aspects of vague language that occur whenever using two-valued logic: the existence of borderline cases, and the indeterminate (inconsistent) truth-values characteristic of borderline cases. The first of these difficulties is addressed by creating an individual category for borderline cases that classifies them separately from clear-cut cases; the second by assigning a truth-value to borderline cases different from the standard truth-values of 'true' or 'false'.

Altering logic in this fundamental way as a means for resolving the sorites, however, runs into two main difficulties. To begin with, every attempt to design a many-valued logic to operate in a truth-functional manner seems to suffer from strange and counterintuitive results. Well established logical principles such as modus ponens and the Law of Non-Contradiction fail for some variants of many-valued logic, while comparisons of independently derived truth-tables constructed by different logicians show wide disagreement with regard to truth-value assignments. Even for the most ordinary operations using the standard logical operators 'and', 'or', 'negation', and 'implication' it is difficult to make definitive truth-value assignments that are not subject to dispute.

The second difficulty with regard to many-valued logics is their inability

to adequately manage the effects of higher-order vagueness. Because there exist not only borderline cases of heaps, but also borderline-borderline cases of heaps (and so on, indefinitely), there is no prospect of ever eliminating each last borderline case just by increasing the number of truth-values. But that is exactly what is needed from logics having more than two truth-values. Otherwise, there is little point in applying these to the sorites argument. (Borderline cases, it must be remembered, are just what give rise to the paradoxical nature of the sorites to begin with.) So, if precluding borderline cases is not possible due to the very nature of higher-order vagueness, many-valued logics ultimately can fare no better than the standard two-valued approach they are intended to improve upon.

Fuzzy logic, the logical extension of many-valued logic to an infinite number, has its own particular difficulties. While not everyone agrees so, it is widely held that fuzzy logic, like many-valued logic, should be truth-functional. Some varieties of fuzzy logic are apparently deficient in this regard, although the kind patterned after probability theory appears to be adequate. The two places where fuzzy logic encounters truly serious problems, however, are (1) with regard to the precision it sets in the assignment of truth-values, and (2) in its conception of truth as allowing for partial truths. If fuzzy logic makes truth-value assignments overly precise, it ends up treating vague concepts as though they were actually somehow quite exact (much as Epistemic Theory does). This mis-characterization results from confusing the descriptive properties of vague language with its prescriptive rules, a point raised in the Black-Hempel debate (see section 1-2). Conversely, if truth-value assignments are not very exact, the problem of the sorites simply reemerges in a slightly modified form, but is never actually eliminated. Either way, the issue of precision is highly problematic for this theory.

Additionally, for fuzzy logic to work as an explanation of the sorites, it needs to interpret fuzzy set theory as a theory of truth, thereby providing a fairly

reasonable account of how a sorites series can transition from statements that are clearly true to ones that are obviously false. But this explanation makes it a requirement that truth actually exist to varying degrees, that it makes sense to think of some assertions, at least, as being <u>partially</u> true. Yet nothing in our ordinary use of the term 'true' outside this particular interpretation of fuzzy set theory suggests this is in any way legitimate. 'Truth' in other contexts seems always to be all-or-nothing, not partial. Moreover, truth in any normal sense is iterative: if a statement is true, then it is true that it is true. Yet, those assertions fuzzy logic contends are properly designated as 'partially true' lack this property; there is no obvious way to allow for there to be partially true statements about partially true statements. This provides added reason to think the notion of 'partial truth', and fuzzy logic, itself, is fundamentally misguided.

The solution to the sorites given by non-classical semantics is based on the related ideas that (1) a disjunction can be true even when no specific individual disjunct is identifiably true; and (2) a universal generalization can be false even if none of the particular existential statements that comprise the generalization are false. If correct, these claims make it possible to construct the sorites in such a way (see <6*15>) that the logical form is recognizably flawed (the Tolerance Principle fails to hold), even though there is no identifiable false step in the argument. Thus, this account attempts to show the sorites does not need a sharp cutoff in order for the argument to be unsound. Before it can be regarded as an adequate explanation of the sorites, however, two important issues regarding non-classical semantics need to be addressed.

The most obvious question to raise with regard to non-classical semantics is whether the basic principles on which its solution is based are, indeed, correct. That is, can a disjunction be true if no disjunct is identifiably true, and can a universal generalization can be false even if no existential statement is false? On the one hand, both Supervaluationism and Intuitionist logic lend some support to this approach, although the theoretical basis for

invoking Intuitionist logic is highly controversial. These two theories provide some basis for adopting non-classical semantics as a possible solution to the sorites. At a more fundamental level, however, it must be admitted that there is really no conclusive independent means of ascertaining whether the logical principles in question ultimately should be accepted. There is nothing to finally settle this issue one way or the other. In the last analysis, it seems, we have nothing more than our intuitions to rely upon in deciding this issue.

What appears more troublesome for non-classical semantics is a problem already identified with regard to Supervaluation Theory. Whereas non-classical semantics would seem to work well in explaining how the results from a set of standard forced march experiments could be understood, this theory doesn't seem equally capable of providing a completely general solution to the sorites, one sufficient to fully address all versions of the sorites. In particular, it doesn't adequately account for what occurs in cases of a pairwise forced march. Because the pairwise forced march proceeds in a manner that strictly adheres to the Tolerance Principle, yet non-classical semantics rejects the Tolerance Principle as the one flawed element in the sorites, non-classical semantics cannot be considered correct as a theory for pairwise forced marches. Consequently, this non-classical account of the sorites comes up somewhat short as a general solution to the paradox.

The remaining alternative, Incoherence Theory, is persuasive just to the extent that the sorites strikes us as a genuinely valid argument. If we are incapable of locating some logical weakness in the sorites, then either its logic form is truly legitimate or we have simply overlooked in what way it is flawed. The problem is that unless we do, in fact, discover where its logic goes wrong, we have no way of determining which of these is the case. Most of those who have examined the paradox strongly suspect the form of the argument is seriously defective, even if the nature of the error has thus far eluded us. Although, strictly speaking, rejecting the sorites should first require us to

indicate exactly how we believe the argument fails, the alternative of simply accepting the sorites as valid is far from an appealing option.

It is fitting to conclude this examination of the sorites paradox by reiterating a comment made by Richard DeWitt (1992**:** 118) back in 1992, a remark that seems just as relevant to the situation as it exists today as it did then**:**

> "The past twenty or so years have seen the sorites paradox receive a good deal of philosophical air-time. Yet, in what is surely a sign of a good puzzle, no consensus has emerged. It is perhaps time to stop and take stock of the current status of the sorites paradox. My main contention is that the proposals offered to date as ways of blocking the paradox are seriously deficient, and hence there is, at present, no acceptable solution to the sorites".

Notes

1. The nonsensical idea of a 'cubic acre' is cited by Hugh W. Thompson (2002**:** 64) as used to measure the size of the money bin belonging to the fictional character 'Scrooge McDuck', the world's richest duck.

2. The terms <u>localized</u> <u>supertruth</u> and <u>global</u> <u>supertruth</u> are non-standard, and are introduced here in the present work to help clarify the sometimes unrecognized distinction between the somewhat different variants of Supervaluationism. Similarly, in section 6-4 the expression <u>extended</u> <u>supertruth</u> is introduced, but this is, again, not established terminology.

3. Although Sorensen (1988c**:** 58) in his previous remark characterizes Machina as attempting to preserve truth-functionality at the expense of classical theorems, in the statement that follows Machina (1976**:** 56) raises doubts about <u>both</u> classical theorems <u>and</u> truth-functionality without giving a clear indication of which he considers more fundamental to maintaining a coherent logic. In any case, the sorites places classical logicians in a difficult position with quite limited options. It appears necessary to give up some aspect of classical logic in order to save others, even though this means making a quite basic revision to classical logic that not everyone will find acceptable.

4. Bertil Rolf (1984**:** 222) and Roy Sorensen (1991a**:** 69) are two others who have also questioned the validity of having partially true statements about partially true statements.

5. The terms 'fuzzy logic' and 'infinite-valued logic' are not necessarily synonymous. Richard DeWitt (1992**:** 107), for example, distinguishes these two, and considers fuzzy logic a special case of infinite-valued semantics, based on a distinction involving the preservation of truth by modus ponens.

The present work does not follow that convention, and simply uses the designations 'fuzzy logic' and 'infinite-valued logic' interchangeably. Similarly, as discussed in the text, 'fuzzy logic' can be a less specific term than is 'degree of truth', but is often used to mean the same thing. In any case, regardless of the particular terminology adopted, it is an essential aspect of this particular theory that modus ponens is not truth preserving in the case of the sorites.

6. An alternative to these two theoretical approaches is to argue that the existential statement 'There is some G that is not H' is not true even when the universal statement 'Not (all G's are H)' is true, but without giving any direct theoretical reason for holding to this position. This is exactly what Hans Kamp (1981: 247) does: "Although I personally find this idea intuitively plausible I have no real argument to show that the principle is correct ...".

7. There is a subtle but important distinction here between non-acceptance and outright denial. Because Intuitionists find no valid proof for inferring an existential truth from denial of a universal generalization, nor any proof for the validity of the Law of the Excluded Middle, neither of these laws are held as true. Conversely, however, Intuitionists do not assert these laws are false. That would mistakenly imply proofs exist that these laws are invalid. Since there is no compelling argument one way or the other, Intuitionists simply find no persuasive reason to be constrained by these classical laws of logic.

BIBLIOGRAPHY

Abbott, W. 1983. A note on grim's sorites argument. <u>Analysis</u>, 43**:** 161-164.

Ackerman, F. 1994. Roots and Consequences of Vagueness,
in J. Tomberlin, <u>Philosophical perspectives, 8, Logic and language</u>.
Atascadero**:** Ridgeview Publishing Company, 1994**:** 129-136.

Ballmer, T. & Pinkal, M. 1983. <u>Approaching vagueness</u>. New York**:** Elsevier
Science Publishing Company.

Black, M. 1937. Vagueness**:** An exercise in logical analysis. <u>Philosophy of
science</u>, 4**:** 427-455.

Black, M. 1963. Reasoning with Loose Concepts. <u>Dialogue</u>, 2**:** 1-12.

Bosch, P. 1983. "Vagueness" is context-dependence, a solution to the sorites
paradox, in T. Ballmer & M. Pinkal, <u>Approaching vagueness</u>. New York**:**
Elsevier Science Publishing Company, 1983**:** 189-210.

Burgess, J. 1990. Vague objects and indefinite identity. <u>Philosophical studies</u>,
59**:** 263-287.

Burks, A. 1946. Empiricism and vagueness. <u>Journal of philosophy</u>,
43**:** 477-486.

Burns, L. 1986. Vagueness and coherence. <u>Synthese</u>, 68**:** 487-513.

Burns, L. 1991. Vagueness: An investigation into natural languages and the sorites paradox. Dordrecht: Kluwer Academic Publishers.

Burnyeat, M. 1982. Gods and Heaps, in M. Schofield & M.C. Nussbaum, Language and logos. New York: Cambridge University Press, 1982: 315-338.

Campbell, R. 1974. The sorites paradox. Philosophical studies, 26: 175-191.

Cargile, J. 1969. The sorites paradox. The British journal for the philosophy of science, 20: 193-202.

Clark, M. 1987. The truth about heaps. Analysis, 47: 177-179.

Code, M. 1995. The sorites paradox, in M. Code, Myths of reason: Vagueness, rationality, and the lure of logic. Highlands: Humanities Press International, 1995: 33-54.

Cooper, N. 1995. Paradox lost: Understanding vague predicates. International journal of philosophical studies, 3: 244-269.

Copilowish, I. 1939. Border-line cases, vagueness, and ambiguity. Philosophy of science, 6: 181-195.

Day, T. 1992. Excluded middle and bivalence. Erkenntnis, 37: 93-97.

Deas, R. 1989. Sorensen's sorities. Analysis, 49: 26-31.

DeWitt, R. 1992. Remarks on the current status of the sorites paradox. <u>Journal of philosophical research</u>, 17**:** 93-118.

Dickens, C. 1980. <u>A tale of two cities</u>. New York**:** Signet Books.

Dummett, M. 1975. Wang's paradox. <u>Synthese</u>, 30**:** 301-324.

Edgington, D. 1992. Validity, uncertainty and vagueness. <u>Analysis</u>, 52**:** 193-204.

Engel, R. 1989. On degrees. <u>Journal of philosophy</u>, 86**:** 23-37.

Fine, K. 1975. Vagueness, truth and logic. <u>Synthese</u>, 30**:** 265-300.

Fraassen, B. 1966. Singular terms, truth-value gaps, and free logic. <u>Journal of philosophy</u>, 63**:** 481-495.

Goguen, J. 1968-1969. The logic of inexact concepts. <u>Synthese</u>, 19**:** 325-373.

Goldstein, L. 1988. The sorites as a lesson in semantics. <u>Mind</u> 97**:** 447-455.

Govier, T. 1982. What's wrong with slippery slope arguments? <u>Canadian journal of philosophy</u>, 12**:** 303-316.

Grim, P. 1982. What won't escape sorites arguments. <u>Analysis</u>, 42**:** 38-43.

Grim, P. 1983. Is this a swizzle stick which i see before me? <u>Analysis</u>, 43**:** 164-166.

Grim, P. 1984. Taking sorites arguments seriously**:** Some hidden costs. Philosophia, 14**:** 251-272.

Haack, S. 1979. Do we need fuzzy logic? International journal of man-machine studies, 11**:** 437-45.

Haack, S. 1996. Vagueness, in S. Haack, Deviant logic, fuzzy logic**:** Beyond the formalism. Chicago**:** The University of Chicago Press, 1996**:** 109-125.

Hardin, C. 1988. Phenomenal colors and sorites. Nous, 22**:** 213-234.

Hausser, R. 1983. Vagueness and truth. Conceptus, 17**:** 29-52.

Heck, R. 1993. A note on the logic of (higher-order) vagueness. Analysis, 53**:** 201-208.

Heller, M. 1990. The sorites paradox, in M. Heller, The ontology of physical objects. New York**:** Cambridge University Press, 1990**:** 68-109.

Horgan, T. 1994a. Robust vagueness and the forced-march sorites paradox, in J. Tomberlin, Philosophical perspectives, 8, Logic and language. Atascadero**:** Ridgeview Publishing Company, 1994**:** 159-188.

Horgan, T. 1994b. Spindel conference**:** Vagueness. The Southern journal of Philosophy, 33, supplement.

Jacquette, D. 1989a. Epistemic blood from logical turnips. Philosophy and rhetoric, 22**:** 203-211.

Jacquette, D. 1989b. The hidden logic of slippery slope arguments. <u>Philosophy and rhetoric</u>, 22**:** 59-70.

Kamp, H. 1981. The paradox of the heap, in U. Mönnich, <u>Aspects of philosophical logic</u>. Dordrecht**:** Reidel Publishing Company, 1981**:** 225-277.

Keefe, R. & Smith, P. 1997. <u>Vagueness**:** A reader</u>. Cambridge**:** The MIT Press.

Khatchadourian, H. 1962. <u>Vagueness</u>. Philosophical quarterly, 12**:** 138-152.

King, J. 1979. Bivalence and the sorites paradox. <u>American philosophical quarterly</u>,16**:** 17-25.

Kohl, M. 1969. Bertrand Russell on Vagueness. <u>Australasian journal of philosophy</u>, 42**:** 31-41.

Koons, R. 1994. A new solution to the sorites problem. <u>Mind</u>, 103**:** 439-449.

Kumar, D. 1968. Logic and inexact predicates. <u>British journal for the philosophy of science</u>, 18**:** 211-222.

Lakoff, G. 1973. Hedges**:** A study in meaning criteria and the logic of fuzzy concepts. <u>Journal of philosophical logic</u>, 2**:** 458-508.

Lukasiewicz, J. 1987. On the principle of the excluded middle. <u>History and philosophy of logic,</u> 8**:** 67-69   .

Machina, K. 1972. Vague predicates. <u>American philosophical quarterly,</u>

9**:** 225-233.

Machina, K. 1976. Truth, belief, and vagueness. <u>Journal of philosophical logic</u>, 5**:** 47-78.

March, P. 1984. Sorting out sorites. <u>Canadian journal of philosophy</u>, 14**:** 445-454.

Mignucci, M. 1993. The Stoic analysis of the sorites. <u>Proceedings of the Aristotelian society</u>, 93**:** 231-245.

Moline, J. 1969. Aristotle, Eubulides, and the sorites. <u>Mind</u>, 78**:** 393-407.

Morgan, C. & Pelletier, J. 1977. Some notes concerning fuzzy logics. <u>Linguistics and philosophy</u>, 1**:** 79-97

Morreau, M. 1999. Supervaluation can leave truth-value gaps after all. <u>The journal of philosophy</u>, 96**:** 148-156.

Mott, P. 1994. On the intuitionistic solution of the sorites paradox. <u>Pacific philosophical quarterly</u>, 75**:** 133-150.

Nadin, M. 1980. The logic of vagueness and the category of synechism. <u>The monist</u>, 63**:** 351-363.

Napoli, E. Is vagueness a logical enigma? <u>Erkenntnis</u>, 23**:** 115-121.

Odegard, D. 1965. Excluding the middle from loose concepts. <u>Theoria</u>, 31**:** 138-144.

Peacocke, C. 1981. Are vague predicates incoherent? Synthese, 46**:** 121-141.

Priest, G. 1979. A note on the sorites paradox. Australasian journal of philosophy, 57**:** 74-75.

Priest, G. 2000. Vagueness**:** How do you stop sliding down a slippery slope?, in G. Priest, Logic**:** A very short introduction. New York**:** Oxford University Press, 2000**:** 70-77.

Putnam, H. 1983. Vagueness and alternative logic. Erkenntis, 19**:** 297-314.

Putnam, H. 1985. A quick read is a wrong wright. Analysis, 45**:** 203.

Quine, W. 1981 What price bivalence? Journal of philosophy, 77**:** 90-95.

Radford, C. 1985. The umpire's dilemma. Analysis, 45**:**109-111.

Raffman, D. 1994. Vagueness without paradox. The philosophical review, 103**:** 41-74.

Rea, G. 1989. Degrees of truth versus intuitionism. Analysis, 49**:** 31-32.

Read, S. & Wright, C. 1985. Hairier than Putnam thought. Analysis, 45**:** 56-58.

Read, S. 1995a. Bald men forever**:** The sorites paradox, in S. Read, Thinking about logic**:** An introduction to the philosophy of logic. New York**:**   O x f o r d University Press, 1995**:** 173-202.

Read, S. 1995b. Whose line is it anyway? The constructivist challenge, in S. Read, Thinking about logic: An introduction to the philosophy of logic. New York: Oxford University Press, 1995: 203-240.

Rolf, B. 1984. Sorites. Synthese, 58: 219-250.

Rolf, B. 1982. Russell's theses on vagueness. History and philosophy of logic, 3: 69-83.

Russell, B. 1923. Vagueness. The Australasian journal of psychology and philosophy, 1: 84-92.

Sainsbury, R. 1986. Degrees of belief and degrees of truth. Philosophical papers, 15: 97-106.

Sainsbury, R. 1988/1989. Tolerating vagueness. Proceedings of the Aristotelian society, 89: 33-48.

Sainsbury, R. 1991. Is there higher-order vagueness? Philosophical quarterly, 41: 167-182.

Sainsbury, R. 1992. Sorites paradoxes and the transition question. Philosophical papers, 21: 177-190.

Sainsbury, R. 1995. Paradoxes. 2nd edition. New York: Cambridge University Press.

Salmon, N. 1982. Vagueness and the paradox, in N.U. Salmon, Reference and essence. Princeton: Princeton University Press, 1982: 240-252.

Sanford, D. 1974. Classical logic and inexact predicates. Mind, 83**:** 112-113.

Sanford, D. 1975a. Borderline logic. American philosophical quarterly, 12**:** 29-39.

Sanford, D. 1975b. Infinity and vagueness. Philosophical review, 84**:** 520-535.

Sanford, D. 1976. Competing semantics of vagueness**:** Many values versus super-truth. Synthese, 33**:** 195-210.

Sanford, D. 1979. Nostalgia for the ordinary**:** Comments on papers by Unger and Wheeler. Synthese, 41**:** 175-184.

Schwartz, S.. 1987. Intuitionism and sorites. Analysis, 47**:** 179-183.

Schwartz, S. 1989. Vagueness and incoherence**:** A reply to Burns. Synthese, 80**:** 395-406.

Schwartz, S. 1990. Intuitionism versus degrees of truth. Analysis, 50**:** 43-47.

Schwartz, S. 1991. Intuitionism and vagueness. Erkenntnis, 34**:** 347-356.

Shapiro, S. 1998. A procedural solution to the unexpected hanging and sorites paradoxes. Mind, 107**:** 751-761.

Simons, P. 1992. Vagueness and ignorance. Aristotelian society, suppl. 66**:** 163-177.

Simons, P. 1994-1995. Multivalence and vagueness**:** A reply to Copeland.

Proceedings of the Aristotelian society, 95**:** 201-209.

Slater, H. 1989. Consistent vagueness. Nous, 23**:** 241-252.

Smith, B. 1998. Vagueness. The monist, 81(2).

Smith, J. 1984. The surprise examination on the paradox of the heap. Philosophical papers, 13**:** 43-56.

Sorensen, R. 1985. An argument for the vagueness of 'vague'. Analysis, 45**:** 134-137.

Sorensen, R. 1987. The vagueness of knowledge. Canadian journal of philosophy, 17**:** 767-804.

Sorensen, R. 1988a. Precisification by means of vague predicates. Notre Dame journal of formal logic, 29**:** 267-274.

Sorensen, R. 1988b. Rival responses to the sorites, in R.A. Sorensen, Blindspots. Oxford**:** Clarendon Press, 1988**:** 217-252.

Sorensen, R. 1988c. Vagueness, bluriness, and measurement. Synthese, 75**:** 45-82.

Sorensen, R. 1989a. The ambiguity of vagueness and precision. Pacific philosophical quarterly, 70**:** 174-183.

Sorensen, R. 1989b. Slipping off the slippery slope**:** A reply to professor Jacquette. Philosophy and rhetoric, 22**:** 195-202.

Sorensen, R. 1991a. Fictional incompleteness as vagueness. <u>Erkenntnis</u>, 34**:** 55-72.

Sorensen, R. 1991b. Vagueness within the language of thought. <u>Philosophical quarterly</u>, 41**:** 389-413.

Sperber, D. & Wilson, D. 1985/1986. Loose talk. <u>Proceedings of the Aristotelian society</u>, 86**:** 153-171.

Sylvan, R. 1993. Ubiquitous vagueness without embarrassment**:** Logic liberated and fuzziness defuzzed. <u>Acta analytica</u>, 10**:** 7-29.

Tappenden, J. 1993. The liar and the sorites paradoxes**:** toward a unified treatment. <u>The journal of philosophy</u>, 90**:** 551-577.

Thomason, R. 1982. Identity and vagueness. <u>Philosophical studies</u>, 43**:** 329-332.

Thompson, H. 2002. <u>Chemical & engineering news</u>, 80**:** 64.

Thorpe, D. 1984. The sorites paradox. <u>Synthese</u>, 61**:** 391-421.

Todt, G. 1983. Fuzzy logic and modal logic, in T. Ballmer & M. Pinkal, <u>Approaching vagueness</u>. New York**:** Elsevier Science Publishing Company, 1983**:** 213-260.

Travis, C. 1985. Vagueness, observation, and sorites. <u>Mind</u>, 94**:** 345-366.

Tye, M. 1989. Supervaluationism and the law of excluded middle. <u>Analysis</u>, 49**:**

141-143.

Tye, M. 1990. Vague objects. <u>Mind</u>, 99**:** 535-557.

Tye, M. 1994. Sorites paradoxes and the semantics of vagueness, in J. Tomberlin, <u>Philosophical perspectives, 8, Logic and language</u>. Atascadero**:** Ridgeview Publishing Company, 1994**:** 189-206.

Unger, P. 1979a. I do not exist, in G. MacDonald, <u>Perception and identity</u>**:** <u>Essays presented to A.J. Ayer, with his replies</u>. Ithaca**:** Cornell University Press, 1979**:** 235-251.

Unger, P. 1979b. There are no ordinary things. <u>Synthese</u>, 41**:** 117-154.

Weiss, S. 1976. The sorites fallacy**:** What difference does a peanut make? <u>Synthese</u>, 33**:** 253-272.

Wheeler, S. III. 1975. Reference and vagueness. <u>Synthese</u>, 30**:** 367-379.

Wheeler, S. 1979. On that which is not. <u>Synthese</u>, 41**:** 155-173.

White, D. 1985. Slippery slope arguments. <u>Metaphilosophy</u>, 16**:** 206-213.

Williamson, T. 1992. Inexact knowledge. <u>Mind</u>, 101**:** 217-242.

Williamson, T. & Simons, P. 1992. Vagueness and ignorance. <u>Supplementary proceedings of the Aristotelian society</u>, 66**:** 145-162.

Williamson, T. 1994. <u>Vagueness</u>. New York**:** Routledge.

Wilson, F. 1987. Resemblence, universals and sorites**:** Comments on March on sorting out sorites. <u>Canadian journal of philosophy</u>, 17**:** 175-184.

Wright, C. 1975. On the coherence of vague predicates. <u>Synthese</u>, 30**:** 325-365.

Wright, C. 1976. Language-mastery and the sorites paradox, in G. Evans and J. McDowell, <u>Truth and meaning**:** Essays in semantics</u>. Oxford**:** Clarendon Press, 1976**:** 223-247.

Wright, C. 1987. Further reflections on the sorites paradox. <u>Philosophical topics</u>, 15**:** 227-290.

Wright, C. 1991. The sorites paradox and its significance for the interpretation of semantic theory, in N.L. Cooper & P. Engels, <u>New inquiries into meaning and truth</u>. New York**:** St. Martin's Press, 1991**:** 135-162.

Wright, C. 1992. Is higher order vagueness coherent? <u>Analysis</u>, 53**:** 129-139.

Zadeh, L. 1965. Fuzzy sets. <u>Information and control</u>, 8**:** 338-353.

Zadeh, L. 1975. Fuzzy logic and approximate reasoning. <u>Synthese</u>, 30**:** 407-428.