MATHEMATICS ANXIETY AS A VARIABLE IN THE CONSTRUCTIVIST APPROACH TO THE TEACHING OF SECONDARY SCHOOL MATHEMATICS

by

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SUMMARY

Mathematics anxiety is a personal characteristic which is widespread and continuing. It has a debilitating effect on mathematics performance and contributes to perceptions and attitudes that perpetuate a dislike for mathematics and a lack of confidence when dealing with mathematical problems.

An investigation of relevant literature on mathematics anxiety identifies sources and symptoms and emphasises a need for a comprehensive approach to remediation. The historical development of an appropriate measuring instrument is documented and statistical evidence is used to create a mathematics anxiety rating scale suitable for measuring anxiety levels of secondary school pupils and student teachers.

The extensive literature interest, research publications and remedial programmes emphasise the problem of mathematics anxiety and thus the need for a comprehensive approach to remediation. Mathematics teaching and curriculum design is expounded to provide the necessary direction to the alleviation of mathematics anxiety. General perspectives on curriculum design are discussed and a cyclical systems approach is recommended. Elements of this approach are detailed and are linked to important personal characteristics to add a humanistic and socio-cultural view of curriculum design in mathematics.

The didactic viability of constructivism as an approach to mathematics curriculum design is investigated. Constructivism embodies a philosophy and a methodology which addresses the critical aspects influencing mathematics anxiety. Classroom topics and activities are reviewed in terms of a constructivist approach and the sources of mathematics anxiety are discussed from a constructivist perspective.

A longitudinal case study of pupils during their five years at secondary school as well as a study involving student teachers was undertaken. Mathematics performance, perceptions, attitudes and levels of anxiety were investigated by means of tests, questionnaires, and mathematics anxiety rating scales. The statistical results of this research provide evidence to support a comprehensive approach to the remediation of mathematics anxiety.

Constructivism is seen as the synthesis of critical aspects of teaching and curriculum development which will stem the perpetuation of mathematics anxiety. Constructivism provides the didactic approach to develop each individual's intellectual autonomy and mathematics power, by instilling a problem solving attitude and a self-confidence when doing mathematics.

KEY TERMS:
mathematics anxiety; constructivist approach; mathematics teaching; mathematics curriculum; secondary school mathematics; systems approach; situation analysis; classroom environment; understanding; self-confidence; communication; social interaction; problem solving; radical constructivism; social constructivism; intellectual autonomy; mathematics power.

Student Number: 251-127-4

I declare that **MATHEMATICS ANXIETY AS A VARIABLE IN THE CONSTRUCTIVIST APPROACH TO THE TEACHING OF SECONDARY SCHOOL MATHEMATICS** is my own work and that all the sources that I have used or quoted have been indicated and acknowledged by means of complete references.

SIGNATURE
Mr P.L. Hawkey

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DATE

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CHAPTER ONE
INTRODUCTION AND ORIENTATION

1.1 Reasons for this research

A number of important issues prompted the research undertaken in this thesis. These issues need to be discussed briefly before formalisation of a statement of intent. In summary, the three major areas of concern influencing the direction of this study are mathematics anxiety, reform in curriculum design and a growing emphasis on reform in instructional practices in the classroom.

1.1.1 The emergence of interest in mathematics anxiety

Mathematics anxiety has received considerable attention over the past number of years. The advent of the computer age and the widening array of courses and vocations that require a theoretical and practical knowledge of mathematics have all contributed to the emphasis on mathematics achievement. This greater emphasis and importance placed on mathematics has influenced the problems affecting mathematics achievement and in particular increased the possibility of mathematics anxiety being more prevalent.

Whilst it cannot be claimed that mathematics anxiety is the only problem affecting mathematics achievement, it is certainly an important variable to be considered. Several researchers have shown that mathematics anxiety has a debilitating effect on mathematics performance (Sarason 1980; Richardson & Suinn 1972; Suinn *et al* 1972). In fact Hembree (1990:37) conducted an analysis of the research findings of first studies of mathematics anxiety and found that higher mathematics anxiety consistently related to lower mathematics performance.

Interest in mathematics anxiety has resulted in the publication of a number of articles and research records. Fortunately, Hembree (1990:35) has identified 151 studies which provide detailed data with product-moment correlation coefficients using validated instruments and samples of at least ten subjects. Hembree's "Bibliography of research on Mathematics Anxiety" (1988) is evidence of the wide interest in this debilitating factor. The bulk of this research has been conducted in England, Canada and the United States of America where programmes have been designed to combat the problem and colleges and private institutions are offering remedial courses. These remedial courses have grown over the years and the consideration of the mathematically anxious pupil has been foremost in the minds of educators for some time.

The problem of mathematics anxiety cannot be studied in isolation because it affects the whole spectrum of mathematics education from curriculum decisions to the instructional methods. The acquiring of sound mathematics knowledge necessarily involves both emotional and intellectual concerns and school programmes and classroom strategies must be designed to take cognisance of this fact.

1.1.2 <u>Changing concerns of curriculum design</u>

The last decade has seen an increasing concern for attitudes, values and experience with an emphasis on inquiry and dialogue in curriculum design and development. The Commission on Standards for School Mathematics in the U.S.A. identifies as one of the goals that students become confident in their ability to do mathematics (1989:5).

The task of the teacher is outlined as follows:

> *Students should be exposed to numerous and varied interrelated experiences that encourage them to value the mathematical enterprise, to develop mathematical habits of mind and to understand and appreciate the role of mathematics in human affairs* (1989:5).

Cockcroft in his report on the teaching of mathematics in the United Kingdom also stipulates the aim to develop enjoyment of mathematics and confidence in its application (1982:195). A learner-centred approach is also evident in South Africa during this time of change. The Committee of Heads of Education Departments states as a continuous aim of pre-tertiary education that a positive and realistic self-image, self-confidence, a sense of responsibility and inner discipline should be developed (1991:13).

At the NECC Mathematics Conference held at Broederstroom, in South Africa, in April 1993, a recurring theme was a call for developing a confidence to do mathematics. The following statement was made by Olivier at the Conference:

> *Most teaching is based on a behaviourist perspective of the learning of mathematics and a formalist view of mathematics, and consequently ignores and even actively suppresses children's informal knowledge* (1993:2).

All of the documents quoted above share a concern about the traditional rigidity of mathematics teaching and learning and it is this rigidity and teacher-centred or content-centred approaches to the curriculum that has contributed to an underlying lack of self-confidence and an anxiety in dealing with mathematical problems. The approach to curriculum development is now more pupil-centred with the focus on learning activities, evaluation and the process of the development of knowledge.

New technological and social developments are taken into consideration to provide a more comprehensive assessment of needs and the curriculum has become more responsive to individual and group concerns. The opportunity for each individual to become more active in constructing his or her mathematical concepts based on prior informal knowledge is an ingredient for less anxiety provoking situations in the mathematics classroom.

A move towards a pupil-centred and problem-centred approach in curriculum design will result in a greater emphasis on formative evaluation and a wider variety of instructional methods. This should provide more positive expressions by teachers and students in classrooms, and more positive attitudes toward school.

1.1.3 <u>The emphasis on teaching methodology in mathematics</u>

There is no doubt that recent developments in curriculum design will be beneficial to the mathematically anxious child. In the United States of America the new "Curriculum and Evaluation Standards for School Mathematics" makes particular reference to teaching in the stated aims of the authors. The following aspects are emphasised:

> *Students need a non-threatening environment in which they are encouraged to ask questions and take risks;*
> *Teachers should view their role as guiding and helping students to develop their mathematical knowledge and power.*
> (Commission on Standards for School Mathematics, 1987:5)

In the South African context, curriculum development in mathematics also reflects a concern for the individual and an emphasis on teaching method. Fostering self-esteem, dignity and self-control is regarded as an essential directive for teachers. The following statement emphasises the concern for the instructional style.

> *These aims should be regarded as a statement of intent and they should be realised as much through the methods of teaching and inquiry as through the mathematical content being taught* (Department of Education and Culture, 1991:1).

Whilst mathematics anxiety is not mentioned, the parallels with studies in this field are once again evident. Mathematics teachers must become aware of the new role they are expected to adopt. Imparting content is generally no longer viewed as the central aim of the teacher. A concentration on process and affective variables has developed which now emphasises aspects of teaching methodology important to the mathematically anxious child. The following nine recommendations from Hawkey (1988:27) focus on the critical aspects of teaching methodology that will alleviate mathematics anxiety in the classroom.

1. Create a positive supportive classroom atmosphere in which taking risks is perceived as acceptable.

2. Stress understanding and not "rote learning". Mathematics is not a set of rules to be learned.

3. Help pupils to develop self-confidence - encourage their intuitive ideas and provide positive feedback.

4. Dispel mathematics myths - make students aware of any irrational reasons they may have for not being good at mathematics.

5. Avoid sex-role stereotyping of mathematics as a male domain.

6. Avoid inflexible or excessively authoritarian teaching.

7. Be aware of the possible negative effects of testing.

8. Make students aware of the usefulness of mathematics.

9. Concentrate on problem solving, spatial skills, practical problems, and the language and symbolism of mathematics.

These recommendations emphasise a view that mathematics teaching should seek critical aspects which lead to improving mathematics performance by restoring confidence in the ability to do mathematics and also improving perceptions and attitudes towards mathematics.

Wain (1989:32) supports these views and also emphasises the teaching of mathematics as the critical factor in the avoidance of mathematics anxiety.

> *One of the key aims in teaching mathematics should be to enable the learners to feel confident in the acquisition of knowledge. The fact that the teaching of the subject has often promoted fear is an awful indictment* (Wain, 1989:32).

The development of personal self-confidence in mathematics should be a priority for teachers. Without a sense of "mathematics power" the individual student will be unable to construct new algorithms by building on his or her informal knowledge. The teacher must become a facilitator in the classroom and the use of a problem-centred and child-centred learning approach must help to develop the conceptual and procedural knowledge of each individual.

Instructional techniques need to be focused on the best possible ways to address the problems facing the learner. It is the intention of this thesis to support the constructivist theory of learning and teaching mathematics by investigating the components of the theory that have a direct bearing on alleviating mathematics anxiety.

Hence, the teaching of mathematics is the central theme of this thesis and the critical aspects of mathematics teaching methodology are of central concern. Thus, variables must be formulated according to constructivist theory and coupled with the concerns of mathematics anxiety researchers, to provide guidelines for teachers in the classroom.

The ultimate goal of mathematics teaching could be stated simply as providing each student with intellectual autonomy and self-confidence in their ability to do mathematics work. However, this is no easy task and requires continuous attention by the teacher to critical aspects of the teaching situation. In this study, these critical aspects have been categorised as follows:

1. Classroom environment
2. Understanding
3. Self-confidence
4. Communication
5. Social interaction
6. Problem solving

These aspects of teaching methodology form a recurring theme in this thesis because they are relevant concerns of both mathematics anxiety arousing situations and the constructivist approach to the teaching and learning of mathematics.

Curriculum design and learning problems such as mathematics anxiety can only be addressed through the teacher. After all the theories and research findings are analysed one single element is evident. Successful implementation of reform in mathematics teaching is dependent on the enthusiasm and dedication of the teacher.

1.2 **Statement of the problem**

Mathematics anxiety is a personal characteristic which has a debilitating effect on mathematics performance and a person's sense of self-worth. In addition, the problem is compounded because it contributes to perceptions and attitudes that perpetuate a dislike for mathematics and a lack of confidence when doing mathematics. A high percentage of children are not performing to their full potential and are being restricted in their mathematical development and intellectual autonomy by a lack of self-confidence when dealing with mathematics. This results in under-achievement and general avoidance of mathematics.

This problem is widespread and continuing. Hence there exists a need to re-emphasise the importance of mathematics anxiety as a problem which affects the mathematical development of the learner. This requires an extensive investigation into the didactic scope of these problems and the further identification and explication of the sources of mathematics anxiety.

Attempts to alleviate mathematics anxiety have concentrated on small group therapy and individual counselling. This is time consuming and has limited success in that it does not provide treatment for enough pupils. There is therefore, a need for a more comprehensive approach to remediation. Such an approach will have to address all the critical aspects of mathematics anxiety and contribute to an improvement of performance coupled with the development of intellectual autonomy, self-confidence and a feeling of self-worth.

Constructivism is a philosophical position which may provide a useful framework for thinking about mathematics learning in classrooms and therefore may contribute in important ways to the effort to reform classroom teaching. However, constructivism does not stipulate a particular didactic approach to the teaching and learning of mathematics.

This study investigates the didactic viability of constructivism as an approach to the mathematics curriculum. In particular, this study endeavours to answer the question, to what extent constructivism can be utilised within the framework of a systems approach to curriculum design, in such a way that it addresses the critical aspects of mathematics anxiety. This, therefore, includes a curriculum study which takes into account the sources of mathematics anxiety as well as the critical aspects of teaching and learning which help alleviate mathematics anxiety.

1.3 Analysis of the problem

A brief analysis of the problem of mathematics anxiety suggests that the direction of research in this area should address the development of a suitable mathematics curriculum and the issue of reform in mathematics instruction.

1.3.1 The problem of mathematics anxiety

The causes of mathematics anxiety are wide and varied. Aspects such as attitudes, beliefs, connotations, content, teaching methods, parental influence and classroom atmosphere all contribute to the problem of developing anxiety. The reasons for concern are poor mathematics performance, lack of self-confidence and sense of self-worth, the perpetuation of the problem through parents and teachers and eventual deprivation of job opportunities.

McNeil (1985:367) suggests that instead of making the search for generalization, researchers should look for unique personal characteristics and uncontrolled events in situations. He goes on to say that a generalization may be made to form a working hypothesis but clues should be investigated in learner variables, teacher variables and school and classroom ambient variables.

McNeil's views are particularly relevant to this study as variables in the contexts he recommends are all discussed. In particular it is the social context which envelops all three categories of variables. The learner and teacher in the school/classroom environment provides the social scene for any proposed teaching reform.

Whatever variables contribute towards mathematics anxiety, two elements of research should be emphasised. These are the affect on mathematics performance and the need to improve attitudes towards mathematics. Hembree (1990:38) integrated 151 studies on mathematics anxiety by meta analysis and found that high mathematics anxiety consistently correlated with poor mathematics performance. Secondly, Hembree's analysis revealed that a positive attitude towards mathematics and towards their parents and teachers consistently correlated with lower mathematics anxiety rating scores.

This study will investigate a need to work towards solving the problem of how to alleviate mathematics anxiety on a wide scale. It is evident from the studies mentioned above that personal characteristics in a social context play an important role in the development of mathematics anxiety and that learner variables, attitudes and teaching procedures should be the focus of one's attention. However, remediation has been limited to small groups and individuals and needs to be expanded to accommodate all pupils through the mathematics curriculum.

1.3.2 **The development of a mathematics curriculum**

The second aspect of the problem of mathematics anxiety that should be considered is the influence that research in this field could have on curriculum planning. Whilst content cannot be termed irrelevant in mathematics, a definite advantage for the mathematically anxious child is a move towards a more pupil-orientated approach. In fact, Wain (1989:33) suggests that the content of the curriculum should be reduced and more emphasis and time be allocated to providing understanding and a confidence to explore mathematical tasks.

In South Africa, movement towards a truly pupil-centred curriculum has made some progress at primary school level but the many ideas involving problem-centred and pupil-centred approaches have been put on hold during this time of transition in the country. However, it is evident from the NECC Conference which was held in Broederstroom in April 1993, that a move towards a constructivist approach in education is an idea shared by leading educationalists in this country.

This reform will not be easy because of many factors which mitigate against constructivism. Most of these factors involve the commitment of the mathematics teacher and a sense of realism from enthusiastic theorists. Gadanidis (1994:91) makes the observation:

> *Mathematics education suffers from a condition that resembles schizophrenia. One of its personalities is exhibited in day-to-day realities of classroom learning; another is evident in journal articles, in-service presentations, and other such forums where educators present alternative realities of learning.*

A plea for realism and a pragmatic approach to constructivism should ensure that reform in mathematics education is beneficial. Too often theorists can become engrossed in an ideal but provide very little insight into the practical implications of introducing the theory to the classroom.

1.3.3 Implementing reform in mathematics instruction

The third consideration in the analysis of the problem of mathematics anxiety is the implementation of the curriculum. Aims and goals are often stated with genuine concern for teaching methods but too often the realities reveal a total lack of appreciation by the teacher in the school situation.

This may be for several reasons such as poor communication, lack of in-service courses for teachers, inadequate teacher training, and lack of teacher enthusiasm for the new ideas. The practical problem of having to complete a set syllabus in a limited amount of time may also discourage teachers from spending time enhancing their pupils' confidence in mathematics.

The simplest answer to limited time for a teacher would be to adopt a "chalk and talk" approach to teaching and this could have disasterous effects for the implementation of any new syllabus. Teachers must adopt an openness to allow pupils to experience ways in which the subject extends into many different types of work. Teachers often agree on the importance of such teaching methods but then "slip back" to old tried and trusted routines.

Yackel *et al* in "Teaching and Learning Mathematics in the 1990s" (1990:13) suggests that children develop a wider variety of solution methods when they are encouraged to solve problems in their own way rather than following procedures presented by the teacher.

Thus the teacher's attitude is crucial to the development of sound instructional methods and a confident problem solving atmosphere in the classroom. This is not an easy task for the teacher, as good instructional reform requires a communication of teaching ideas as well as a system which provides material, support and continued provision of information for teachers. The goals should not be simply to implement a new curriculum but also to monitor and maintain the required success level of this system.

1.4 Definition of terms

Before proceeding with this study certain terms need to be defined. The terms used in the title are essential ingredients of the research and the definitions provided here clarify their meaning in the context of this study.

1.4.1 Mathematics anxiety

A simple definition would be:

> *Anxiety in the presence of mathematics* (Aiken, 1976:295).

However, a more comprehensive definition is provided by Richardson & Suinn (1972:551):

> *Maths anxiety involves feelings of tension and anxiety that interfere with the manipulation of numbers and the solving of mathematical problems in a wide variety of ordinary life and academic situations.*

For the purpose of this study, mathematics anxiety is regarded as a functional psychological term representing a specific anxiety state. This anxiety state has a debilitating effect on mathematics performance in formal and informal situations.

1.4.2 A variable

A variable in the context of the title refers to a specific design element which influences a learning outcome. The purpose of studying a variable is to consider the degree of its effect on the learning outcome. Mathematics anxiety is one such variable which influences instructional and learner procedures.

1.4.3 A constructivist approach

A constructivist approach to mathematics is that mathematics is construed as a way of thinking rather than a body of fixed knowledge or content (Bishop, 1985:24). The root assumption of constructivism is the idea that learners construct their own interpretation of their experiences.

This approach represents a move away from a belief that knowledge is usually transmitted by the teacher and that learning is the passive absorption of knowledge.

Constructivism provides a basis for a learner-centred approach which allows children to become actively involved in constructing their understanding of mathematics. It is both a philosophical theory of knowledge or cognitive position and a pragmatic methodological position (Noddings 1990:18) and as such combines the didactic disciplines of theory and practice of teaching and learning.

1.4.4 Teaching of secondary school mathematics

In South Africa the traditional move to secondary school from primary school takes place after seven years of schooling and is called the Standard 6 year. The mathematics in the primary school involves arithmetical computation and the learning of basic numerical skills.

Secondary school mathematics introduces the child to a world of new language, new symbols and a more abstract subject. Number concepts become more generalized and the introduction of geometry tests the pupil's spatial awareness.

The subject becomes more removed from everyday number manipulation and real life problems. Algorithms need to be established early and there is often an urgency to use drill and practice to establish a sound knowledge of basic algebra. The content of the mathematics curriculum is full and varied and the time allocated to transmitting this content is often limited.

Bishop (1989:190) says that a curriculum is only as good as the quality of its teachers. Without the teacher's co-operation all the curriculum planning and theory and suggestions to alleviate mathematics anxiety is a waste of time and effort.

The didactics of mathematics teaching has changed over the years. Mathematics teaching is no longer simply transmitting a body of knowledge but rather the creating of desirable learning situations which facilitate the pupils' access to knowledge.

Driscoll & Lord (1990:239) describe a new mathematics teacher as a guide, coach and psychologist. As a guide the teacher leads students into important areas of investigation. As a coach the teacher inspires persistence and as a psychologist recognises attitudes, values and beliefs of the student as a crucial aspect of their learning process. Emotional and intellectual progress are inseparably intertwined in the mathematics classroom, and mathematics teaching should reflect this fact.

> *By the end of the primary years a child's attitude to mathematics is often becoming fixed and will determine the way in which he will approach mathematics at the secondary stage* (Cockcroft, 1982:101).

This statement implies an onerous task for mathematics teachers in the secondary schools. At secondary school an awareness of the differences in attainment levels has important implications for teachers. As pupils grow older these differences become more evident and influence curriculum goals and teaching methods. The fact that mathematics is a hierarchical subject necessarily implies that level of attainment at the beginning of the course is a critical component for future success.

Teaching at secondary school must provide attention to the affective variables associated with mathematics. The classroom atmosphere and providing positive mathematical experiences are the cornerstones of providing a base in which students at different levels may all reach their full potential. In essence this requires that all students develop "mathematics power" and the development of this power becomes the central concern of the teacher. Mathematical power is the ability to explore, conjecture and reason logically as well as the ability to use a variety of mathematical methods effectively to solve non-routine problems. In addition, it is the development of personal self-confidence (Commission on Standards for School Mathematics, 1989:5).

1.5 Aims and objectives of the study

It is important in a study such as this that aims and objectives are clarified. The direction of this study is determined by five aims that are stated in terms of the central elements of this thesis and the integration of these elements. These aims are all directed towards providing a proposed comprehensive approach to the alleviation of mathematics anxiety and the development of each individual's "mathematics power" (see 1.4.4).

1.5.1 Providing a clear understanding of mathematics anxiety as a variable in mathematics teaching

The first aim of this study is to provide an overview of mathematics anxiety as an important variable in mathematics teaching. By this means it is envisaged to establish that mathematics anxiety is a serious developmental problem which hinders the normal educational progress of a high percentage of children.

This aim leads to the following objectives being pursued:

a) To establish that mathematics anxiety is widespread and continuing.

b) To establish that mathematics anxiety affects mathematics performance in the classroom.

c) To establish that mathematics anxiety contributes to perceptions and attitudes that perpetuate the problem.

d) To identify critical elements of mathematics anxiety that should be addressed by mathematics educators.

e) To propose that it is necessary to establish a comprehensive approach to remediation of mathematics anxiety which includes curriculum planning and instructional techniques.

1.5.2 To provide an overview of mathematics teaching and curriculum design in mathematics

The second aim of this study stems from the first in that the need to combat mathematics anxiety leads to a need to explore the components of curriculum design that best address the problem. Thus, the second aim is to provide an overview of mathematics teaching and curriculum design to establish a frame of reference for good and sound mathematics teaching which will alleviate mathematics anxiety.

This aim leads to the following objectives being pursued:

a) To establish to what extent various views on curriculum design has influenced mathematics teaching.

b) To propose that a cyclical systems approach provides the structure to address the problem of mathematics anxiety.

c) To identify the components of a systems approach that provide a frame of reference for mathematics teaching and the alleviation of mathematics anxiety.

d) To establish a need for the use of situation analysis to identify critical aspects of curriculum planning that will influence mathematics anxiety.

e) To identify critical elements of mathematics teaching that influence mathematics anxiety.

1.5.3 <u>Providing a view of the constructivist approach to the teaching and learning of mathematics</u>

The third aim of this study is to provide an overview of the constructivist approach with the purpose of establishing whether or not this approach could be adapted to focus on the critical elements affecting the progress of the mathematically anxious child.

This aim leads to the following objectives being pursued:

a) To identify the tenets of constructivism which provide the direction and framework for mathematics teaching.

b) To identify aspects of the sources of mathematics anxiety that are addressed by the tenets of a constructivist approach.

c) To establish a link between the constructivist view of teaching mathematics and the critical elements of mathematics teaching that alleviate mathematics anxiety.

d) To propose that constructivism provides both a cognitive position and a methodological approach which is compatible with the requirements of the mathematically anxious child.

1.5.4 <u>To conduct empirical research on mathematics anxiety and achievement to support literature views</u>

The central aim of the empirical research is to provide confirmation for the views on mathematics anxiety which have been established in other research reported in the literature. In particular, that it is widespread and continuing, that it affects performance and that it influences perceptions and attitudes which perpetuate the problem.

Thus, the following objectives are pursued:

a) To establish that mathematics anxiety does exist at the start of secondary school and is widespread.

b) To establish that mathematics anxiety does exist at the end of secondary school and is widespread.

c) To establish that mathematics anxiety exists amongst young trainee teachers.

d) To investigate any change in mathematics anxiety during the secondary school years.

e) To establish that mathematics anxiety has a negative effect on mathematics performance.

f) To establish whether mathematics performance deteriorates over the secondary school years.

g) To establish whether perceptions and attitudes perpetuate mathematics anxiety.

h) To identify critical elements of teaching methodology that affect mathematics performance and cause anxiety.

1.5.5 <u>To Analyse and Synthesise the implications of the literature and empirical research for mathematics teaching</u>

The fifth aim of this study is to analyse and synthesise the central elements of the thesis. The overriding aim of this thesis is that the critical aspects identified in the research on mathematics anxiety, constructivism and the curriculum are integrated to formulate an approach to mathematics teaching that incorporates the didactic principles which will alleviate mathematics anxiety and lead to the development of intellectual autonomy and self-confidence.

The objectives pursued to achieve this aim are as follows:

a) To identify the elements of situation analysis that incorporate aspects of mathematics anxiety concerns and aspects of a constructivist approach.

b) To identify the components of a systems approach which emphasise constructivist views and address the problem of mathematics anxiety.

c) To identify and propose a teaching methodology as a function of the systems approach that adopts the tenets of constructivism.

d) To propose a constructivist approach as a comprehensive didactic approach which will address social, cognitive and emotive sources of mathematics anxiety through the medium of the classroom.

e) To establish the critical elements of teaching and learning through constructivism that will develop the mathematics power of each individual.

1.6 Programme of study

To achieve the above stated aims this study will be structured to provide a literature study of mathematics anxiety, teaching and the curriculum and the constructivist approach. This will be followed by the empirical research on mathematics anxiety and achievement. Finally certain conclusions will be reached in light of the information provided in this study and directed by the aims and objectives stated in Section 1.5.

A brief outline of the modus operandi for this study is provided below.

1.6.1 Literature study of mathematics anxiety

The literature available on mathematics anxiety is vast and varied and could provide volumes. However, for the purpose of this study, theories and research findings which are relevant to curriculum design and the teaching process will be of primary concern. In Chapter Two a brief historical perspective will provide details of past studies in this field as well as details of recent research developments pertinent to this study.

The literature will be analysed to identify why mathematics anxiety is viewed as an important variable in mathematics teaching. Mathematics anxiety is widespread, affects performance, causes mathematics avoidance and creates a vicious circle effect which perpetuates the problem. These factors all establish the wide interest and concern in mathematics anxiety and emphasise the high percentage of people who actually suffer from some form of mathematics anxiety.

The literature also provides details of research on mathematics anxiety and this will be discussed under three main categories, namely, measuring, sources and symptoms. The measuring of mathematics anxiety is investigated to provide a history of the research into this sphere and also to establish the principles by which the measuring instruments for this study were compiled. The sources of anxiety are many and varied but for the sake of convenience in this study they are categorized under socio-cultural, emotive, cognitive and educational factors. Following these causes of mathematics anxiety, the symptoms as they appear in the classroom are discussed.

Finally, the literature research will be analysed to provide important implications for this study and to synthesise those areas of mathematics teaching and learning that must play a significant part in addressing the problem of mathematics anxiety by influencing curriculum design and the didactic approach to mathematics.

1.6.2 <u>Literature study of teaching and the curriculum</u>

A review of the developments in curriculum studies in Chapter Three will provide a background to the present thinking in this field. These ideas have influenced mathematics curriculum design in a way which has had much benefit for the mathematics anxious student by moving towards a more pupil-centred approach.

An analysis of the historical aspects of curriculum design and a study of a number of approaches to curriculum planning provides the necessary background for establishing a didactic design which contributes positively to the needs of the mathematically anxious child. A cyclical systems approach is investigated as a framework for curriculum planning and the problem of mathematics anxiety is discussed within this framework.

A number of elements from relatively new approaches to curriculum will be emphasised as beneficial for alleviating mathematics anxiety. These include aspects of child-centred, humanistic and process approaches as well as aspects of cognitive psychology and the affective and socio-cultural domain. A number of these aspects will become evident in the constructivist approach to the teaching of mathematics. Critical aspects of mathematics teaching that affect mathematics anxiety are identified and categorised to establish a direction for curriculum planning and a comprehensive didactic approach to alleviate mathematics anxiety.

1.6.3 <u>Literature study of constructivism</u>

Relevant aspects of the constructivist theory will be analysed. Emphasis will be given to the teaching and learning of mathematics as a proactive formula for the successful development of mathematical understanding.

Constructivism embodies a philosophy of how pupils come to learn as well as a design for teaching strategies. The aim of this study is to link this cognitive position and methodological position to the requirements of the mathematically anxious child. This will involve an emphasis of a child-centred approach that believes in the individual's ability to construct knowledge and a methodological approach which encourages such development.

The constructivist approach suggests certain reforms in curriculum design which emphasise developing mathematical skills by developing understanding and a problem solving attitude. The didactic design becomes of central concern as processes important to teaching and learning are emphasised above the need for particular content.

The intent of this study is to review the literature on a constructivist approach which embodies the principles of child-centred education and the need for problem solving skills as well as a methodology which promotes deeper understanding through social interaction, reflection and communication.

The benefits for the mathematically anxious pupil in a constructivist programme will be detailed in Chapter Four as aspects of constructivism are linked to the problems of mathematics anxiety.

The teaching process proposed by constructivists will be outlined and the crucial issues relevant to curriculum design and the avoidance of mathematics anxiety will be given particular attention.

1.6.4 **Empirical research programme**

Chapter Five reports on a longitudinal research programme which was structured for the purpose of this study. It takes the form of a case study in which the progress of 100 pupils from Standard 5 to Standard 10 has been monitored and in which these pupils have been subjected to various tests and questionnaires. The questionnaires were constructed to measure the following:

1. Mathematics anxiety at Standard 5 level
2. Mathematics performance at Standard 5 level
3. Mathematics anxiety at Standard 10 level
4. Mathematics perceptions and attitudes at Standard 10 level

The data from these instruments would be used to substantiate the literature claims on mathematics anxiety as outlined in Section 1.5.4.

A parallel study of 148 student teachers at a College of Education was carried out and the data collected is used to investigate the following:

a) The existence of mathematics anxiety amongst prospective mathematics teachers.

b) The correlation between mathematics anxiety and past school performance in mathematics.

c) The competence of the prospective teachers to teach at secondary school level.

This data would also be used to substantiate the literature claims on mathematics anxiety as outlined in Section 1.5.4.

1.6.5 Conclusions

Inevitably this study will provide teachers, pupils and curriculum designers with recommendations concerning the teaching and learning of mathematics. Chapter Six will provide a detailed discussion on the implications of this study and how the findings of the literature study and empirical research suggest certain changes to curriculum ideas. What is certain is that these ideas will impact mainly on the mathematics teacher as a specialist in the secondary schools. This research specifically intends to provide guidelines for mathematics teachers in their approach to mathematics instruction.

CHAPTER TWO
MATHEMATICS ANXIETY

2.1 General perspective

The concept of mathematics anxiety has received increasing attention over the past number of years and research on the nature and aetiology of the problem has been widespread. Suggested methods of combating this problem has led to the establishment of many programmes and institutions offering to alleviate mathematics anxiety.

Most of these programmes are in the United States of America where the term "mathematics anxiety" has become a popular educational phrase which is often used without enough knowledge of the problem. A number of factors may influence the learning of mathematics. Mathematics anxiety is one such factor which educators are now recognizing as a very important variable in the didactic studies of mathematics.

It is important that students do not view mathematics anxiety as just another theoretical basis for research but rather a genuine, growing concern for a widespread debilitating developmental problem.

There are many reasons why there is growing concern for mathematics anxiety and this chapter will provide some insight into these reasons.

2.1.1 Why mathematics anxiety?

The development of the term mathematics anxiety evolved from two directions. Firstly, the psychological interest in general anxiety led to studies of more situation specific anxieties such as public-speaking anxiety, anxiety when travelling, test anxiety and eventually mathematics anxiety. Whilst mathematics anxiety can provide many answers to questions about general anxiety it is not conclusive that the two are related and, indeed, many people suffering from mathematics anxiety do not suffer from general anxiety (Richardson & Suinn, 1972:551).

However, there does appear to be a strong connection between mathematics anxiety and test anxiety. Stipek (1988:10) supports this belief because she reports that over a third of college students applying for a behaviour therapy program to reduce test anxiety indicated that their primary difficulty was related to mathematics and simply being asked to do a mathematics problem was viewed as a test in itself.

It is true to say that studies of state anxiety were probably much more easily arranged by using test or mathematics anxiety because it is more difficult to simulate such situations as public speaking or air travel.

The second direction from which mathematics anxiety evolved is from mathematics educators and researchers. For many years mathematics as a subject has been of great importance but none more than recent years. The advent of the computer age and the widening array of vocations that require a theoretical and practical knowledge of mathematics have all contributed to an added emphasis on mathematics achievement.

From the mathematics educators point of view, mathematics anxiety is an important variable to consider because it is a factor which can cause an inadequacy in mathematics as much as any real lack of mathematical ability. The implication is, therefore, that the alleviation of mathematics anxiety and the promotion of positive mathematics attitudes should be one of the primary tasks of mathematics educators.

Carpenter *et al* (1981:25) and Brush (1979:5) both express concern for the decline in positive attitudes towards mathematics in the secondary school years, which their respective research revealed. In addition, research by Hembree (1990:38) has revealed that mathematics anxiety is on the incline during the secondary school years.

Mathematics anxiety may be an individual trait but it must necessarily influence mathematics educators. Attention must be given to all aspects which may debilitate mathematics performance and undermine the confidence and attitude of students. Whilst the psychologist may only be interested in the study of the emotions of anxiety, mathematics teachers are concerned about the outcomes or consequences of feeling anxious in doing mathematics.

2.1.2 What is mathematics anxiety?

The simplest definition of mathematics anxiety is provided by Aiken (1976:295) who says:

> *It is anxiety in the presence of mathematics.*

Other researchers have elaborated on this definition and have tried to describe the anxiety. Lazarus (1974:16) describes it as:

> *An unnatural and impeditive dread of mathematics.*

He goes on to say that it also has a self-sustaining syndrome whereby the individual's low level of functioning leads to difficulty and failure which in turn leads to further anxiety.

Definitions by Auslander and Richardson & Suinn are more descriptive. Auslander (1979:17) defines mathematics anxiety as follows:

> *Mathematics anxiety can be defined as the experience of mental disorganization panic, and fear that prevents a person from learning mathematics..*

Richardson & Suinn's (1972:551) definition of mathematics anxiety is:

> *Mathematics anxiety involves feelings of tension and anxiety that interfere with the manipulations of numbers and the solving of mathematical problems in a wide variety of ordinary life and academic situations.*

In these two definitions the focus is on the consequences of anxiety rather than the emotion itself.

As mentioned earlier (see 2.1.1) , the debilitating effect that mathematics anxiety has on mathematics performance is of central concern in this study. Whilst recognizing the fact that mathematics anxiety is a functional psychological form representing a specific anxiety state, it is the consequences of this state which have far-reaching implications for the mathematics educator. These implications become evident when research and literature on this topic is studied. The remainder of this chapter provides a clearer perspective of the concerns of educators for the problem of mathematics anxiety.

For the purpose of this study, mathematics anxiety is seen as a developmental problem which has a debilitating effect on mathematics performance and as such should be an important consideration of curriculum designers and mathematics teachers.

2.1.3 <u>The importance of mathematics anxiety</u>

Schonell & Schonell (1957:7) came to the conclusion that normal emotional reactions are more important than normal intellectual ones to progress in mathematics. This concern for the emotive factors influencing mathematics performance is expressed by a number of researchers.

Crumpton (1977:11) says that a majority of individuals probably suffer from some form of mathematical anxiety. She goes on to say that this may occur at primary school level but for some people it may only start at university level.

Stipek (1988:111) came to the conclusion that mathematics anxiety develops more over the critical secondary school years and she expresses concern for the quality of teaching during this time in a child's life.

Stodolsky (1985:132) supports this view with her report from an observation study of mathematics and social studies classes. She found that mathematics instruction had characteristics that would lead students to perceive their role in learning mathematics as primarily passive. Stodolsky (1985:132) believes that the type of teaching provided a basis for mathematics anxiety because, students who did not succeed in the instructional format did not have the opportunity to explore mathematics in an alternative learning format.

The views expressed by the researchers quoted above provide important reasons for mathematics teachers to note of the anxiety syndrome. Important implications for teachers are as follows:

1. Mathematics teaching cannot only be content related. Cognitive and emotional factors play an important part in the development of a child's mathematical progress.

2. Mathematics teachers cannot assume that because the child has not had a problem with anxiety previously that this cannot develop at any future stage during schooling.

3. Mathematics teaching must continuously seek methods to provide the ideal instructional atmosphere for the child and to strive for instructional formats which explore the potential of each child.

Good mathematics teaching is essential to mathematics progress, but teaching can only be good if the teacher is concerned about mathematics anxiety and the affective and cognitive variables that are connected to mathematics anxiety and factors hindering the progress of their pupils.

Research has identified four most important reasons why mathematics anxiety is a critical factor in child development and why it is an important variable for the teachers' attention. Mathematics anxiety is widespread, it affects performance, causes avoidance of mathematics and has a vicious circle effect by which the problem is passed on from generation to generation.

2.1.3.1 **Mathematics anxiety is widespread**

From the very early literature on mathematics anxiety, researchers have emphasised the fact that it is a debilitating psychological trait which is more widespread than suspected and that it is not restricted to any particular group of people or to any given time-span in a person's life.

Some people may not feel anxious until they reach university level while others may panic at primary school. Zacharias (1976:22) stated that mathematics anxiety afflicts almost everyone and Lazarus (1974:17) felt that:

> *the children and adults who should be judged mathophobic outnumber by far those who are comfortable with mathematics.*

Mathematics has unfortunately acquired the reputation of being an extremely difficult subject. This perception together with socially acquired negative attitudes all contribute to the extent in which students develop mathematics anxiety.

There is little need to research the numbers of people who feel anxious in the presence of mathematics. One only needs to discuss the topic in any company to uncover a hate or loathing or unashamed boastfulness for past or present bad performances in mathematics or past bad experiences in mathematics. These are all signs of a presence of mathematics anxiety which leads to people developing their own defence mechanisms to explain why they are unable to cope with the subject.

Low levels of anxiety can enhance a child's ability to think and act (Kemp, 1990:2) and this positive effect can be used to motivate the learner and improve his or her performance. However, the inhibition produced by mathematics anxiety appears to swamp any motivating effect (Biggs, 1963:59) and unless special attention is given to building confidence there will be little progress made in developing mathematics performance.

In addition, anxiety appears to be more easily aroused in learning mathematics than it is in any other school subject. The reasons for this are discussed later when the sources of anxiety are investigated (see 2.2.2).

2.1.3.2 Mathematics anxiety affects performance

Research concerning mathematics anxiety and learning seems to indicate that high levels of anxiety hinder academic progress. Negative correlations have been found between mathematics anxiety ratings and performance on the mathematics component of the Differential Aptitude Test (DAT) (Richardson & Suinn, 1972; Suinn *et al*, 1972).

Hembree (1990:42) reviewed thirteen previous studies of the relationship between mathematics anxiety and performance. He reports that in all cases the low-anxious students always performed better than those with high-anxiety levels.

Hembree (1990:43) also discovered that treatments that resulted in significant mathematics anxiety reduction were accompanied by significant increases in mathematics test scores.

Because Hembree (1990:44) integrated the results of one hundred and fifty one studies on mathematics anxiety his conclusions are extremely important for any further research on this topic. On the question of mathematics anxiety depressing performance, he comes to the following conclusions:

1. Reduction in mathematics anxiety resulted consistently in higher achievement.
2. Treatment can restore the performance of formerly high-anxious students to the performance level associated with low mathematics anxiety.
3. There is no compelling evidence that poor performance causes mathematics anxiety.
4. Special work to enhance students' competence failed to reduce their anxiety levels. (Hembree, 1990:44)

Hembree's research has considerable significance because he has integrated the findings of such a wide range of previous research reports on mathematics anxiety. The evidence that appears in his conclusions suggest that whilst mathematics anxiety does affect performance, poor performance does not necessarily cause the anxiety. It is also significant that remediation of the problem may be better through the alleviation of anxiety rather than an enhancement of competency in mathematics.

2.1.3.3 **Mathematics anxiety causes mathematics avoidance**

Children and adults who are anxious about mathematics tend to avoid mathematics altogether or take an easy option. Tobias (1976:56) noticed that:

> *some of her students were even contemplating changing their majors to avoid mathematics prerequisites..*

This problem appears to be universal as even students in Kwazulu-Natal schools appear to select courses which provide an "easy way out". Boys are generally encouraged to take mathematics but in fact are too quick to resort to the standard grade level. Girls may give up mathematics altogether or opt for a standard grade course which is far below their capabilities. Mathematics avoidance leads to a limited choice of courses at university as even fields that are thought of as non-mathematical have become more mathematized. This course restriction subsequently limits the career development of the student.

Mathematics anxiety may be viewed as a cause or an effect of mathematics avoidance. Negative and self-defeating feelings generated by mathematics anxiety could lead to mathematics avoidance whilst people who have avoided mathematics for some time may find that the unfamiliarity, awkwardness and forgetfulness they experience would lead to mathematics anxiety. An important distinction to make is that mathematics avoiders are not necessarily mathematics anxious, but may have chosen to avoid mathematics because of such factors as ignorance of its importance in careers or the stereotyped view of mathematics as a male domain.

Whatever the reasons, mathematics avoidance causes added tension and complications to the problem of mathematics anxiety because of a spiralling effect which is created. Anxiety results in avoidance - which results in reduced ability to perform mathematical operations - which generates negative feelings and increased anxiety - which promotes further avoidance and so on.

Sovchick (1989:118) is concerned that mathematics anxiety of teachers causes them to avoid topics not because they are difficult for students to understand but rather because the teacher is fearful of teaching the topics incorrectly. Thus, avoidance of mathematics may be traced to a teaching inadequacy which will have a negative effect on the performance and confidence of the students.

The potential cause and effect relationship of mathematics anxiety and mathematics avoidance suggests some overlap in their origins which stems from both social and cultural attitudinal variables. These origins may be traced to problems at home but are often traced to poor classroom techniques.

The chain of events causing avoidance needs to be identified if it is being provoked by mathematics anxiety. By addressing the problem of anxiety and attempting to alleviate anxiety, the compulsion to avoid mathematics at all costs will be reduced. This in turn will ensure that more students offer mathematics in their school courses and thus have a wider choice of career opportunities once they leave school.

2.1.3.4 **Vicious circle effect**

People who feel highly anxious about mathematics often rationalise that it is unimportant and the fact that they are incompetent in various areas of mathematics is of no consequence. These people tend to pass on these feelings of anxiety. Parents who are convinced that mathematics is unessential and unimportant are likely to pass on these feelings to their children. Likewise, a teacher with anxiety towards mathematics could drastically affect the attitude of the children he or she teaches. If this "vicious circle" effect is not stopped, mathematics anxious children will continuously be emerging from our schools and finding themselves as parents and teachers ready to perpetuate a similar syndrome.

Part of the vicious circle effect is a self-fulfilling prophecy which parents and teachers may bestow on their children. Predictions of failure in mathematics may be transmitted by parents and/or teachers in rather subtle or even unintended ways. These predictions are perceived by the child and inevitably he or she fulfils the prophecy.

To avoid the self-fulfilling prophecies and to break the chain of the vicious circle, teachers of mathematics and parents need to develop skills in helping children to change such negative frames of reference which perpetuate mathematics anxiety.

2.2 <u>Research on mathematics anxiety</u>

Early research in mathematics anxiety was often limited to observations of a widespread disability. However, recent interest in mathematics anxiety has resulted in the publication of research findings on its nature and aetiology, suggested methods of approaching the problem and the establishment of programmes to combat the problem. This research will be referred to throughout this review of the literature. Studies and research findings have now reached a stage where it is fairly safe to assume that many people learn far less mathematics than they otherwise might, owing to mathematics anxiety (Dougherty, 1981:1).

It is now crucial that this research is used to guide curriculum developers and teachers to a consideration of both emotional and intellectual concerns in classroom strategies and school programmes.

The literature research study discussed in this section concentrates on the following aspects which are relevant to this thesis.

1. Research into the measurement of mathematics anxiety is detailed and investigated to provide a background to the development of a measuring instrument suitable for the South African situation.

2. The sources of mathematics anxiety are identified and categorised as socio-cultural, emotive, cognitive and educational for future reference.

3. The symptoms of mathematics anxiety in the classroom are identified and classified as non-task orientated behaviour, dependence on teacher, non-social behaviour and physical reactions.

4. Finally, the concerns expressed in terms of causes and symptoms are integrated to express the implications for this study. In essence, this emphasises the teaching of mathematics to alleviate mathematics anxiety.

The fundamental intention of this literature research is to establish that the question of whether or not mathematics anxiety exists is no longer an issue. The literature reveals that it is widely accepted that mathematics anxiety is a reality for many people and that researchers must now concentrate on an assessment of the levels and intensity of mathematics anxiety, the source of the problem and curriculum innovations that may assist in alleviating the problem.

2.2.1 **Measuring mathematics anxiety**

The instruments used for measuring mathematics anxiety have an interesting history. These measuring scales have evolved from psychological testings of various origins with fairly vague connections to mathematics anxiety.

The descriptions provided below will show how these scales evolved from general anxiety and test anxiety scales. It will also show how mathematics anxiety scales may be associated with measures of mathematics performance and attitude or personality traits. Mathematics anxiety scales have developed from fairly simple questionnaires to much more sophisticated instruments. The following provides a brief description of how mathematics anxiety scales developed.

2.2.1.1 **The Three Number-Anxiety Item from the Taylor Scale**

The Taylor Scale consists of forty seven anxiety provoking situations which required a "whether or not" response to each situation. One of the earliest mathematics specifications for investigating specific number anxiety was the use of three items from the Taylor Scale. These three items were used by Dreyer & Aiken (1957:346) to identify number anxious pupils amongst a college population. The three items asked the subject to respond to the questions:

(a) whether or not arithmetic made him nervous,

(b) whether or not he would 'freeze up' whenever he saw a mathematics problem, and

(c) whether or not he was as good in mathematics as in other subjects.

This early development in mathematics anxiety scales was fairly brief but it did indicate a move from more general studies of anxiety to an investigation of a specific anxiety trait.

2.2.1.2 <u>The National Longitudinal Study of Mathematics Abilities</u>

Crumpton (1977:18) describes two mathematical scales which were developed for use in the National Longitudinal Study of Mathematics Ability which was carried out in the United States of America. These scales are an example of how test anxiety measures were used to develop mathematics anxiety scales. More specifically, Crumpton (1977:18) describes the Achievement Anxiety Test (AAT) developed by Alpert & Haber (1960) which was modified. The AAT is said to measure *facilitating and debilitating effects of anxiety on achievement performance*. These tests were modified to relate more specifically to mathematics anxiety. The result was two scales, one which measures facilitating mathematics anxiety or the degree to which students' mathematics achievement is facilitated by stressful conditions, and a second scale which measures debilitating anxiety or the degree to which students' mathematics achievement is harmed by stressful conditions. However, there is no information about reliability or validity data for any of these scales and they are not widely used (Crumpton, 1977:18).

2.2.1.3 <u>Fennema and Sherman Scales</u>

Richardson & Woolfolk (Sarason, 1980:272) describe the Fennema - Sherman Scale. It is a set of nine 12-item scales designed to measure a number of different attitudes and feelings about the learning of mathematics by female and male High School students. These scales asses the student's confidence in learning mathematics; the view of mathematics as a male domain; usefulness of mathematics; attitudes of mother, father and teachers towards the student's learning of mathematics; and several other factors. One of the scales measures mathematics anxiety. Students respond to a five-point Likert-type scale in order to indicate the extent to which they agree or disagree with 12 statements that express feeling anxious, tense, or at ease with mathematics problems and tests. Six items are scored positively and six negatively. Male and female norms based on two large High School samples are presented, and a split-half reliability co-efficient of 0,89 is reported for the scale. The battery of nine scales are highly respected and widely used in America and deserve close study by anyone interested in mathematics anxiety treatment or research (Richardson & Woolfolk in Sarason, 1980:273).

However, Richardson & Woolfolk in Sarason (1980:273) point out that:

> *No information is given concerning test-retest stability or validity of the scale, and it would be desirable to have more precise information than given about internal consistency reliability to ensure that each item of this very brief scale correlated substantially with total scores on the instrument.*

2.2.1.4 **Mathematics Anxiety Rating Scale (MARS)**

The MARS test was specifically constructed by Suinn to provide a measure of anxiety which solely concentrated on mathematical concepts and number manipulations (Richardson & Suinn, 1972:551). The MARS is undoubtedly the best known scale for measuring mathematics anxiety and it is frequently referred to in the literature on the subject.

Two forms of MARS have been used in research: A 98-item scale and a 94-item scale. The items include brief descriptions of ordinary life and academic situations involving the manipulations of numbers or solving of mathematical problems that may arouse anxiety. A total mathematics anxiety score is calculated by assigning a value of 1 to 5 corresponding to the level of anxiety checked (1 for "not at all" anxious to 5 for "very much" anxious), and then summing all the values.

The research in this study makes use of the adaptations of the MARS test (See Appendix 8). This instrument was selected as a model due to its popularity and the high regard researchers have for MARS in the United States of America. The main reason for its widespread use is the normative data which Richardson & Suinn (1972:552) and Suinn *et al* (1972:374) have provided. Three studies have been conducted to collect normative data on the MARS. Richardson & Suinn (1972:553) reported that on the 98-item form, data on a sample of 397 students enroled in a large state university in Missouri, had been collected. The mean score for this sample was 215,38 (possible high score of 490, low score 98), the standard deviation was 65,38; the test-retest reliability co-efficient was 0,85 and the internal reliability co-efficient was 0,97. A separate sample of 30 junior and senior students were administered the MARS and then the Differential Aptitude Test (DAT). The

Pearson product-moment correlation co-efficient between subjects' scores was -0,64. Richardson & Suinn (1972:551) contended that:

> *Since high anxiety interferes with performance and poor performance produces anxiety, this result provides evidence that MARS does measure mathematics anxiety.*

Suinn *et al* (1972:374) reported that normative data had been collected on a sample of 119 students enroled in a large state university. The mean and standard deviations were 197,3 and 55,5 respectively, on the first testing and 179,9 and 55,9 respectively on the second testing. The test-retest reliability co-efficient was 0,78 after 2 weeks. Correlations between MARS and DAT were -0,35 and -0,32 indicating that high anxiety as measured by MARS is associated with low performance on DAT tasks (Suinn *et al*, 1972:303). The mean score of students who sought therapy for mathematics anxiety was 256,9 indicating a high level of mathematics anxiety and providing evidence of the validity of MARS.

Richardson & Woolfolk (Sarason, 1980:274), performed a principle components factor analysis with a varimax rotation of the factors on the 397 students' MARS scores from the Richardson & Suinn (1972) study and also determined the item-total correlation for each MARS item. They found that:

> *Almost all MARS items correlate with total scores above 0.40 and that all the items describing evaluative academic and problem solving situations correlate more highly with total MARS scores than do items concerning everyday, non-evaluative number manipulations* (Sarason, 1980:274).

Richardson & Woolfolk (Sarason, 1980:274), selected 40 MARS items with the highest item-total correlations (from 0,74 to 0,56). They claim that this 40 item scale is:

> *Presumably at least as reliable, stable and valid as the original MARS and is almost certainly dominated by a single homogeneous factor of anxiety concerning evaluative test taking and problem solving mathematics situations* (Sarason, 1980:274).

The normative data associated with the MARS test makes it a most reliable instrument to use in research.

The MARS test and more specifically the modification of this test by Richardson & Woolfolk (Sarason 1980:275-278), provide an ideal basis for research into mathematics anxiety. The tests of reliability, stability and validity have been established and there is no reason to doubt the claims of Richardson & Woolfolk stated above. The research instruments used in this study were developed with these recommendations in mind. Details of how items were adapted is described fully in Chapter Five.

2.2.2 Sources of mathematics anxiety

It is difficult to discover precisely why mathematics anxiety occurs. Reasons could be found within the individual, the classroom or the society. Over the last two decades mathematics anxiety has received considerable attention and gradually answers are emerging.

Whilst some factors obviously overlap, the sources of mathematics anxiety are categorised in this study under the headings socio-cultural, emotive, cognitive and educational (or school) factors.

2.2.2.1 Socio-cultural factors

Tobias (1978b:7) claims that the sources of mathematics anxiety are more political and social and remediation should focus less on curing the individual and more on rectifying the conditions which foster anxiety.

Tobias (1987:4) feels that often the anxiety towards mathematics develops from early when children are given the impression that certain people can do mathematics and certain people simply cannot.

Stipek (1988:111) supports this notion and adds that if they believe they lack ability in mathematics they also believe that there is nothing to be done about it. There is no doubt that researchers believe that mathematics anxiety develops from perceptions and attitudes which are transmitted in society.

It is becoming socially acceptable to not be able to do mathematics. Many professional men who are otherwise proud of their achievements will shamelessly admit to being no good at mathematics. Parents who may find themselves unable to cope with the "new maths" or are very committed to old methods and ideas, may express negative feelings to their children. There is often quite a strong conflict between homes and schools because parents do not understand the logical progress of their child's mathematics course.

Kemp (1990:2) is also concerned about the socio-cultural "reputation" of mathematics. He emphasises the fact that parents and teachers transfer their own negative feelings of mathematics onto young pupils.

He goes on to say that whilst adults do not talk about failure in any other subject they admit unashamedly how poorly they have done in mathematics. Kemp feels that, very subtly children are brainwashed to fear mathematics long before they have had an opportunity to explore it for themselves (Kemp, 1990:3).

Parents may also cause anxiety by the pressure they place on their children to do well. In general, parents are more concerned that their son copes well academically, whilst daughters are often not expected to do as well as sons. The daughter may feel anxious if she does not live up to this subordinate role, whilst the son may become anxious under the pressure being put to bear on him to do well (Jacobs, 1978:125).

Williams (1988:95) found that many parents and teachers pass on their anxieties to the children. She quotes parents and teachers statements implying an uncomfortable feeling with and/or anxiety about mathematics. She says that, too often the parent presupposes that the child will have these same feelings and attitudes towards mathematics.

Lewis (1987:62) identifies three major causes of mathematics anxiety in children and in each of these cases the heart of the problem lies in the excessive and unrealistic nature of:

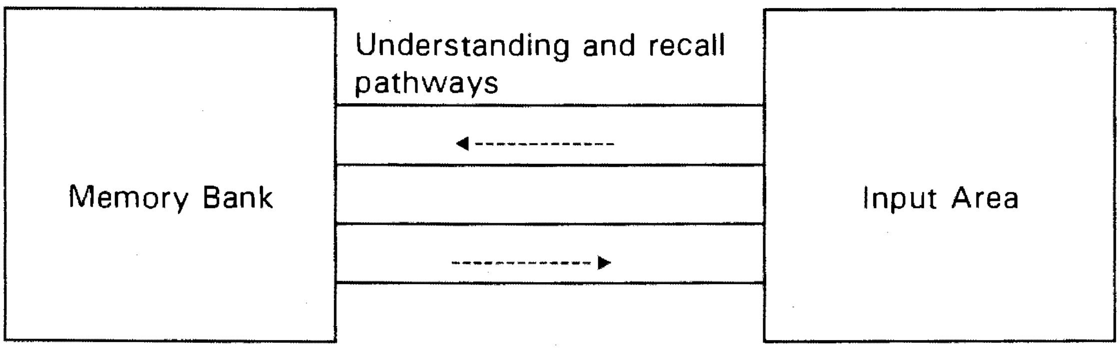
1. the challenges of the subject;
2. the social demands;
3. the expectations of parents and/or teachers.

Whilst mathematics will always remain an intellectually challenging subject, it is the perceptions and attitudes developed socially that perpetuate the problem of mathematics anxiety. Society demands a certain level of competence in mathematics which is reinforced by parents' expectations.

2.2.2.2 **Emotive factors**

Mathematics anxiety is a highly emotive state and it is important, therefore, to consider certain emotive reactions of this condition. A person may develop psychological blockages which will seriously impair his or her mathematics performance.

These debilitating reactions are sometimes difficult to detect because they are only aroused when mathematics is involved. For example, Richardson & Suinn (1972:551) have shown that mathematics anxiety can afflict an individual who does not normally feel anxious. To describe these feelings of anxiety Tobias (1987:6) uses a very simple diagram of the functioning brain having an input area, a memory area and some kind of understanding and recall pathways.



Tobias (1987:6)

She feels that often all parts of the brain are able to function efficiently until the emotions come into play. Faced with a mathematics problem panic sets in and the understanding and recall pathways become immediately cluttered by emotions (Tobias, 1987:7).

Researchers such as Stipek (1988:111), Crumpton (1977:11), Zacharias (1976:22) and Lazarus (1974:17) have also shown that mathematics anxiety is not necessarily related to general intelligence and people who perform poorly in mathematics may be highly successful in other subjects. Some of the factors which may be considered to make mathematics an anxiety provoking subject are listed below. The connotations attached to mathematics and the myths that people have developed and believe, contribute to the emotiveness in mathematics.

<u>Connotative meanings</u>:

Richardson & Woolfolk (Sarason, 1980:271) emphasise the fact that mathematics has a unique and important component which appears to be its connotative meanings for many people. They say that ***being good at*** or ***liking*** mathematics connotes:

Certainty
Perfection
High intelligence
Genius
Some arcane wisdom
Highly specialised knowledge remote from common sense
The essence of practicality
A characteristically masculine activity

These connotations cause people to develop certain emotive reactions to the subject and each individual may be affected in a different way by the various connotative meaning connected with mathematics.

Certain pupils may assume that doing mathematics is beyond them and only a subject for the gifted few. The fact that an individual who is good at mathematics is set apart from others and that being good at mathematics is seen as a characteristically masculine activity can lead to several emotive reactions. Males who are not good at mathematics may feel inadequate whilst those that excel may feel out of place. Certainly there has been evidence of girls who do well at mathematics and are not eager to reveal their aptitude for fear of being regarded as strange (Jacobs, 1978:28).

<u>The Myths of Mathematics</u>:

Kogelman & Warren (1978:30) believe that people develop certain emotive reactions towards mathematics because of 12 myths which have gradually become entrenched in our society. These myths are:

1. Men are better in mathematics than women.
2. Mathematics requires logic, not intuition.
3. In doing mathematics you must always know how you got the answer.
4. Mathematics is not creative.
5. There is a best way to do a mathematics problem.
6. It's always important to get an answer exactly right.
7. It's bad to count on your fingers.
8. Mathematicians do problems quickly in their heads.
9. Mathematics requires a good memory.
10. Mathematics is done by working intensely until the problem is solved.
11. Some people have a "mathematics mind" and some don't.
12. There is a magic key to doing mathematics.

Kogelman & Warren (1978:30) emphasise the mystification that surrounds mathematics and see this as a major cause of the manifestation of anxiety. They also claim that many of these myths are formed at an early age at school and are often because of misguided attitudes of both parents and teachers.

In summary, it is probably fair to state that mathematics raises questions of various emotive influence which are often not present in the perceptions of other subjects. Connotations and myths become entrenched in their association with mathematics and the emotions are called into play more often than not when the question of doing mathematics is involved.

2.2.2.3 **Cognitive factors**

The content and nature of the subject mathematics is often also a source of anxiety. As pupils progress through school certain cognitive factors unique to mathematics may cause a setback and hence contribute to feelings of anxiety. Some of these factors to consider are:

a) Mathematics gets more difficult (more abstract) as one progresses at school.

b) Students spend more and more time learning what is already known and less and less time contributing their own ideas.

c) After arithmetic, the everyday uses of the material learned in class seems limited.

d) Working with numbers does not seem to contribute to a better understanding of ones self or of society; the subject seems very impersonal.

e) Ability in mathematics is more measurable than ability in most other subjects and this often leads to anxiety due to fear of exposure.

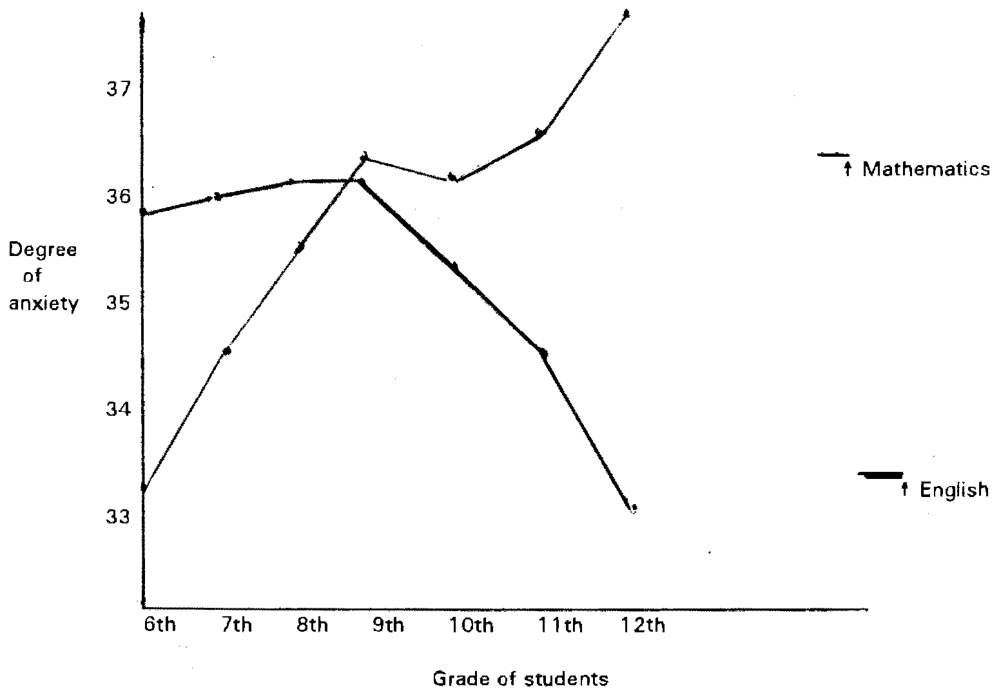
f) The continuity aspect of mathematics makes it more difficult as one continues through school. Without a good grounding mathematics will become more and more difficult and hence more anxiety provoking.

g) The language and notations used in mathematics is unique and the need to learn this new language before one can understand mathematics is a source of anxiety.

Whilst many of the factors mentioned above may also be true of other subjects they are accentuated in the teaching and learning of mathematics. There is little doubt that the content is difficult and that the volume of required conceptual links requiring comprehension and assimilation into constructs is greater than in any other subject.

Brush (1981:37) uses a comparison with English as a school subject to illustrate the higher rating of mathematics as an anxiety provoking subject. Her research comparing the anxiety related to mathematics and English revealed an interesting pattern which is shown clearly in the graph illustrated below.

Self ratings of amount of anxiety experienced by students in situations involving mathematics and English. The range of possible values for each is 12 to 60, with a higher number representing more anxiety.



(Brush, 1981:38)

Brush (1981:38) explains that in sixth grade (standard four) the English tasks are fairly daunting. They involve spelling tests, grammar exercises, comprehension, etc. However, English becomes more a study of literature and there are more opportunities for students to express their own ideas in later years. There is also a de-emphasis of evaluation situations.

In contrast, mathematics is becoming more abstract, an increase in testing is often evident and pupils have little chance to express their own ideas. In addition, the usefulness of the mathematics at high school is often questioned whereas the usefulness of reading and writing skills is obvious.

These contrasts may be over simplified but they do provide a guide to some of the essential features affecting cognitive progress at school. If the specific causes of increased mathematics anxiety can be traced to elements of secondary school teaching then this is where the solutions to the problem must receive attention.

2.2.2.4 **Educational or school factors**

Many aspects discussed under the previous three headings may be termed "educational". However, for the purpose of this study educational factors will be those relating more specifically to the school. The school has an obligation to society and to the individual to present mathematics in such a way that each individual can develop an understanding of and an enjoyment of the subject. When considering the sources of mathematics anxiety, educational factors within the school is the obvious place to start one's investigation.

The interpretation of the curriculum, the attitude of the school and the proficiency of the teachers are all important factors influencing mathematics anxiety. In the past, too much emphasis was given to content and inadequate concern was shown for the nature of the student learning. The school must become an institution which is more student-orientated.

Discussions of classroom procedures and routines that can be used to prevent negative experiences and create more enjoyment with mathematics are essential. Learning how to alleviate the anxiety of their future pupils will help the student teacher eliminate the fear that he or she personally feels. Teaching to avoid mathematics anxiety should become a matter of concern for all those involved in education.

There are many factors involved in education at school level which can influence the teacher's approach to the teaching of mathematics. The school usually conforms to the normal hierarchical structures of control:

(a) the education department (regional or national);

(b) the headmaster;

(c) the head of the mathematics department within the school;

(d) the teacher.

Whilst the teacher has some autonomy in the classroom it is often ruled by the parameters set by the other three players. With teachers having an increasing say in curriculum design it is important that mathematics teachers provide the force behind reform in the classroom. Ultimately it is the teacher in the classroom who is seen as the main source of anxiety and hence the one best able to address the problem.

A number of researchers make reference to the importance of the mathematics teacher in providing the correct classroom atmosphere for their pupils to develop a healthy attitude towards mathematics. Kogelman & Warren (1978:18) relate a number of incidents where students recall some traumatic experience they have had with mathematics teachers. Johnson (1981:2) states that:

> *Research shows that the attitude of the teacher is a prime determinant of the attitude of the child.*

Research by Widmer & Chavez (1982:276) also supports this viewpoint and emphasises the need for a supportive mathematics teacher who is able to create a healthy learning environment in the classroom.

Eisenberg (1991:157) expresses concern for classroom teaching of mathematics and feels strongly that one of the major goals of school and university mathematics is to impart to students the self-confidence to be able to think.
He feels that teachers should make a conscious effort to introduce what he calls recreational problems. An example of one of his problems is:

> *A fly and a spider are in a room 12x12x30. The fly is on the middle of one of the 12x12 walls one foot from the floor. He is so scared that he does not move. The spider is in the middle of the other 12x12 wall, one foot from the ceiling. What is the minimum distance the spider must crawl to get to the fly?* (Eisenberg, 1991:157).

By building confidence in doing this type of problem Eisenberg (1991:154) claims that his students developed self-confidence in their mathematical ability.

Stodolsky (1985:13) observed a distinct pattern in the way mathematics is taught in the classroom. She was particularly concerned with the rigidity of lessons and she believed that this lack of variety may contribute to anxiety. She found that mathematics classes were characterised mainly by:

1. A reliance on recitation and setwork pattern of instruction.

2. A reliance on teacher presentation of new concepts or procedures. (Stodolsky, 1985:130)

Greenwood (1984:663) stated that:

> *The principal cause of mathematics anxiety lies in the teaching methodologies used to convey the basic mathematical skills to our youngsters.*

He further asserted that the explain-practice-memorize teaching paradigm is the real source of the mathematics anxiety syndrome. He believes that whilst accepting the importance of the content of a mathematics curriculum the emphasis has to be more on a process that takes cognisance of variables such as mathematics anxiety. A curriculum which considers the psychological problems of learning mathematics needs to be pupil-centred with an emphasis on instructional methods.

Stipek (1988:113) observed that:

> *Anxiety debilitates performance most when a task is introduced in a way in which poor performance could damage a student's ego.*

She stresses that it is essential that teachers address this problem by ensuring that tasks are not introduced as tests of ability and that attention is not orientated to the evaluative aspect of a task. Seemingly simple tasks, such as answering a question in class or being asked to complete a problem on the chalk board in front of the class, can be extremely anxiety provoking.

In her discussions with students Tobias (1987:5) noted that they often complain that there is little opportunity for debate or discussion in the mathematics classroom.

She claims that this is often a source of trauma for many young people. She goes on to say:

> *More often than not, however, mathematics is presented as a fixed set of rules to be digested whole and without dispute, which may discourage students from learning.*

It is clear that many researchers see the classroom and teacher as important aspects of a child's environment that impact on anxiety. Developing positive attitudes towards mathematics should therefore be a major goal of mathematics teachers. Pre-service primary and secondary school teachers must be adequately prepared in mathematics in order for a mathematics subject didactics course to be fully effective. Teachers need to be well versed in teaching methods which alleviate anxiety and this means a good knowledge of pupil-centred teaching styles. In addition, pre-service teachers must be provided with self-confidence themselves in dealing with mathematics and illustrating the usefulness and importance of mathematics.

The school and the classroom are an important part of the child's social environment. The teacher is the major influence on this environment and yet he or she is not trained to counsel students. However, what they can do is ensure that they provide the correct atmosphere in the classroom. Students should be comfortable with the interactive processes in the classroom and they should feel confident enough to participate in the lesson.

The emphasis in this study is focused on teaching methods specifically because this is of major concern to researchers. The school and especially the mathematics classroom is also the logical place to confront the problem. Parents will influence pupils in the socio-cultural environment and emotive and cognitive factors will contribute to anxiety. However, the teacher in the classroom can counter these factors and the school factors by creating the correct classroom environment and using teaching methods that counter those which cause anxiety.

Thus, the critical factors involved in mathematics anxiety have essential implications for mathematics teaching as expressed in Section 1.1.3. These factors are summarized later in Section 2.3 and will be fully discussed throughout this thesis (see 3.4.1 and 4.3).

2.2.3 Symptoms of mathematics anxiety

It is important for teachers to recognise firstly, that mathematics anxiety exists and secondly that it manifests itself in certain recognisable symptoms in the classroom. The following symptoms should be noted by the secondary school teacher:

1. Non-task orientated behaviour
2. Dependence on teacher
3. Non-social behaviour
4. Physical reactions

It is difficult to departmentalise these symptoms and often there is some overlap in the four categories that are presented here. However, in recognising these symptoms in classroom behaviour it will become obvious that mathematics anxiety is widespread and can be identified by these common reactions.

2.2.3.1 Non-task orientated behaviour

The mathematically anxious student may be able to perform basic facts or rote learning tasks but they have difficulty with problem solving. Sovchick (1989:116) describes this inability as students being unable to suspend judgement or reflect on solution paths. They have difficulty looking at alternatives or developing a plan.

These students are extremely cautious when faced with unique problems to solve or when creative thinking is necessary. They become distracted from the task by engaging in other activities such as selecting paper, cleaning desk, sharpening a pencil, drawing a diagram in geometry over and over again and generally doing things unrelated to tackling the given problem.

The difficulty stems from having attempted to memorise rules and routines that appeared to be useful in their earlier mathematics years. However, this rote learning gradually became more inadequate as the number of rules and routines that they would be required to remember became overbearing. Not having developed a problem solving attitude they now become anxious when faced with a problem that requires independent thought.

2.2.3.2 **Dependence on teacher**

Mathematics anxiety often reveals itself in the behaviour of a child who is more dependent on the teacher to describe solutions or how to start a problem. This problem is not solved by the teacher who moves around the classroom offering assistance to each student. Students should be left to work out their own solutions and persevere with their attempts to solve problems.

The dependent pupil is often looking to the teacher for confirmation and the teacher often perpetuates this by emphasising right answers without concern for the process involved. Sovchick (1989:119) says:

> *Students need much experience working with concrete materials, doing experiments and solving problems with more than one solution.*

He believes that by emphasising process rather than product the teacher will be providing a strategy which helps to alleviate mathematics anxiety.

2.2.3.3 **Non-social behaviour**

The mathematically anxious child often appears to daydream but is in fact using this as a defense mechanism. Working in isolation is not beneficial because of the tendency for these anxious students to become non-participants. It is important that they are included in group work as they need the experience of working with classmates on mathematics work.

They may have also developed certain social traits from the family situation. For example, an extreme fear of failure may stem from a home environment where high goals are set and the child equates parental love with success at school. The child may also come from a "broken home" where no support is available or a home where very little interest and no praise for good work is evident. These factors all impact on the child's sense of self-worth and will often manifest themselves in non-social behaviour at school. They avoid contact with others and with the teacher in fear of revealing the source of their problems. Their anxiety becomes more deep-rooted because they do not have the confidence to communicate and socially interact with their peers when doing mathematics work.

2.2.3.4 **Physical reactions**

Physiological symptoms such as headaches, eye tics and stomach aches may all be evident in the mathematically anxious child. However, the anxiety may also be displayed by a child becoming rather angry when faced with a problem he cannot do.

Compulsive behaviour during tests may be revealed by non-task orientated behaviour such as arranging of coloured pencils, placing a lucky mascot on the desk, repeated checking of the time on a personal clock or asking the teacher for non-task related assistance such as "What date is it today?" or "Do we need to underline our answers?".

Teachers need to involve students in more task orientated projects which involve solving problems, working with manipulative materials and making estimations (Sovchick, 1989:117). In addition, the teacher needs to display a calm approach to the task at hand so as to display a lack of anxiety and a need to still feelings of anger.

The four categories describing symptoms of mathematics anxiety all lead to implications in the teaching process. It is all very well to believe that mathematics anxiety exists and that it manifests itself in certain symptomatic behaviour in the classroom but what should be done about it should be the question foremost on the minds of curriculum developers.

Classroom strategies and teaching methods must become a central consideration in any study of mathematics anxiety and therefore have further implications for this study.

2.3 **Implications for this study**

In this chapter it has been shown that mathematics anxiety is widely recognised as a very real factor and an important variable of mathematics teaching. Studies in mathematics anxiety have a long history and cover a comprehensive range of issues involving mathematics performance and the teaching of mathematics.

The remediation of the problem of mathematics anxiety has also received attention and institutions have been established to combat the problem and mathematics educators have acknowledged the need to pay attention to the problem when teaching mathematics. The volume of research and overwhelming interest in mathematics anxiety as a developmental problem, leaves one in little doubt that this problem is widespread and continuing.

The answer to the alleviation of mathematics anxiety can be formulated from the ideas on research into the aetiology and the symptoms as they appear in the classroom situation. Whilst socio-cultural, emotive and cognitive sources of mathematics anxiety can be traced to areas outside the school domain, they all manifest themselves in the classroom and thus become the concern of the teacher. It is the manner in which the subject is presented, in particular classroom environments, by particular teachers and to certain types of children that establishes the arena in which mathematics anxiety manifests itself.

The purpose of this study is to highlight the problem of mathematics anxiety to a level where curriculum developers would agree that there is a need to draw the attention of mathematics teachers to the problem. Curriculum developers should also agree that teaching method is the critical component of curriculum design. Whilst not neglecting its interrelationship with the other components, teaching methodology must be directed to influence teachers to adopt a didactic position which will help to alleviate mathematics anxiety.

Sections 2.2.2.4 and 2.2.3 emphasised aspects of the classroom situation and the teaching methodologies which affect mathematics anxiety.A summary of these concerns for teaching methods may be listed as follows:

1. Providing the correct classroom environment and atmosphere (Widmer & Chavez, 1982:276).

2. Ensuring a true understanding of mathematics and not a rote learning of rules and algorithms (Greenwood, 1984:663).

3. Developing each individual's self-confidence in the mathematics classroom and in doing mathematics work (Eisenberg, 1991:157).

4. Providing ample opportunities for pupils to communicate their ideas and possible solutions (Tobias, 1987:5).

5. Allowing for social interaction in the classroom by encouraging debate and discussion (Johnson, 1981:2).

6. Providing opportunities for problem solving which allow for reflection on solution paths and developing an attitude that is task orientated (Sovchick, 1989:116).

By addressing these issues in the classroom the other sources of anxiety will receive attention. The empathic attitude of the teacher and a classroom situation that allows for debate and discussion will provide the student with the opportunity to question connotations and myths surrounding mathematics (see 2.2.2.2). In addition, the student will understand why mathematics progress requires a certain cognitive style (see 2.2.2.3) and how parents and others are influencing attitudes towards mathematics (see 2.2.2.1). The teacher is encouraged to discuss these topics as part of the social interaction in the classroom.

The six aspects of teaching methods listed above are fully explained in Section 3.4.1 as critical variables of mathematics teaching. Bell (1978:9) stresses the fact that whilst it is important to understand the theories of learning presented in curriculum studies, it is the ability to apply these theories in the teaching of mathematics that is a prerequisite for effective mathematics teaching.

2.4 **Summary and synthesis**

The aim of this chapter was firstly to identify the variable mathematics anxiety and then to establish why it is an important variable in the teaching and learning of mathematics. This was expressed in terms of the fact that many researchers believe it is widespread (see 2.1.3.1) and that it perpetuates itself (see 2.1.3.4) by parents and teachers developing mathematics anxiety at an early age and then passing it on to the next generation.

The importance of mathematics anxiety as a variable is also emphasised by the research findings on performance and avoidance. Hembree's (1990:42) work of integrating one hundred and fifty one studies on mathematics anxiety proved invaluable and is strong evidence that mathematics performance is affected by mathematics anxiety (see 2.1.3.2). The fact that mathematics anxiety is viewed as a cause or effect of mathematics avoidance emphasises the intricate concerns of researchers such as Sovchick (1989:118) and Tobias (1976:56) who discuss teacher avoidance and pupil avoidance of mathematics (see 2.1.3.3).

The evidence of research in the literature on mathematics anxiety provides a convincing argument for emphasising it as an important variable in mathematics teaching. The fact that mathematics anxiety affects performance is a factor which alone should motivate curriculum designers to take cognisance of the problem.

The literature research on the measurement of mathematics anxiety provides an historic view of the development of certain instruments and specifically culminates in a report on the Mathematics Anxiety Rating Scale (MARS) developed by Richardson & Suinn (1972:551) and described in Section 2.2.1.4.

A modification of the MARS instrument was developed by Richardson & Woolfolk (Sarason 1980:274), and their recommendations were discussed. The objective of this was to provide background to the development of the measuring instruments to be used in the empirical research in Chapter Five. This measuring instrument is also evidence of the widespread interest in mathematics anxiety as it is widely used in research on mathematics anxiety.

The sources of mathematics anxiety have been discussed with reference to the literature research and categorised in terms of socio-cultural, emotive, cognitive and educational factors. Whilst providing details of aetiology in several areas the emphasis was placed on educational factors and in particular the classroom situation and teaching methodology. For this reason the symptoms of mathematics anxiety and how it manifests itself in the classroom or teaching situation are examined (see 2.2.3). Factors under the headings non-task orientated behaviour, dependence on teacher, non-social behaviour and physical reactions give an indication of what the teacher is confronted with in the classroom.

This chapter provides the impetus for the remainder of this study in that mathematics anxiety has been established as an important variable in the teaching of mathematics. It is widespread and continuing and manifests itself in some recognisable behaviour in the classroom. Clearly the alleviation of such a problem requires a comprehensive strategy to be developed within the framework of curriculum design.

Chapter Three investigates the possible curriculum designs that may provide the required strategies and in particular the mathematics teaching methods as part of mathematics curriculum design. This investigation culminates in a further synthesis of the concerns of mathematics teaching and mathematics anxiety which were established in Section 1.1.3 and expressed again in Section 2.3. These concerns involve the classroom environment, understanding, self-confidence, communication, social interaction and problem solving (see 3.4.1).

The concerns of researchers involved in mathematics anxiety are similar to the tenets of the constructivist approach. Hence the six factors referred to as concerns of mathematics teaching also become concerns of the constructivist approach. These elements will be viewed from the constructivist perspective in Chapter Four (see 4.3).

The empirical research in Chapter Five provides support for the arguments put forward in this chapter on mathematics anxiety and in Chapter Four on constructivism. A correlation of ideas on mathematics anxiety and constructivism with the support of the empirical evidence will provide a synthesis of recommendations for the teaching of mathematics.

CHAPTER THREE

MATHEMATICS TEACHING AND CURRICULUM DESIGN IN MATHEMATICS

3.1 Introduction

In Chapter Two it was established that mathematics anxiety is widely recognised as a real problem in the mathematics development of the child. It follows, therefore, that curriculum planning must take cognisance of the needs of the mathematically anxious child.

It is not the intention of this study to become immersed in theories and arguments on curriculum but rather to give emphasis to mathematics teaching and classroom techniques as a critical aspect of the alleviation of mathematics anxiety.

Learning strategies and teaching strategies all form part of the modern mathematics curriculum and the views of both learners and educators are important when new curriculums are designed. In the overview of mathematics anxiety presented in Section 2.3 it was emphasised that classroom techniques and teaching style are critical factors in the alleviation of mathematics anxiety.

Conceptions of the curriculum depend on a particular orientation. McNeil (1985:1) categorises these orientations as humanistic, social reconstructionist, technological and academic. In essence the approach adopted in this thesis could be categorised as humanistic and certainly the emphasis is on the individual's personal growth. However, it is difficult not to find value in the other categories. Indeed, the expectations of a humanistic approach should include some aspects related to the other three orientations.

By developing an individual's personal competence one would be influenced by societal and technological needs. The need for independent thinkers, a sense of creativity and a problem solving attitude is not in conflict with the needs of a modern technological society in which we now live. In addition, whilst the methodology in the classroom may differ if one adopts a humanistic approach, it does not necessarily de-emphasise the academic interest that each individual may develop towards mathematics. Therefore it is the contention of this thesis that it is not necessary to categorise a curriculum orientation as suggested by McNeil but to rather use elements of various orientations to formulate a suitable approach.

In this study a constructivist approach is proposed because it constitutes such a formulation of different elements from a number of curriculum orientations. Gordon (1994:131) emphasises this fact when he states that:

> *Different psychological and philosophical paradigms underpin theoretical formulations of constructivism and the resulting differences in pedagogics which claim to be based on constructivist paradigms.*

Gordon (1994:131) goes on to say that this view of constructivism has led to it being criticised as little more than an umbrella construct covering a range of practices. This is hardly a criticism if the constructs embraced are based on sound didactic principles.

In Chapter Three and in greater detail in Chapter Four it will become evident that a constructivist approach is essentially a problem-centred and pupil-centred approach to curriculum and thus very much a humanistic orientation. However, the teaching and learning of mathematics in the secondary school is the focus of this study and therefore a broader view of the concept of curriculum and curriculum development will provide a basis for a teaching and learning philosophy and methodology which provides a solution to the problem of mathematics anxiety.

This chapter will provide some background and historical perspective to curriculum design before expounding fully the concept of a mathematics curriculum that addresses the problem of mathematics anxiety and adopts a constructivist approach to the teaching of mathematics.

3.2 <u>A general perspective on curriculum design</u>

It is difficult to categorise the many theories of curriculum design and even more difficult to define the concept. Historically it appears that curriculum design has developed in two important directions. Firstly, it has taken on a broader portfolio as opposed to the old narrow view of planned content and outcome. Secondly, it has developed away from a content orientated approach towards a more pupil-centred and process approach.

These two developments require careful and comprehensive planning in the construction of a new curriculum. The more comprehensive view ensures that all aspects pertaining to the development of the child are considered and that a careful analysis of the situation is undertaken. The more humanistic approach ensures that the rights and needs of the individual are considered when constructing a curriculum. In both instances the process of teaching and learning is an essential aspect of the curriculum.

It is not the task of this study to analyse the merits and demerits of various curriculum theories. However, a synopsis of important curriculum trends will serve to provide a basis for initiating a curriculum design which incorporates the didactic principles of constructivism.

3.2.1 <u>A traditional view</u>

Sears & Marshall (1990:7) Describe the traditional approach as:

> *The most dominant, persistent and generally accepted theoretical approach to curriculum.*

They go on to explain that this approach has been called by a number of terms but the curricular outcomes and the ways to develop curriculum to achieve these outcomes are similar. Sears & Marshall (1990:7) list the terms that are used together with the proponents of this traditional approach: traditionalists (Pinar), structuralists (Huenecke) and behaviourists (Klein).

A traditional view of curriculum design is one which emphasises the role of organised subject matter. The skills and body of knowledge each student should acquire are predetermined and the outcomes desired deal primarily with the development of their intellectual capacities. With the emphasis on content the affective domain of the learner is often neglected even though it might be mentioned. The traditional curriculum is carefully planned and organised prior to classroom engagement. Goals and objectives are determined, content is selected and organised and text books are often prescribed (Sears & Marshall, 1990:7).

The teacher is seen as a conveyer of the curriculum and they are trained to be authorities in what students are expected to learn. There is little variety in the work and the pupil is detached rather than involved in the engaging of higher level thinking (Sears & Marshall, 1990:7).

The organisation of subject matter provides direction for the teacher and a systematic and organised approach for the learner. The student is able to build a store of knowledge most efficiently and economically. The teacher is the source of the knowledge and the desired methods and these will be conveyed to the student as a series of rules to be memorized for future reference (Lerman, 1990:56).

Moodley *et al* (1992:2) describe the traditional approach as a teaching of the disciplines which means that:

> *Pupils knowledge has to be built up in accordance with structure and contents independently of the learners.*

Teachers are therefore prepared to be authorities in what students are expected to learn and the teacher's task is to impart their knowledge with competent, professional and didactic skill.

According to Zais (1976:400) the traditionalist point of view ignores any human psychological processes and gives little regard to the future use of the knowledge in any real-life situations. Stimulation, maturation and motivation are not regarded as factors which have a bearing on intellectual development. Building a store of knowledge is seen as the ultimate goal and the individual's learning processes and ability to assimilate and apply knowledge is not taken into account (Zais, 1976:400).

The teacher's role is restricted to the classroom and the teacher is not expected to become a more extended professional. Input into curriculum planning is discouraged and discussions with subject and didactic experts is not viewed as an essential part of the teacher's role (Parsons, 1987:36).

The fact that a traditional view is a narrow view of curriculum with little regard to psychological processes and further use of the knowledge, mitigates against its value in this study. However, a traditional approach to teaching does not necessarily consist of an authoritarian teacher issuing a stream of instructions and rules to an apprehensive class who are trained to repeat what they have learnt by memory and without understanding. The teacher may be caring and interested in each child's development yet not adopt a humanistic or child-centred approach.

A number of teachers may view themselves as non-authoritarian and emphasise a democratic teaching style which favours learning through discovery and problem solving. However, in the practical situation of the classroom they may resort to what would be termed "traditional methods" due to a pressure of time or a need for a more systematic approach to the acquisition of the knowledge needed at a particular point in the pupil's development.

3.2.2 **A comprehensive view**

Curriculum theorists later adopted a more comprehensive view of the curriculum. The idea of prescribing content and outcomes became intertwined comprehensively with aspects of teaching and learning.

Niebuhr (1986:99) provides a comprehensive view of the curriculum in the following definition:

> *A curriculum is a scientific written programme for teaching and learning in which are included the aims and corresponding selected and organised subject matter together with didactic guidelines for the creation of learning opportunities and for evaluation.*

This definition embodies the four components that Zais (1976:438) listed as essential to a comprehensive view of curriculum. These four components are:

1. Aims, goals and objectives
2. Content

3. Learning activities

4. Evaluation

He stresses the importance of recognising the inter-relationship of these components and that decisions about any one will have a bearing on decisions about the others.

With all the advances in society and technology it is naive to assume that merely describing content is sufficient curricular planning. A comprehensive approach ensures that not only all the components of the learning process are considered but that goals are continuously re-assessed to ensure that the pupil is being suitably prepared for the world in which he lives (Kelly, 1989:16).

Whilst most educationists agree with this comprehensive approach to curriculum planning, it is often the differential emphasis placed on the ingredients of the curriculum that causes most argument. Whilst the aims, goals and objectives may be similar, the emphasis on content, learning activities and evaluation may differ drastically. These components of the curriculum are analysed in terms of a systems approach in Section 3.3.1.

3.2.3 A systems approach

The systems approach described by Kelly (1989:15) adopts a comprehensive view of curriculum planning in that it is seen as consisting of four components similar to those proposed by Niebuhr (1986:99).

> Objectives - what are we hoping to achieve?
> Content or subject matter - what will we cover to achieve it?
> Methods or procedures - what activity and methods will be most effective?
> Evaluation - what devices should be used to measure achievement?
> (Kelly, 1989:15)

Kelly (1989:15) suggests that as a linear model this approach is too simplistic. However, as a cyclical model where each component is interrelated with the other three and the interaction of the four components leads to continuous review of the situation, the systems approach becomes a dynamic model for curriculum design. Golby *et al* (1983:358) describe proponents of the linear model as "hardliners" who believe in a "feedforward" role of the systems approach. The proponents of a cyclical and successive adjustment to the systems approach are entitled "softliners" by Golby *et al*.

According to Golby *et al* (1983:358) the "hardliners" view of the systems approach is simple in that it adopts a more traditional view of education. Goals are set to provide certain prescribed behavioural objectives through the use of organised subject matter presented by the teacher.

Alternatively the "softliners" are viewed by Golby *et al* (1983:359) as not focusing specifically on required behaviouristic outcomes. The "softliners" adopt a stance that requires continued analysis of the situation and hence a comprehensive assessment of Kelly's components of the curriculum.

Kelly (1989:16) believes that we must acknowledge the dynamics of curriculum planning and the fact that the four components are constantly being modified. However, whilst objectives are thus continually reviewed, this model is essentially one in which the criteria by which objectives are selected is a prior consideration of curriculum planning.

It is difficult to find an argument against Kelly's point of view of a cyclical systems approach. The idea that the curriculum consists of a system of components which constantly interact to provide a frame of reference for good and sound teaching is a necessary paradigm for teachers. This more comprehensive view emphasises the various components so that teachers do not only concentrate on teaching methodology but also consider the other components which interact with teaching methods. The questions of what teachers are trying to achieve, what they will cover and how they measure achievement must also form part of a teacher's frame of reference if teaching is to benefit.

In contrast, the traditional approach does not emphasise this comprehensive view of curriculum and does not provide for teacher development by expecting the teacher to be more widely involved in, and knowledgeable about, the various components of the curriculum.

A narrow view of curriculum leads to a traditional approach to teaching which is essentially orientated towards training pupils to be competent performers. A systems approach requires significant insight from educators and this provides an informed basis in which a number of components will impact on their teaching as they are made aware of variables which are important to curriculum design.

3.2.4 **A process approach**

Whilst some models of curriculum planning have placed the emphasis on content, a more popular emphasis in recent years has been on procedures (Kelly, 1989:16). The main concern of a process approach is on methods of learning rather than the content of what is learnt.

An interesting analogy is made by Goodsen & Dowbiggin (Taylor, 1993:145) when they state that:

> *The form of subject knowledge has grown increasingly irrelevant to the experience of learning, just as psychiatric knowledge in the 19th century grew increasingly irrelevant to effective therapy.*

The point that Goodsen & Dowbiggin make is that with all the content knowledge one may acquire, it is of little use if one cannot apply the knowledge to any given practical situation. Thus, the rote learning of subject content does not prepare the pupil for the applications he or she would be faced with in real life.

A traditional emphasis on content can invariably lead to rote learning, whilst the methods of acquiring knowledge and the utilization and communication of the content will lead to greater understanding and an ability to apply the knowledge in a variety of contexts.

Zais (1976:327) points out that it is difficult to separate content and process as they are mutually inclusive. However, it is often convenient to make a distinction for the purpose of analysis. Whilst one accepts this interrelationship between content and process, for the purpose of this study it is deemed not only convenient but necessary to make a distinction and to emphasise the positive contribution of the process approach.

3.2.5 **Synthesis of curriculum design**

The important components of a systems approach to curriculum design have been expressed in terms of a traditional view versus a comprehensive view and a process approach as opposed to a content approach. These issues have been stressed because of the relevance that they have to this study.

In evaluating the merits of a comprehensive approach it is essential to recognise that its major strength is the involvement of teachers in the understanding and development of a curriculum. The role of the teacher in the classroom is a reflection of their view of curriculum ideas and predicates on their understanding of the teacher-student relationship. If the teacher does not recognise the value of all the components of the curriculum then the teacher-student relationships formed will consist of a traditional, rigid, behaviouristic outlook. This view normally accepts cognitive development as a simple matter of continued association and memorization.

The danger of a comprehensive approach is that it could be fairly unstructured with an unbounded philosophy of romanticism (Sears & Marshall, 1990:41). The systems approach offers a more structured framework for the comprehensive view by emphasising components of the curriculum that will impact upon teaching. In this way important variables are included in curriculum planning and teaching can draw on an integration of ideas and even conventional approaches but at the same time keep within the boundaries of a curriculum plan that is potentially productive.

The process approach is also a reaction to traditionalism in that it discards the outdated behaviouristic views of teacher-student relationships. The idea that the teacher is the "fountain of knowledge" and the students are "empty vessels" to be filled, is rejected. The process approach requires the teacher to form a more trusting relationship with the student. The teacher as a facilitator in the classroom develops a mutual interaction with the student which helps the student to reflect on solutions and negotiate new methods. A process orientation encourages the teacher to develop a comprehensive approach to the understanding of teaching and learning and not to be bound by an approach that emphasises content and the transmission of this content by the most convenient method. A process approach necessitates the teacher continuously reviewing the teaching and learning situation and incorporating aspects that facilitate long term understanding and provide opportunities in the classroom for students to question and investigate.

A comprehensive view of curriculum demands an overview of all the major components of curriculum and an understanding of the interrelationship between aims and objectives, content, teaching methodology and evaluation. A cyclical approach to the elements of this curriculum planning model need not be in conflict with the process approach but would rather place emphasis on teaching methodology whilst recognising that it cannot be isolated from its relationship with the other three components of curriculum planning.

In the following section a number of specific curriculum perspectives which have influenced mathematics curriculum design will be linked to the general perspectives outlined in this section.

3.3 Perspective on the mathematics curriculum

In Section 3.2 the two central arguments of curriculum design have been discussed. These are a traditional view versus a more comprehensive view and the view that content or process should be stressed. It is necessary to view these aspects in the context of the mathematics curriculum and to explore further curriculum concerns that may influence mathematics curriculum design. For example in the U.S.A. the comprehensive approach is evident in the Curriculum and Evaluation Standards for

School Mathematics (Commission on Standards for School Mathematics, 1989). These standards were developed by mathematics educators for mathematics educators and the word standard is used to judge both curriculum and evaluation on their value. The curriculum is defined as follows:

> *A curriculum is an operational plan for instruction that details what mathematics students need to know, how students are to achieve the identified curricular goals, what teachers are to do to help students develop their mathematical knowledge, and the context in which learning and teaching occur* (Commission for Standards for School Mathematics, 1989:1).

Essentially, as far as the mathematics curriculum is concerned, the arguments on content and process arise out of views held about the nature of mathematics and how children learn mathematics (Moodley *et al*, 1992:4).

A formalistic or traditional view of mathematics is governed by a belief that mathematics is a body of knowledge consisting of a set of concepts and rules. It follows that this view would expect the teaching of mathematics to provide a sound acquisition of algorithms and routine mathematical skills to solve problems.

In contrast, a process approach emphasises the learning of mathematics as a process that develops the concepts of generalisations and relationships and provides the student with the ability to enquire, investigate and solve problems through utilising their own thought processes. The teacher facilitates this learning by providing opportunities in the classroom for students to question, challenge, investigate and discover for themselves.

Traditionally mathematics curriculum design has not associated itself with process theories. The emphasis of teacher activities and the nature of the teacher's role is not a model which was generally associated with mathematics curriculum planning. However, mathematics educators are becoming more aware of the importance of the role of the teacher, their didactic approach and how pupils learn mathematics (Cockcroft, 1982:71; Commission on Standards for School Mathematics, 1989:4; Department of Education, 1995).

A shift to a more process-orientated approach and an emphasis on teaching methods is significant for the mathematically anxious child because it highlights the need to understand how each individual thinks about mathematics and how each individual comes to know mathematics. This requires better communication between teacher and pupil and a greater empathy towards the pupil by the teacher.

Mathematics has a unique situation in curriculum design because it is influenced by philosophy, psychology and didactics (Moodley *et al*, 1992:2). For this reason, one's views on the nature of mathematics as well as the psychological aspects which influence mathematics learning will be necessary considerations for the didactic approach and the overall curriculum design.

3.3.1 **General perspectives**

Inevitably each person involved with curriculum development will be influenced by a particular philosophy and will consequently endeavour to convince others that this philosophy is the best suited for the situation. The concern of this study is the development of a mathematics curriculum which addresses the affective domain and in particular the alleviation of mathematics anxiety.

It is the ultimate intention of this study to propose that a constructivist approach serves the best interests of curriculum developers who are concerned about the affective domain of the students. Before examining the constructivist approach in Chapter Four it is necessary at this stage to consider the procedures involved in the development of a mathematics curriculum.

The traditional view of the mathematics curriculum would not be an adequate approach in today's world. There are many variables influencing mathematics teaching and learning and curriculum designers must, of necessity, take a more comprehensive view of the curriculum. Within this comprehensive view a different emphasis will be given to the various concepts.

Harley (1983:4) proposes that a comprehensive view of the mathematics curriculum would involve the following components:

situation analysis,
aims and objectives,
choice of subject matter,
teaching methods (learning experiences and learning opportunities), and
evaluation. (Harley, 1983:4).

Whilst accepting the various components of a comprehensive view of the curriculum, this study is essentially focused on the teaching methodology of mathematics. However, as each component is interdependent it is important that teaching methods are not totally divorced from the other aspects of the curriculum and that all the components provide a system which ultimately forms the paradigm for mathematics teaching.

These components of the mathematics curriculum will be discussed briefly at this stage and will necessarily form part of the discussion when concerns of mathematics anxiety become the focus of curriculum in Section 3.3.3.

3.3.1.1 **Situation analysis**

Situation analysis developed from a needs assessment and a needs analysis process of curriculum planning. Thus the needs of the major players and the socio-cultural basis of the education environment became the focal point of analysis.

Different researchers may emphasise different areas that need to be analysed but in general lists of factors which are common to situation analysis theory include the following:

Social and cultural values and expectations.

Physical amenities, resources and finances.

Course content and subject structure.

Internal factors and psychological aspects.

Forms of knowledge, teaching and learning processes.

(Golby *et al*, 1983:2)

Situation analysis is not regarded as a component of the curriculum by all curriculum development theorists (Harley, 1983:8). The nature of this study supports the need for situation analysis and specifically those elements of situation analysis which give consideration to the psychological aspects of learning and teaching mathematics.

According to Harley (1983:8) situation analysis includes a wide range of aspects which must be considered and which set the parameters for curriculum design. His list of the elements of situation analysis covers those issues expressed by Golby *et al*:

> Philosophy of life - educational ideals
> Nature and structure of the subject
> History of the subject
> Ability and training of teachers
> Psychological aspects
> Logistic consideration
> Developments elsewhere in the world
> Demands of society
> (Harley, 1983:8)

This list is typical of situational analysis but does not define what aspects will be analysed under each heading. In addition, it does not necessarily cover all situations that may require analysis. A more comprehensive view would provide certain requirements under the heading "Logistic Consideration". For instance economic situation, physical amenities, etc. Additional situations may also be included as curriculum designers deem necessary for the mathematics curriculum.

With the exception of "Psychological aspects", Harley's elements of situation analysis do not focus on the learner as an individual but tend to emphasise external factors such as world trends, educational ideals, physical amenities and societal demands. Whilst these aspects are important for mathematics curriculum development they do not give enough attention to the social, affective and cognitive domain of the learner.

This study emphasises the humanistic and child-centred elements of curriculum development. Thus, the impact of socio-cultural factors and affective variables on the cognitive processes is of central concern (see 3.3.2.6). The affective variables such as fear and anxiety may be included in the analysis of psychological aspects. However, socio-cultural, affective and cognitive factors are closely interwoven because affective learning is concerned with the personal and social development of each individual (see 3.3.2.4).

Clearly the socio-cultural environment of the learner as well as the learner's personal characteristics need to be included in situation analysis (see 3.3.3.1). The home environment has a profound impact on the learner's personal and social development (see 2.2.2.1). In addition, the home and the school provide the environment in which the learner's characteristics are moulded. The potential cognitive ability of the individual is influenced by his or her perceptions and experiences. Within this domain the needs and aspirations of the learner are nurtured to develop mathematical power (see 1.4.4) and, in particular, a confidence when doing mathematics.

Each element of situation analysis should be of equal interest in curriculum planning. It is assumed, however, that those providing the input may have a particular bias in their research and/or point of view and this should be considered positively if curriculum planning involves a broad spectrum of interested participants.

3.3.1.2 **Aims and objectives**

The components of situation analysis provide the thought for reflecting on aims and objectives. However, Harley's (1983:1) exposition of curriculum planning is cyclical and each component of the curriculum is interdependent on the others (see 6.3). In other words, aims and objectives need to be re-evaluated as other components of the curriculum are developed.

Aims and objectives are often referred to merely as "goals" (Dreckmeyr, 1989:80) and these goals are a directive for the activities taking place and provide a focus for the entire educational program. In the mathematics curriculum these goals would be based on a particular philosophy of the nature of mathematics and hence the view of what constitutes the required outcome of the mathematics student.

Mathematics teaching is the function of a systems approach to curriculum design and therefore goals would be centred around sound teaching practice and on the development of the student.

The aim of the mathematics curriculum in this study could be simply stated as providing each individual with intellectual autonomy and a self-confidence in doing mathematics. The achievement of this aim would require a focus on certain objectives for teachers in the classroom. In summary, the stated objectives of the mathematics curriculum should cover the following critical aspects:

1. To provide students with the ability to communicate mathematically.
2. To guide the student into becoming a mathematical thinker.
3. To enable the student to become confident in his or her own ability.
4. To provide students with the opportunities to become problem solvers.

These objectives provide a broad guideline for teaching methodology and are not as specific as the aims expressed in the Department of Education (1995) core syllabus for South African secondary schools. This document includes five "societal aims", eight "general teaching and learning aims" and twelve "specific aims of mathematics education". For the purpose of this thesis the four objectives described above provide an adequate background for mathematics teaching (see 6.4.1) and generally cover the concerns of the Department of Education (1995) core syllabus.

3.3.1.3 **Choice of subject matter**

In South Africa, and indeed the literature from other parts of the world indicates that the common problem of the mathematics curriculum is the emphasis on content. Howson & Kahane (1986:10) make special mention of developing countries where mathematics curriculum remains essentially unchanged from that which was planned for a small academic elite in earlier years. What is in the syllabus has become mathematics and any other extensions are considered unnecessary and a waste of time by teachers and learners. It is difficult to blame teachers for this approach when an overfull syllabus is presented to them at the beginning of each year.

It was emphasised in Section 3.2.4 that a move towards a process approach has been noted in recent years (Kelly, 1989:16). For this reason the traditional emphasis on content in mathematics would be questioned. It is inevitable that a content based curriculum will lead to a traditional style of teaching and an emphasis on rote learning methods. The process approach is favoured in this study and the mathematics curriculum would therefore take cognisance of a move away from content emphasis and give particular attention to process.

The International Commission on Mathematics Instruction (ICMI) recognised the fact that content would need to be sacrificed if the level of students' understanding was the main concern and if the growth of other types of knowledge than rote learning was to be fostered (Howson & Kahane, 1986:55).

The recommendation of the ICMI group is that any new mathematics curriculum be viewed more in the form of restructuring and defining aims related to students' learning patterns, rather than introducing new content. However, they do recognise the fact that this restructuring will meet with some resistance from curriculum planners and teachers (Howson & Kahane, 1986:55).

The new Interim Core Syllabus for South African Schools (Department of Education, 1995) has not made any provision for a more concerted effort on the part of teachers to pay attention to application, understanding and exploration. No thought is given to a reduction in content as proposed in Section 3.3.3.3 or of addressing the problem of assessment (see 3.3.3.5) of a pupil's mathematical progress. Whilst researchers remain mathematically idealistic in the amount of mathematics which must be presented, little time is available for the teacher to be creative and innovative.

Content of the mathematics syllabus must provide the necessary material to advance the intellect of each pupil. However, the way the pupil interacts with the content will determine the level of understanding he or she reaches. Curriculum design needs to include in its operational plan for instruction what teachers need to do to help students develop their mathematical knowledge.

3.3.1.4 **Teaching methods**

The component of teaching methods in the mathematics curriculum is of central concern in this study. A number of critical aspects pertaining to teaching methods were identified in Section 1.1.3 and will be a recurring theme in this thesis (see 2.3, 3.4 and 4.3).

Rakgokong (NECC Mathematics Commission, 1993) describes teaching in South Africa as:

> *Sit down, don't talk and above all don't think for yourselves.*

Simpson (NECC Mathematics Commission, 1993) asks the question:

> *Why is it that a school mathematics teacher spends much of his/her time trying to convince pupils that the mathematics which they study is relevant to their everyday life?*

The International Commission on Mathematical Instruction (ICMI) met in Kuwait in 1986 and the subsequent document entitled "School mathematics for the 1990s" emphasises the need to consider both the teaching and learning of mathematics.

> *Instruction or teaching will always remain a key issue for us, but as is emphasised in this volume, learning demands equal consideration.*
> (Howson & Kahane, 1986 : Foreword)

All the above comments indicate a support for the emphasis on the "teaching methods" component of the curriculum. As stipulated in Section 3.2.4 a process orientation to teaching methods is proposed in this thesis whilst taking account of the humanistic and child-centred orientations. In adopting this stance it is important that content is de-emphasized (see 3.3.1.3) and the traditional views on the teaching of mathematics are opposed. Thus the following orientations should guide curriculum planners when considering teaching methods.

(a) A process orientation

Curriculum designers in South Africa have recognised the need to pay attention to the processes involved in teaching and learning mathematics. In the syllabus publications of the Department of Education and Culture (1991:2) one of the stated principles is:

> *The mathematics curriculum should make provision for the mathematical knowledge, skills and concepts which the pupils need to acquire, and also for the mathematical processes by which they are actively and productively involved in the learning ...*

Burkhardt (1981:1) believes that skills learned in secondary school mathematics have little use for most pupils. He believes that the few applications used in the mathematics classroom are *highly stylised, artificial situations* of little concern to students and certainly no help in the effective use of mathematics.

(b) A humanistic and child-centred orientation

The new Interim Core Syllabus for secondary school mathematics in 1966 (Department of Education, 1995) is the first mathematics syllabus in South Africa which applies to all South Africans. It is obvious from the stated aims of this Interim Syllabus that fostering teaching and learning aims has been given considerable attention. This document's message is perhaps best expressed by the first stated general teaching and learning aim:

> *To develop independent, confident and self-critical citizens.*

In a document entitled "A Curriculum Model for Education in South Africa" (Committee Heads of Education Department, 1991:13), the first stated aim of pre-tertiary education is the development by learners of:

> *A positive and realistic self-image, self-confidence, a sense of responsibility and inner discipline.*

c) <u>Opposing the traditional view</u>

The traditional approach to teaching mathematics still has a strong following. A typical lesson will still consist mainly of the teacher explaining a new technique on the chalk board and then setting the class the task of doing similar examples in their books. The lack of recognition by curriculum designers (and in South Africa this includes practising teachers) of the fact that the content of the present mathematics syllabus is overfull, is leading to inadequate teaching methods which in turn generates failure, despair, lack of confidence and anxiety when faced with applying the knowledge gained.

Teachers are often entrenched in using traditional methods of "chalk and talk" presentations, with questions and answers related to assigned materials. Repetition of skills for mastery and a strict adherence to text book examples is the usual procedure in a mathematics lesson. Bell (1978:9) says that understanding content is not sufficient and that:

> *An outstanding teacher will know, understand, and apply various theories about how people learn mathematics in his or her teaching.*

Bell made this comment some time ago but it is still relevant for teachers today. Teacher training courses and in-service courses must provide the necessary background for teachers to understand the new approaches to the teaching of mathematics and why these approaches are considered important.

The teaching and learning of mathematics does not only involve what mathematics students must learn but rather how it should be taught and how students acquire mathematical knowledge.

Whilst accepting the various components of a comprehensive view of the curriculum, this study is essentially focused on the teaching methods of mathematics. However, as each component is interdependent it is important that teaching methods are not totally divorced from the other aspects of the curriculum and that all the components provide a system which ultimately forms the paradigm for mathematics teaching.

Hence, teaching methods need to receive particular attention in curriculum design and the critical aspects of teaching which were identified as classroom environment understanding, social interaction, communication, self-confidence and problem solving in Section 1.1.3 need to provide teachers with guidelines by which mathematics teaching will benefit. These critical aspects form a continuing theme in this thesis and will be discussed in detail in Sections 3.4, 4.3 and 6.4.3.

3.3.1.5 **Evaluation**

Evaluation is not of central concern to this thesis but as Harley (1983:42) points out it is closely linked to aims and objectives and interdependent with the other components of the mathematics curriculum. Teaching methods remains the emphasis in this study but as Du Toit (in Moodley *et al*, 1992:111) exclaims:

> *Much that is wrong in teaching has been ascribed to bad practices engendered by the examinations system.*

Writing an examination successfully was not one of the stated goals in Section 3.3.1.2 and formal mathematics testing will not reveal if all the goals have been achieved. Dreckmeyr (1989:89) says that:

> *Evaluation is directed to the assessment of the teaching and learning events themselves as well as of their outcomes in terms of particular goals.*

In his definition of evaluation Dreckmeyr distinguishes between formative evaluation and summative evaluation. The formative evaluation provides pupils with continuous feedback and provides a measure of their success in their studies and a motivation for these studies. The summative evaluation would focus on the outcome of the teaching course and would be directed towards achieving the goals set for the teaching programme (Dreckmeyr, 1989:89).

For the purpose of this study it is not possible to embark on long discussions on evaluation as the main focus is on teaching and learning. However, suffice it to say that evaluation is a component of the curriculum and it is an integral part of the teaching and learning process. As such it should not take place on its own but rather always be planned and used as a teaching aid (Dreckmeyr, 1989:89).

Du Toit (in Moodley *et al*, 1992:112) agrees with Dreckmeyr's description as he also views evaluation as a continuous process and not a separate procedure. He explains that the continuous process must be closely related to each aspect of the total programme of pupil development.

In this description of evaluation it is therefore imperative in this study that the evaluation procedures determine the success of the theory and methodology in the teaching of mathematics. This will be ascertained by measuring the success in the achievement of the goals stated in Section 3.3.1.2 and not by the marks obtained in any given formal mathematics test or examination.

3.3.1.6 **A synthesis of mathematics curriculum components**

This section has emphasised the interdependence and interrelationships of the various components of the mathematics curriculum. At the same time it is important to note that an extensive study of each curriculum component is too vast and is beyond the scope of this study. The essential component of the curriculum for this study is that of teaching methods (learning experiences and learning opportunities) and in particular how a variable such as mathematics anxiety and a constructivist approach can be integrated to formulate recommendations for the teaching of mathematics in the secondary school.

Thus, the attention of this study is more focused on the psychological aspects of situation analysis and the teaching methods implicit in the other categories of situation analysis. In addition, the "teaching methods" component of the curriculum has been emphasised and its interrelationship with the other components has been documented.

The components of the mathematics curriculum will be discussed further when mathematics anxiety concerns become the focus of the situation analysis in Section 3.3.3. At this stage it is important to first review some important trends in mathematics curriculum design.

3.3.2 Approaches to curriculum design in mathematics

A number of viewpoints historically or presently have influenced mathematics curriculum design. These views are mainly based on philosophical, psychological, social and didactic issues.

3.3.2.1 Cognitive psychology

The fundamental change in curriculum thinking has been a move away from a behaviouristic approach in psychology. Behaviourism lost its dominant role in the early 1960s and a "cognitive revolution" took place (Greer & Mulhern, 1989:19).

Cognitive science has to do with the psychology of the mind and involves the study of cognitive models or cognitive processes. Cognitive science researchers are seeking answers to questions such as:

> How do people think about mathematics?
> How do people come to understand mathematical concepts, processes and skills?

As mathematics educators are also interested in developing a better understanding of the nature of mathematics learning and instruction, it is easy to understand why an alliance would be formed between cognitive scientists and mathematic educators (Schoenfeld, 1987a:3).

From the cognitive standpoint, people construct their own personal mental representations which are made up of cognitional models for aspects of the real world. Psychologists have become particularly interested in analysing these constructs and mathematics has become the subject which is able to provide answers for cognitive psychology studies (Skemp, 1971; Greer & Mulhern, 1989; Schoenfeld, 1987a).

Greer & Mulhern (1989:21) view cognitive science as a study of how the mind processes information and point out that information processing theories are not restricted to mathematics but apply to thinking in general. However, information processing theorists find mathematics a subject which provides a natural domain for the application of their theories. Greer & Mulhern (1989:21) explain this alliance by pointing out that:

> *Mathematics, after all, often looks exactly like the processing of information symbolically represented.*

Whilst this alliance with psychologists and renewed interest in mathematics learning processes may be beneficial to the mathematics educator, it is also viewed with concern. Information-processing represents a much too narrow view of mathematical thinking and of mathematics in general. The mathematics educator needs to be also concerned with mathematics as a human activity in a socio-cultural context.

3.3.2.2 **A humanistic approach**

As mentioned in Section 3.3.2.1, from a mathematics educator's point of view the cognitive psychologists do not answer all the questions about teaching and learning. Their view is focused essentially on the information processing aspect of doing mathematics and neglects any human features such as affect, context, culture and history.

A humanistic approach to curriculum design represents a reaction by psychologists to a purely cognitive analysis of learning. It gives attention to important aspects of human behaviour such as interaction with other people and the environment. The person's history and culture are also deemed important aspects influencing learning and any special problems and issues facing each individual and affecting intellectual functioning are also given attention.

Gardner (1985:44) states that:

> *The kind of systematic, logical, rational view of human cognition that*

pervaded the early literatures of cognitive science does not adequately describe much of human thought and behaviour.

There is now a growing realization that human thinking does not conform to normative formal models. Emphasis for cognitive studies does become more humanistic as theorists give more consideration to individual thought processes and an analysis of underlying reasoning. Cognitive processes are not ignored by humanists but rather tempered with other important aspects of human behaviour.

Piaget's theory of cognitive development was purely an interest in the construction of knowledge within the child rather than the acquisition of knowledge by transmission. His theory was void of any humanistic considerations. However, Piaget did contribute to the way we think about childrens' acquisition of knowledge and he characterised the child as an active constructor of his or her own knowledge (Greer & Mulhern, 1989:289). The theories of cognitive processes such as Piaget's have led to a closer collaboration between cognitive psychologists and mathematics educators.

However, mathematics educators also heeded the important theories of humanistic psychology which postulates that individuals constructed versions of their experience through personal perceptions that in turn influenced their view of the world and their actions within it (Beane, 1990:43). Thus, the humanistic approach recognizes the need for attention to the development of a clear self-concept and a positive self-esteem as crucial aspects of a fulfilling life (Beane, 1990:43).

Whilst the cognitive psychologist may contribute to an understanding of the development of knowledge, the mathematics educator needs to use this information to provide background to a more humanistic approach in the presentation of the subject. There needs to be more consideration of other aspects of human behaviour. Teaching mathematics requires an individualistic approach which understands each persons interaction with other people and with the environment. A humanistic approach also requires a consideration of the history of the person, his or her socio-cultural situation as well as any special problems each person may need to overcome.

3.3.2.3 **A child-centred approach**

For the purpose of this thesis it is important to define a child-centred approach in the context of a constructivist's view to teaching mathematics. A child-centred curriculum does not mean individualised teaching or a laissez faire approach to the curriculum. What is proposed is a more comprehensive knowledge of the pupil, his or her thought processes and emotional concerns. It also involves a more understanding view of the socio-cultural environment of the individual and the school.

Marsh (1992:93) describes a child-centred approach as:

> *The development of the whole personality of children, to satisfy their curiosity and develop their confidence, perseverance and alertness.*

This short description of child-centred education emphasises the mathematical empowerment of the child as the required outcome of curriculum planning. This emphasis is echoed by Cockcroft (1982), The Department of Education (1995) and The Commission on Standards for School Mathematics (1987).

The Commission on Standards for School Mathematics (1987:5) in the U.S.A., provides the best synthesis of goals which reflect the emphasis on the child-centred approach.

1. Learning to value mathematics through varied experiences related to their own cultural, historical and scientific perspectives.

2. Becoming confident in one's own ability to do mathematics and make sense of new problem situations in the world around them.

3. Becoming a mathematical problem solver is an essential ingredient for a future productive citizen.

4. Learning to communicate mathematically using the signs, symbols and terms of mathematics.

5. Learning to reason mathematically by making conjectures, gathering evidence and building an argument to support mathematical notions.

In summary, a child-centred approach to mathematics teaching requires a provision of mathematical literacy for each child which promotes a self-confidence and an ability to utilize mathematics in problem solving and communication.

3.3.2.4 **The affective domain**

A move towards a more democratic form of curriculum planning should include a sensitivity to personal characteristics. Whilst this appears to be the case in modern curriculum ideals, Beane (1990:xi) argues that it is still very much a peripheral issue in schools and that:

> *affective concerns were more and more relegated to separate courses on affective education.*

Curriculum planning can only claim to be humanistic or pupil-centred if attention is given to the effects of school experiences on the self-concept and self-esteem of the pupils. Beane (1990:xi) describes the idealised school as one which portrays the following:

> *Pursued diversity, co-operation, personalness, interaction, problem solving, participative governance and other features.*

Thus the argument for a consideration for the affective domain of pupils becomes an all encompassing ideal and not simply a treatment programme for any particular dysfunctional problem.

Beane (1990:3) argues that whilst the term "affect" has generally been used to refer to mental aspects of human nature such as fear, anxiety, anger, love, etc., these states of emotion occur simultaneously with cognitive processes. These cognitive processes form part of each individuals personal constructs which have developed from his or her socio-cultural perspective.

Mathematics anxiety is an affective variable in that it impacts on each individual's self-concept and self-esteem. In addition, mathematics anxiety causes a blockage in the understanding and recall pathways of the brain (see 2.2.2.2). For these reasons it is important that the argument by Beane for recognition of the affective variable in curriculum design is acknowledged.

Whilst Beane makes a convincing argument for an affective consideration in curriculum planning he does not neglect to establish the link between personal interests and social relations and, in fact, emphasises the importance of a child-centred or personal approach and an approach which considers the social development of each individual.

The constructivist approach to teaching mathematics adopts a similar stance to the proposals of "affective learning" theorist such as Beane (1990:10). Affective learning is concerned with the personal and social development of each individual. This includes knowledge, skills, behaviour and attitudes related to personal interests and social relations and the integration of those two (Beane, 1990:10).

3.3.2.5 **<u>The socio-cultural domain</u>**

It is not the intention of this study to become involved in radical arguments of the influence of society on educational goals. It has been argued that schools reproduce the values and attitudes needed to maintain dominant social groups and that little opportunity is given to pupils to generate their own meaning of knowledge.

Whilst not wishing to over emphasise the societal factors influencing curriculum reform there are aspects which have important implications for this study. The socio-cultural factors influencing mathematics anxiety have been discussed in Section 2.2.2.1. Here some discussion of the other influences of society is necessary.

Changes in society, especially in technological advancement and in employment requirements must be given consideration. However, modern society does not require a stereotype output which prescribes that curriculum design provides workers for the present day situation alone. The rapid changes in societal needs indicate that educators must prepare with a measure of flexibility and a willingness to adapt.

The Commission on Standards for School Mathematics (1987:3) states that:

New societal goals for education include:

> 1. *Mathematically literate workers;*
> 2. *Lifelong learning;*
> 3. *Opportunity for all; and*
> 4. *An informed electorate.*

Implicit in these goals is a school system organized to serve as an important resource for all citizens throughout their lives.

Also implied in this statement is a need for schools to change their approach to the teaching of mathematics. A strict adherence to algorithms and a learning of specific rules for a given situation will not prepare the individual for the society he or she will eventually have to face. As the rules no longer apply, so the individual will become more anxious about the ability to cope with simple mathematical tasks. Mathematics teaching has a responsibility to provide a basis for students to develop their problem solving skills in a non-threatening environment and to ensure that the future use of these skills will facilitate the needs of a changing society. As society changes so must the schools, the curriculum and the teaching method.

The school and the classroom are microcosms of society and the interaction between student and student, and student and teacher are crucial aspects of the learning process. It is these social aspects of the constructivist approach that will receive the most attention in the next chapter.

3.3.2.6 Synthesis of mathematics curriculum design

In Section 3.2.5 it was established that a more comprehensive view of the mathematics curriculum was intended in this study and a process-orientated approach was emphasised. The interrelated components of the systems approach was seen as providing the necessary framework for a mathematics curriculum. These components of situation analysis, aims and objectives, content, teaching methods and evaluation will be discussed in the context of providing for the mathematically anxious child in Section 3.3.3.

The systems approach also allows for an integration of theories discussed in this section (see 3.3.2.1 to 3.3.2.5). In fact, McCutcheon (1982:19) recommends such an integration for curriculum design and Moodley *et al* (1992:2) emphasises the fact that mathematics curriculum design is influenced by philosophy, psychology and didactics. Hence, the mathematics curriculum must draw on all the theories discussed in this section.

Cognitive psychology provides essential knowledge of how people think about mathematics and in particular how they would come to understand mathematical concepts, processes and skills. This knowledge has to be tempered with some humanistic considerations which pays attention to aspects of human behaviour which are not purely cognitive but which will influence the cognitive performance. The person's history, culture and environment are important influences in their life and special problems which may affect the self-concept and self-esteem are seen as debilitating factors in the intellectual development of the individual.

A child-centred orientation does not mean a total disregard for cognitive processes and is essentially a humanist orientation. However, the emphasis is on the child and the promotion of the development of the personality of the child.

The influence of a consideration of affective variables and socio-cultural variables must form an integral part of the mathematics curriculum. Affective variables such as fear, anxiety and anger impact on cognitive processes and socio-cultural environments, in particular, the home, also have a major influence on a child's performance in mathematics. More importantly, the demands of society in an ever changing technological world accentuates the need for a dynamic mathematics curriculum which provides for change.

3.3.3 **Mathematics anxiety and the mathematics curriculum**

A comprehensive view of the mathematics curriculum will benefit those concerned with mathematics anxiety if all the components are interpreted in a way which provides specific emphasis to the process approach and affective variables whilst also giving attention to humanistic and child-centred concerns.

A systems approach that requires a comprehensive, cyclical view of the mathematics curriculum can provide a basis for the needs of an affective variable such as mathematics anxiety.

McNeil (1985:358) says that:

> *If there is a systems approach with procedures for setting goals, assessing needs, specifying objectives and priorities, and using evaluation to guide improvement, there will be more progress toward broad democratic goals.*

McNeil (1985:358) believes that the use of a systems approach will provide a more formative evaluation of the situation and emphasise instructional methods which allow for more positive expression from teachers and students in the classroom and ultimately a more positive attitude towards school. It follows that this more positive attitude will increase self-confidence and reduce anxiety.

The comprehensive view of the curriculum was described in Section 3.3.1 and will now be discussed with an emphasis on mathematics anxiety in the mathematics curriculum. The components of situation analysis, aims and objectives, choice of subject matter, teaching methods and evaluation form the framework of curriculum planning. The main emphasis will be on teaching methods but as explained in Section 3.3.1 each component is interdependent and it is therefore important that teaching methods are linked to all aspects of curriculum planning.

3.3.3.1 **Situation analysis and mathematics anxiety**

The elements of situation analysis were discussed in Section 3.3.1.1 where it was noted that these elements could be categorised under various headings. It was also recommended that Harley's list of elements should be supplemented by adding headings which provide for an analysis of more humanistic and learner-centred aspects of mathematical development. For this reason two elements are added here for discussion. These are entitled "home environment" and "learner characteristics" and have particular relevance for this study.

The heading "home environment" provides a category for analysing the development of mathematics anxiety outside the school environment. The influences at home can negate any real effort at school to provide for the alleviation of anxiety. The liaison between school and home, between teacher and parent, is a crucial aspect of the personal and social development of the learner.

The heading "learner characteristics" is a necessary addition to situation analysis because it also provides for a focus on the personal and social aspects of the learner. Under this heading the needs, the cognitive ability, the perceptions and experiences and the aspirations of the learner receive the focus of attention. The influence that these learner characteristics have on the alleviation of mathematics anxiety is of central concern.

The elements of situation analysis are composed by the curriculum planners and as such will cover a wide range of critical aspects which would all receive equal attention. However, this study does have a particular bias in that mathematics anxiety is the focus of attention. For this reason each element will be discussed but those pertaining more to mathematics anxiety concerns will receive prominence.

a. **Philosophy of life-educational ideals**

This aspect of situation analysis essentially determines the frame of reference the curriculum will adopt. For those concerned about mathematics anxiety the educational ideals should be centred on the child and specifically on the child's affective domain.

This stance assumes that affective variables such as mathematics anxiety occur simultaneously with cognitive processes. Impacting on the affective variables are other considerations of a humanistic nature such as, the social and cultural context of the learner and his or her self-concept. The philosophy that believes that individuals construct their own version of reality through their personal perceptions must be based on democratic and dignified principles. Democratic in that each individual is seeking self-fulfilment and dignified in that each individual attains self-esteem, self-confidence and a clear self-concept (Beane, 1990:57).

For the mathematically anxious child this emphasis on humanistic ideals and the affective domain will provide the atmosphere for the non-threatening development of mathematics.

b. **Nature and structure of the subject**

There is little doubt that the teacher's approach to mathematics teaching and learning in the classroom is guided by the view that he or she holds about the nature of mathematics (Moodley *et al*, 1992:1; Goldin, 1990:44).

Moodley *et al* distinguishes between two views of the nature of mathematics. A formalistic view which sees mathematics as a body of knowledge consisting of concepts, rules and algorithms that require certain skills and routines to master. Then there is the view that mathematics consists of processes like generalising, classifying, formalising, abstracting and exploration.

It is this second view of mathematics as a process-orientated approach that is important to the alleviation of anxiety. By adopting a view that promotes learning and memorizing of rules and routines the teacher will be developing a climate for potential anxiety (see 2.2.2.4). This mathematics anxiety will be aroused when the student reaches a stage where the rules and routines no longer work and a deeper understanding is required.

The main concern of mathematics educators should be to provide pupils with opportunities to construct their own mathematical ideas and develop a problem solving attitude to doing mathematics work. This type of climate will provide the pupil with the correct view of the nature and structure of the subject and ensure that he or she is prepared for the more abstractness of mathematics in later years at school. This will in turn eliminate the chances of developing mathematics anxiety because certain sections were not remembered or clearly understood.

c. **Aspects from the history of the subject**

For the purpose of this thesis it is not necessary to delve too deeply into the history of mathematics. However, the issues of importance to this study have had little attention in the past and it is pleasing to note that the new ideas in mathematics are concerned with process and give consideration to the mathematical autonomy and self-confidence of the child (Cockcroft Report, 1982; Commission on Standards for School Mathematics, 1987; Department of Education, 1995).

The influence of the internationally recognised Cockcroft Report and the curriculum and evaluation standards for school mathematics (Commission on Standards for School Mathematics, 1987) reflects the growing concern to move from a traditional view of mathematics and mathematics teaching and learning. Mathematics education is unique in that in recent history it has received the benefit of research by mathematicians, educators and psychologists. Thus, theoretical frameworks and methodological ideals have been formed and transformed over the years.

Much research has been done on mathematics anxiety (see 2.2) and this has led to curriculum planners taking more note of such affective variables and how a curriculum could include elements which would combat such developmental problems.

Curriculum designers should also take note of aspects in the history of the subject such as the term "New Maths" from the 1970s and in particular the connotation associated with this term. The mere utterance of the words "New Maths" can cause unfounded and unreasonable levels of mathematics anxiety.

Mathematics is also unique in that it has a number of myths and connotations which have historically been associated with mathematics (see 2.2.2.2). The existence of these myths and connotations need to be acknowledged by mathematics educators as they are a constant source of mathematics anxiety which develops mainly through social intercourse.

d. **Ability and training of teachers**

A constructivist approach and a concern for mathematics anxiety in the classroom has a number of important implications for teachers and hence the training of teachers.

In terms of mathematics anxiety the ability and training of teachers is a critical element of situation analysis because of the vicious circle effect discussed in Section 2.1.3.4. The teacher's own anxiety as well as her approach to the anxiety of the child are factors which will influence the perpetuation of the problem. Research in this study showed that teachers in training suffered from high levels of anxiety which had a negative correlation with their school achievement level. In addition, the level to which they expressed a confidence to teach was unrealistic in terms of their own ability (see 5.6.2.4).

The negative influence that teachers contribute to mathematics anxiety was fully discussed in Section 2.2.2.4 where researchers such as Stodolsky (1985:13), Greenwood (1984:663) and Tobias (1987:5) all expressed concern about the ability of teachers and their teaching skills being a major contributing factor to mathematics anxiety.

Kogelman & Warren (1978:18) reported that in their dealings with mathematics students, the students would often recall traumatic experiences with teachers which had manifested into anxiety in the presence of mathematics. This anxiety is often carried through to further generations as the child becomes an adult and passes on anxieties and attitudes onto his or her children.

The teacher is the prime determinant of the child's attitude and anxieties and they are often based on the teacher's own attitudes and anxieties (Johnson, 1981:2). The school and especially the classroom are an important part of the child's social environment and the teacher is the major influence on this environment. However, the teacher is not trained to counsel students or to be aware of their personal anxieties and attitudes. These aspects should form a critical part of the teacher's training because the presence of a supportive teacher and a healthy classroom climate are crucial elements in reducing anxiety at school.

e. **Psychological aspects of learning mathematics**

Many theorists agree that there are two distinct types of knowledge associated with learning mathematics (Hiebert & Lefevre, 1986:1). These types of knowledge have been given various names by the respective theorists. Skemp (1971:46), distinguishes between "Instrumental Understanding" or rote learning of particular classes of examples and "Relational Understanding" which requires an ability to relate given procedures to a more general knowledge of mathematics.

Instrumental understanding would be the term used by Skemp (1971:46) to describe the initial learning of a mathematical topic which may or may not be related to any other mathematical concepts. However, Skemp (1971:46) claims that without relational understanding knowledge does not lead to meaningful learning. The important idea here is that knowledge is a process of assimilating new material into networks or structures so that it becomes part of an existing network.

Hiebert & Lefevre (1986:4) believe that networking occurs at two different levels, a primary and reflective level. At the primary level information is constructed at the same or lower level as the presented material. At the reflective level *relationships are constructed at a higher, more abstract level than the pieces of information they connect* (Hiebert & Lefevre, 1986:5).

Researchers have constantly debated which area of teaching should receive greater emphasis and the importance of the learning of skills and procedures and acquiring conceptual knowledge have been seen as two opposing objectives. Hiebert & Lefevre (1986:2) report that this debate is becoming more productive with the realisation that both procedural and conceptual knowledge are important. Procedural knowledge refers to the idea of teaching the necessary mathematics skills whilst conceptual knowledge implies an understanding of the mathematics and the underlying relationships of concepts. A recognition of a need to prescribe instructional programmes that link both types of learning from an early age is a distinct advantage for those pupils suffering from mathematics anxiety.

Hiebert & Lefevre (1986:3) say that whilst rote learning may make an individual pupil feel secure in his knowledge of mathematics, this sense of security will only be short-lived if, at a later stage, he or she cannot adapt or assimilate this knowledge. On the other hand, a teacher that insists on only emphasising the need for conceptual understanding of relationships and neglects the basic requirements of the necessary skills or procedures, will also affect the confidence of the pupil.

The Cockcroft Report (1982:70) emphasises the need to teach in a way which will help to develop long term memory and understanding. However, this need in no way be in opposition to, or at the expense of, the development of skills.

For the mathematically anxious child, the idea of mastering the necessary routines in mathematics is essential. This will then allow more time to concentrate on the higher-order thinking for problem solving and developing a clearer understanding of the conceptual relationships involved in mathematics.

Understanding is not the only aspect of teaching that curriculum design should emphasise but it does direct the concerns to the main issue in mathematics education which is the mathematical development of the child. If our goal is to produce pupils who have intellectual autonomy and self-confidence, then understanding is the key factor.

f. **Logistic considerations**

Whilst not of major concern in this thesis it must be mentioned that the cost of teacher training and the upgrading of present teachers is of paramount importance. Teachers who are not adequately trained or under-qualified are unlikely to offer any time to counselling of mathematically anxious pupils. In addition, they are likely to experience anxiety themselves when teaching work that they do not fully understand.

Any inadequacy in schools, classrooms and books will not facilitate a correct environment which is anxiety free. Good facilities, less crowded classrooms and the correct teaching material are the basic needs of teaching. Only when these needs are met will higher order needs of developing self-confidence and self-actualization and alleviating anxiety be addressed.

g. **Developments elsewhere in the world**

There is a definite shift in the emphasis in mathematics curriculum from the idea that pupils are empty vessels to the idea that they are active mathematical thinkers who try to construct meaning and make sense for themselves of what they are doing (Moodley *et al*, 1992:5). This has led psychologists to give attention to cognitive processes as well as to affective variables that may influence these cognitive processes.

One such affective variable is mathematics anxiety which has received a considerable amount of attention over recent years (see 1.1.1). Over one hundred and fifty institutions in the U.S.A. offer courses to alleviate mathematics anxiety and research on this problem is prolific (Hembree, 1990:38). The proceedings from various national and international mathematics conferences also provide evidence of the attention given to mathematics anxiety. Of particular relevance the International Congress of Mathematics Education (ICME) and the International Group for the Psychology of Mathematics Education (PME) have included mathematics anxiety topics and discussion groups over the past ten years.

The Cockcroft Report (1982) in England and The Curriculum and Evaluation Standards for School Mathematics (1987) in the U.S.A., are two documents which have had significant influence on present day thinking. Both documents support a need for a more process orientated approach to mathematics and an emphasis on problem solving abilities. The proceedings from the latest International Congress of

Mathematics Education (ICME) conferences and from the International Group for the Psychology of Mathematics Education conferences are evidence of the world wide support for constructivism and concern for mathematics anxiety.

h. **Demands of society**

The need for reform in school mathematics arises from the rapid changes in technology and a shift from an industrial society to an information society (Commission on Standards for School Mathematics, 1989:3). This shift has necessitated the transformation of mathematics with a consideration of the type of society that the student has to face when he or she leaves school. To cope in an ever-changing world all students need to be able to solve problems, to adapt to various situations and to feel self-confident in their ability to handle mathematics and interact with others in mathematical pursuits.

The greater emphasis on mathematics is potentially an anxiety stimulating environment as extra pressure is placed on students to do well in mathematics because it is important for their future academic development and possible employment.

The teacher is thus faced with the problem of emphasising the usefulness and importance of mathematics whilst at the same time developing a good understanding and a self-confidence in his or her students.

i. **Home environment**

The home environment of both the teacher and the student will impact on mathematics anxiety. This was emphasised in Section 2.2.2.1 where it was noted that there is a general concern for parents and teachers who pass on their anxieties.

It was also noted in Section 2.2.3 that the symptoms of mathematics anxiety can manifest themselves in pupils who have experienced particular problems at home and may come from an insecure home background or a "broken home".

This may not appear to be in the realm of curriculum planning but remediation of mathematics anxiety will be mainly at school and schools can offer programmes which assist parents and keep them informed of what is expected of children doing mathematics work at school and at home. The school should accept responsibility for involving parents in the programme by providing information brochures and parent evenings (Hawkey, 1986:156).

Hawkey (1986:156) suggests four areas that parents could be made aware of as their contribution to helping alleviate mathematics anxiety.

1. Encourage their children to make use of mathematics during normal family activities. This could include weighing and measuring at home and calculations of petrol consumption, tips or tax when on a trip.

2. Parents should be made aware of the harm that unrealistic expectations may cause. Exerting too much pressure on a child may lead to failure or dislike of the subject.

3. Parents should be aware of the negative effects that their remarks and the remarks of other significant people could have on children. They need to promote a positive attitude towards mathematics.

4. It is essential that parents are made aware of any new methods being used in mathematics teaching and the requirements of the mathematics curriculum.

In general, parental support for what is done at school is an essential aspect of the success of the mathematics curriculum and will provide invaluable support to any mathematics anxiety remediation that takes place at school

j) **Learner characteristics**

Whatever their level of attainment, the needs of the pupils are centred around providing them with mathematical power. This mathematical power can be described as the ability to explore, conjecture and reason logically as well as the ability to use a variety of mathematical methods effectively to solve non-routine problems. In addition it is the development of personal self-confidence in one's ability to do mathematics (Commission on Standards for School Mathematics, 1989:5).

The mathematically anxious child needs a classroom environment which serves to promote the concept of mathematics power. They need to learn to value the usefulness of mathematics and be taught to feel confident in their mathematics work. Problem solving will be an essential part of their mathematical development and the ability to reason mathematically and to communicate mathematically are essential aspects of a confident and competent mathematics pupil (Commission on Standards for School Mathematics, 1989:6).

Perceptions and attitudes may also influence the learning capabilities of the child. These are often derived from the perceptions and attitudes of a teacher or parent. In particular, the myths of mathematics postulated by Kogelman & Warren (1978:30) in Section 2.2.2.2 describe a mind set that fosters mathematics anxiety. "Mathematicians do problems quickly in their heads" and "some people have a mathematics mind and some don't" are two myths that need to be dispelled urgently to avoid anxiety. The perception of how mathematics is done can lead students to believe that one requires a certain arcane wisdom (Sarason, 1980:271) and leads to a lack of confidence in one's ability and a belief that mathematics is a subject for the select few. This factor will become the sole reason why a person is not successful and a valid reason for not trying to master mathematics.

The major influence on learner characteristics stems from learner experiences. Whatever their level of attainment, pupils should not have to experience repeated failure. An intense dislike and avoidance of mathematics is often a consequence of repeated failure (Cockcroft, 1982:8). These failures could be exacerbated by the perceptions of inadequacy and could reinforce beliefs that only a few have a mathematics brain. Cockcroft (1982:8) reports on other experiences that lead to failure and subsequent anxiety. These experiences include a change of teacher, prolonged absence from school, irascible or unsympathetic teacher and over expectation of parents.

The needs, abilities, perceptions and experiences of students all foster feelings of mathematics anxiety. These learner characteristics are identified in the classroom and develop from social interaction in the school and at home. They form an essential part of situation analysis (see 6.3.8) but it is important to note that mathematics anxiety remediation should not focus on curing the individual but rather on rectifying the conditions which foster these feelings (Tobias, 1978b:7).

3.3.3.2 <u>Aims and objectives and mathematics anxiety</u>

With the focus of this study on the alleviation of mathematics anxiety, the stated aim could be simply expressed as to reduce the anxiety level of students to a level which enables them to cope with mathematics and be competent problem solvers. However, this is no simple matter and this single aim requires a combination of competencies for students (see 3.3.1.2). The sources of anxiety were categorised as socio-cultural, emotive, cognitive and educational (see 2.2.2). Thus the objectives required to alleviate anxiety will be found in these four categories but developed through curriculum programmes as follows:

1. Socio-cultural anxieties reduced by introducing parent information programmes.

2. Emotive issues addressed by school counselling, teacher support and teacher awareness.

3. Cognitive issues would be addressed by initiating teaching programmes that take cognisance of the nature of mathematics and how pupils come to learn. Teaching for understanding and not memorization (see 3.3.3.1.e).

4. Educational issues will address the classroom environment and the creation of a non-threatening atmosphere in the presence of mathematics work.

3.3.3.3 <u>Choice of subject matter and mathematics anxiety</u>

This thesis is not particularly focused on subject matter as an issue affecting mathematics anxiety. However, there are certain implications involving content. Attention to alleviating mathematics anxiety requires a caring attitude from the teacher and a general building of self-confidence in a child's ability.

The time required by teachers to listen to pupils solutions and methods and to facilitate communication and problem solving opportunities all puts pressure on the teacher's time available to cover the contents of the syllabus. Thus, the implication for curriculum design is a reduction in content to allow for more time for teachers to ensure that the goals stated in Section 3.3.3.2 are achieved.

An overfull syllabus will mitigate against a caring environment as it can often lead to rushed work and a reversion to traditional teaching approaches of "chalk and talk" whilst pupils try to memorize rules and routines.

3.3.3.4 <u>Teaching methods and mathematics anxiety</u>

Teaching methods are of central interest in this study in that many sources of mathematics anxiety can be traced back to poor teaching methodology (see 2.2.2.4).

Teaching methods are crucial to mathematics anxiety (see 2.3) and the proposal of a process orientated approach is important if mathematics anxiety is to be alleviated. Process implies a deeper understanding as opposed to the learning of rules and algorithms. The emphasis on problem solving will also enhance understanding. In both instances elements of anxiety are reduced as pupils do not feel anxious when confronted with new problems. A humanistic and child-centred orientation would ensure that the child receives empathy in the classroom and an opportunity to express any concerns that may cause anxiety. The critical aspects of mathematics teaching methodology are discussed in Section 3.4.1.

3.3.3.5 <u>Evaluation and mathematics anxiety</u>

Once again it is not the intention of this study to become involved in lengthy discussion on the merits and de-merits of various methods of evaluation. However, the emphasis on mathematics anxiety reduction should lead to a de-emphasis on formal examinations and rigid testing techniques. It is revealed later in Section 5.6.3.5 and 5.6.4.6 that the formal testing situation is a high anxiety provoking situation.

Whilst formal testing will probably always form part of the evaluation procedures in mathematics a more continuous process which monitors the child's progress as an individual thinker is needed to instill self-confidence. To achieve the aims and objectives stated in Section 3.3.3.2 the school and in particular the teacher has to continuously evaluate a number of areas in the progress of the child. Socio-cultural and emotive issues should be monitored and teaching methods must essentially provide for a non-threatening environment and a teaching for understanding. It follows that evaluation must be developed to provide non-threatening opportunities to reveal their mathematics competence. This could be achieved by introducing project work, investigational work and by being aware of the negative effect of time pressure in formal testing.

3.3.3.6 **A synthesis of the mathematics curriculum and mathematics anxiety**

The critical aspects involving mathematics anxiety are all addressed in a situation analysis which is comprehensive. Whilst elements such as "psychological aspects", "home environment" and "learner characteristics" are emphasised, it is important to note that mathematics anxiety concerns are related to a wide range of situations.

The situation analysis impacts on the four other components of the curriculum as the interrelationship of these components in a systems approach to curriculum design is evident. The aim of providing children with intellectual autonomy and a self-confidence to do mathematics work is a simple statement but requires a broad spectrum of application in the curriculum. In particular, teaching methods must receive the focus of attention as the alleviation of mathematics anxiety is achieved mainly through the classroom situation. However, the choice of subject matter and the methods and results of evaluation impact on teaching methods and ensure that a continuous re-evaluation of aims and objectives is an ongoing process of the mathematics curriculum. In this way an assessment of the remedial progress being made concerning mathematics anxiety is continuously under review and that the critical aspects of socio-cultural, emotive, cognitive and educational sources are all given due consideration.

These aspects of the mathematics curriculum are of central concern in this thesis and will be discussed again in Section 6.2 and 6.3 where the literature studies and empirical research on mathematics anxiety is integrated with the literature study on constructivism.

3.3.4 <u>Constructivism and the mathematics curriculum</u>

Constructivism is a reaction to the traditional view of mathematics in the curriculum in that it views mathematics as fallible, changing and a product of human construction (Davis & Hersch, 1980; Lerman, 1990; Ernest, 1991).

Moodley *et al* (1992:2) contends that mathematics education is a discipline on its own but it is also related to mathematics psychology, pedagogy and the practice of mathematics teaching. It is this multi-faceted view of mathematics education that has led to the formation of a constructivist approach.

In the first instance constructivism as a philosophy provides a theory of how pup[ils learn and sets the paradigm for the teaching situation. This paradigm formulates the constructivist approach that emphasises a shift from a traditional view to the teacher as a facilitator who teaches by negotiation rather than by imposition.

Secondly a constructivist approach evolves from a process orientation where thought processes are given prominence over content and problem solving attitudes are developed in an investigational and discovery environment.

Constructivism also developed from psychological studies and in particular the cognitive psychology which became dominant after behavioursim lost its dominant role (see 3.3.2.1). In addition, states of emotion described in the affective domain (see 3.3.2.4) have been recognised as occurring simultaneously with cognitive processes and traits such as anxiety form part of each individual's personal constructs (Beane 1990:3).

Whilst recognising the cognitive processes involved in the learning of mathematics, the constructivist approach is not void of humanistic considerations. As a child-centred approach it proposes a more caring attitude by the teacher and a comprehensive knowledge, not only of thought processes and emotional concerns but also an understanding view of the social environment of the individual and the school situation.

Constructivism views mathematics as a human activity which is accessible to all as knowledge comes about by the interaction of the individual with the world. The indulging objective of a constructivist approach is to develop intellectual autonomy and self confidence. Lerman (1990:32) says that the constructivist objective should be that:

> *People see themselves as having the power to engage in problems that dominate their lives, pose questions for themselves and develop solutions.*

To achieve this objective and to hold a constructivist view of mathematics education, has a number of consequences for the school mathematics curriculum.

Ernest (1991:283) expressed these consequences as follows:

1. School mathematics should be centrally concerned with human mathematical problem posing and solving.

2. Inquiry and investigation should occupy a central place in the school mathematics curriculum.

3. The fact that school mathematics is a fallible and changing human construction should be explicitly admitted and embodied in the school mathematics curriculum.

4. The pedagogy employed should be process and inquiry focused.

Ernest (1991:283) explains that unless the process approach is adopted the previous implications are contradicted.

Davis *et al* (1990:188) say that:

> *Constructivism does not offer pedagogical recipes of convenience.*

They stress that teacher education must also be based on a constructivist approach if teachers are to be trained to cope with the classroom requirements of this approach. Teachers will need to develop new skills that enable them to introduce new techniques in the classroom. As a facilitator of learning the teacher needs to learn the following techniques:

1. Allowing for discussion in groups or as a class.
2. Developing a communication between each pupil and themselves.
3. Encouraging exploration and investigation.
4. Expecting reflection and adaption of solutions and methods.
5. Developing a problem solving attitude amongst pupils.

(Greer & Mulhern, 1989:5)

Davis *et al* (1990:191) would add one more element to the above list, and that is the ability of the teacher to develop a caring relationship with his or her students. Davis *et al* believe that constructivism is brought into perspective by a teacher with a caring attitude.

> *When we open ourselves to caring relations, we learn to listen. Then we become convinced that constructivism is fundamentally right.*
> (Davis *et al*, 1990:191)

The consequences of a constructivist view of mathematics will be fully explored in Chapter Four where a more in depth discussion of constructivism is presented. The important issues of a constructivist approach will also be linked to those issues of concern regarding mathematics anxiety.

3.4 <u>**Mathematics teaching and mathematics anxiety**</u>

It has been shown in Section 2.1.3 that mathematics anxiety is an important variable in the teaching of mathematics. In particular, researchers such as Stipek (1988:111) and Stodolsky (1985:132) directed their concerns towards the actual teaching of mathematics.

The motivation to provide for the mathematically anxious child in curriculum design was explained in Section 2.1.3.1 to 2.1.3.4 where it was established that mathematics anxiety is widespread, it affects mathematics performance and causes avoidance of mathematics.

Whilst the sources of mathematics anxiety were categorised in Section 2.2.2 as socio cultural, emotive, cognitive and educational. The remediation is mainly through the medium of the teacher in the classroom. The mathematics teaching process must provide the correct structure to address the problem of mathematics anxiety. In Section 2.3 the areas of concern for mathematics teaching when considering mathematics anxiety were introduced. These areas of concern are now more fully explored within the context of curriculum planning. Once again it must be stressed that mathematics teaching is the integrating theme of this thesis as it is the teaching of the curriculum that best addresses the problem of mathematics anxiety and provides the medium for constructivism.

3.4.1 <u>Mathematics anxiety and critical variables in mathematics teaching</u>

In Sections 1.1.3 and 2.3 a number of elements of mathematics anxiety were identified as critical to the teaching process. These elements have certain implications for curriculum design as they provide direction for teaching methodology that will alleviate mathematics anxiety. In Chapter Four it will become clear that the issues concerning mathematics anxiety coincide with the tenets of a constructivist approach. Hence, a synthesis of these ideas will be pursued in the final analysis in Chapter Six.

It should be pointed out that a number of these elements are interrelated and overlap in certain instances. However, to provide clarity and a structure to the discussion it is convenient to categorise these important factors of teaching that impact on mathematics anxiety.

3.4.1.1 <u>Classroom environment</u>

Developing the necessary classroom environment is the most important task in combating mathematics anxiety. This is particularly relevant in the secondary school years because as Brush (1981:37) shows, mathematics becomes a more difficult subject with a number of cognitive factors that may affect performance. Kogelman

& Warren (1978:16) have also identified the early secondary school years as the period when negative mathematics experiences most frequently occur. They report that prior to this period people often recall being more comfortable with their mathematics performance.

The classroom is a public place where the child interacts with his peers and with the teacher. Being singled out as a pupil to answer a question in mathematics is an anxiety provoking situation (Buxton, 1981:101) and being called to the chalkboard to perform in front of the class also causes anxiety (Tobias, 1987:5). Working under time pressure is also an element of the mathematics classroom that causes anxiety as students who are unable to complete tasks are identified by the teacher and the others in the class.

Derogatory comments by the teacher can inhibit pupils. A teacher may comment "You will never be able to do mathematics" or "You should know this section" or "This should be an easy section". All of these comments may cause various degrees of anxiety as the pupil may give up after hearing the first two comments or may perceive himself as inferior if he does not understand the section referred to in the third comment. The mathematics anxious students are especially sensitive to this type of criticism and often their history will reveal that they have encountered such lack of empathy somewhere in the past.

A classroom environment that is unsupportive inhibits the pupils from taking risks. Creativity, unique methods and perseverance should be supported with regular positive reinforcement. Even if answers are wrong a pupil who has the courage to investigate an inventive idea should be commended.

3.4.1.2 **Understanding**

Teachers will acknowledge that process and product are important in teaching methodology but often their methods are influenced by a rigid examination system. Their teaching is often judged on the evidence provided by examinations and memorization and rote learning are often successful methods for pupils in the early secondary school years.

Greenwood (1984:663) states that:

> *The principal cause of mathematics anxiety lies in the teaching methodologies used to convey the basic skills to our youngsters.*

He expresses concern that using the explain-practice-memorize paradigm does not develop understanding and reasoning. This teaching methodology concentrates on the procedures for obtaining answers and is not particularly concerned with the development of logical thought. By its nature this type of teaching isolates fact from reason and from the important process of problem solving.

The problem of memorization and rote learning are not immediately evident but rather manifest themselves as the pupil progresses through school. Breaking the rote learning syndrome is often very difficult and is often identified by a pupil inquiring "Do you always do these problems like this?" In other words, they are hoping to establish a rule or rigid method to apply. This attempt to always revert to rote learning methods causes anxiety. The number of routines to be memorized becomes overwhelming and they find it difficult to adapt their mathematical knowledge to other problems. Once the pupil realizes his methods are ineffective it is often too late to adopt alternative strategies and severe anxiety may result.

3.4.1.3 <u>**Self-confidence**</u>

Developing self-confidence in a child's ability to do mathematics is obviously the most important task for the alleviation of mathematics anxiety. Pupils' feelings towards mathematics are largely a reaction to mathematics teaching and classroom activities and a number of aspects can be most anxiety provoking and confidence shattering. Teachers need to display a confidence in each pupils ability to do mathematics. Intuitive methods should be encouraged and positive feedback and reinforcement must be an important aspect of a teacher's inventory. The mathematics teacher needs to be aware of a number of features of the school and classroom situation that can destroy self-confidence. The teacher should take note of the following:

a) Be sensitive that derogatory or discouraging remarks such as "that is a stupid question" or "you should know this" can destroy confidence.

b) Be aware that tests and assignments with time restraints can be anxiety provoking. Sometimes being given as much time as needed to complete a test restores confidence and reduces anxiety.

c) Listen intently to questions and answers and always be willing to answer them supportively. Children often feel intimidated by the teacher and refuse to ask questions or have any discussion with the teacher.

d) Do not create humiliating experiences for the pupil. This often occurs when the child is singled out to answer a question or write on the chalkboard.

e) Be less authoritative, less rigid in approach and more flexible. Allow more time for debate and discussion in the classroom.

f) Provide as many positive mathematics experiences as possible in order to develop in the pupils a willingness to try a problem. Too often pupils avoid starting a problem as a defensive mechanism to protect themselves from what they are sure will be defeat or humiliation.

g) Give positive feedback to tests and assignments. Instead of just marking answers right or wrong make comments indicating correct procedures and emphasise analysis of errors and understanding of solution paths.

The above positive actions by the classroom teacher will help the students to develop and increase their confidence when confronted with a mathematical task.

3.4.1.4 Communication

Two important factors involving communication are the main cause for anxiety in the mathematics classroom. The first factor teachers must note is that the language of mathematics can cause confusion in the mind of the child and if pupils are not properly initiated into its vocabulary it can cause anxiety (Tobias, 1978a:48). Learning the language and symbolism is crucial to the effective mastery of mathematics and many students find this extensive use of symbols dehumanizing and offensive (Kogelman & Warren, 1978:27). A dislike for the language and symbolism will inevitably lead the student to simply memorize what is immediately needed and never really internalize the important concepts. There remains a sense of not understanding or a faulty cognition of mathematical language notation that can seriously impair mathematics ability and hence cause anxiety.

The second important factor is that mathematics is often presented from a text book perspective and hence as a rigid and austere discipline. Kogelman & Warren (1978:25) state that:

> *There is no talk of how people do mathematics, only explanation and logical presentation of concepts and methods.*

The teacher often disguises the fact that solution paths are often negotiable and not always correct the first time. The impression is created that mathematics truths have always been there and are not something discovered and developed.

If mathematics is portrayed as not negotiable, inflexible and rigid, pupils will feel that they are not required to debate or discuss mathematics issues. In other subjects such as English, History, Geography etc. the development of healthy discussion is encouraged and provides a sense of participation and hence enhances the pupil's enjoyment of the subject. This valued participation is also significant in contributing to the pupil's feeling of self-worth and self-confidence. As described in Section 3.4.1.3, listening to solutions and encouraging intuition are important aspects in developing self-confidence and hence reducing anxiety.

3.4.1.5 **Social interaction**

The social interaction involved in the child's mathematics development could be related to attitudes. Parent attitudes and teacher attitudes toward mathematics will influence the development of the child's attitude towards mathematics. In essence, teaching must take cognisance of these attitudes that prevail in society and provide the right climate in the classroom to develop a more positive social acceptance of mathematics.

Lazarus (1974:20) makes the point that parental values and attitudes are important determinants of childrens' attitudes and that what is important to the parent will usually become important to the child.

The teaching of mathematics is made much more difficult by the fact that pupils have preconceived ideas and often negative feelings about mathematics. Parents and "significant others" in the child's life often express a dread or dislike of mathematics or a defense of the fact that they could not do mathematics (see 2.2.2.1). This is a cause for anxiety before the child encounters mathematics.

In addition, a number of myths and connotations of mathematics exist in society (see 2.2.2.2) which help to determine attitudes and are a cause of emotive reactions such as anxiety.

These social issues all manifest themselves in the child's attitude in class. Negative attitudes moulded by parents, relations and friends as well as the myths and connotations that have been entrenched by society result in the child being anxious about mathematics before entering the classroom situation. These preconceived ideas may happen at primary school level but are often related to the start of secondary school when mathematics becomes more abstract (see 2.2.2.3).

The first task of the mathematics teacher is to dispel these feelings and myths and be careful that they do not add to them by expressing their own negative feelings or poor attitude. Emenalo (1984:456) expressed the concern that:

> *Fear of mathematics is attributable to many a teacher of mathematics.*

The teacher is an authority figure in the classroom and must be aware that this role can be decisive in forming attitudes and creating anxieties.

Buxton (1981:81) identifies three forms of authority that are essential elements of the social interaction between teacher and pupil in the classroom. He terms these as structural authority, sapiential authority and personal authority. Structural authority is derived from a position and obviously a teacher has a designated position of authority in the class. Sapiential authority is implied when referring to someone being an authority in a particular subject and personal authority relates to a person being more formidable than others and hence more likely to be paid attention to even outside their official position or field of expertise.

It is essential that the teacher is aware of all three of these forms of authority and exercises them with care. All of them can lead to anxiety if not handled correctly.

The teacher should present himself or herself as a facilitator of knowledge and a person who has an empathy for the child's situation. Structural authority allows the teacher to make judgements, ask questions and generally single out the individual and cause anxiety.

Sapiential authority can cause anxiety if the teacher tries to make solutions seem easy to boost their personal ego. By not revealing how a mathematical problem is tackled a teacher is hiding the most important aspect of mathematics, the task of problem solving. The pupil forms the opinion that the solution to all problems is immediately recognised and correctly transmitted to paper without any hesitation, negotiation or rethinking. As it is not possible to do mathematics in this manner the child becomes demoralized and anxious when he or she cannot find a solution immediately. It also inhibits the pupil from trying to investigate or explore various solution paths.

The personal authority of the teacher can also be an inhibiting factor if misused and if the teacher happens to be a formidable figure in the classroom. The teacher needs to be aware of this situation and take care not to be overpowering in the classroom. A rigid inflexible approach does not allow for pupils exploring new areas. The pupils are also unlikely to interact with the teacher and reveal their problems and anxieties.

3.4.1.6 **Problem solving**

Teachers need to provide adequate problem solving situations for pupils to acquire a knowledge of what is required to solve problems. It is important that these skills are developed by allowing the pupil enough time and progressing from problems that are well within the scope of the pupil. They must achieve and experience a feeling of success to believe that problem solving is not a formidable task but an activity within their capabilities.

Sherard (1981:109) reports that an important misconception amongst pupils is that there is one guaranteed right way to solve all problems. The teaching of problems must reveal the many and varied ways of finding solutions and provide the opportunity for discussion between teacher and pupil or pupil and pupil to negotiate the idea of a more elegant solution.

Bulmahn & Young (1982:55) report that many prospective school teachers admitted to a real anxiety when confronted with word problems. This often leads to an avoidance of problems or a rigid approach to the solution of problems. As problem solving is a basic and critical part of the mathematics curriculum, poor teaching methods will cause inadequate techniques and a growing anxiety as the child progresses through school.

In her work with mathematically anxious people, Tobias (1978a:129) found that inadequate problem solving skills were at the heart of mathematics anxiety because they cause frustration and feelings of tension.

3.4.2 **A synthesis of mathematics teaching and mathematics anxiety**

The important aspect of the critical variables in mathematics teaching is that they are all interrelated. It would be difficult to isolate one variable requiring more attention than the others. However, it should be stressed that the classroom environment was discussed first because it is here that the teacher establishes the social scene which will provide impetus to the other five.

Teaching for understanding requires a classroom environment that allows for group work, investigational opportunities and a freedom of students to ask questions and pose problems. Ensuring that memorization and rote learning is not the dominant technique the classroom must reflect a more democratic approach to the solution of problems.

This democracy also impacts on the development of self-confidence as this is directly related to the level of authoritarianism in the classroom. In fact, the critical variable entitled self-confidence in this study could be viewed as anti-authoritarian in the classroom. All the issues discussed in this category relate to the teacher providing a positive classroom atmosphere which encourages participation and offers positive feedback to students.

The communication variable emphasises an important aspect of a mathematical skill required outside the classroom and in later life. Without being able to use mathematics in communication with others the individual will lack a basic life skill that will have a debilitating effect on his or her progress and will lead to anxiety in the midst of any conversation involving mathematics. This communication is developed initially in the classroom and needs to be encouraged in open discussion. The valued participation of each individual will give them confidence to use their communicative abilities and provide an opportunity for the teacher to alleviate any anxiety associated with the use of mathematics.

The importance of social interaction is well documented in Section 3.4.1.5 and again the classroom environment provides the microcosm of society in which children first learn to deal with mathematics. An anxiety free environment will allow for students to interact freely with each other and with the teacher. Not only will this give the student confidence in dealing with mathematics problems, it will also give the student an opportunity to express any fears or anxieties that may have arisen in his or her social situation or home environment.

The problem solving element also depends on the classroom environment and the teaching of mathematics in a way that develops an inquisitive mind. Students should not want to see solutions but rather have the perseverance to work to solve problems by themselves. The fact that mathematics anxiety has a negative effect on mathematics performance can be traced to problem solving skills (Tobias, 1978a:129) as anxiety is often the result of frustration as a student lacks the required knowledge. However, it is also true that a student who possesses the knowledge but has not developed a problem solving attitude will express feelings of anxiety when confronted with a new problem because it does not form part of his or her required learning pattern.

There is no doubt that mathematics teaching involves all the important critical variables which will address the problem of mathematics anxiety. It is essential that these variables become an integral part of curriculum design and a guide to teaching methodology.

3.5 Constructivism and anxiety in mathematics teaching and learning

It is no coincidence that the tenets of constructivism provide substantial arguments under similar categories to those variables discussed on mathematics anxiety in Section 3.4.1. Constructivism is a reaction to the traditional approach to teaching which emphasises rules and algorithms. It provides a philosophy which requires teachers to give more thought to the processes that children use to do mathematics and how they construct their knowledge.

In Chapter Four, constructivism will be fully discussed but it is important to note, at this stage, that there is a close relationship between constructivist theory and mathematics anxiety concerns.

In the first instance the humanistic view and child-centred approach that is adopted by constructivists, ensures that elements of mathematics teaching such as classroom environment, understanding and self-confidence are given attention.

It will also be noted in Section 4.1.2 that the social aspect of constructivism is favoured in this thesis because it provides the structure to address the aspects of classroom environment, understanding and self-confidence by means of attention to communication and social interaction. The role of the teacher as a facilitator will be emphasised and the idea of pupil-pupil interaction as well as pupil-teacher interaction is a critical factor in providing the correct climate for mathematics work.

The concerns of mathematics anxiety researchers are also addressed by the constructivist approach being essentially a problem solving approach. This is not in the sense that problem solving strategies are taught or recommended but rather that a problem solving attitude is developed. This involves an approach that requires the child to develop an understanding of how problems are solved by reflecting on their solution paths. The teacher facilitates this process by analysing the various methods put forward by the class and negotiating the most acceptable solution.

Finally, the idea of reflection (see 4.4) as a cornerstone of constructivism integrates all the concerns of mathematics anxiety because it provides the platform for a caring teacher who listens to his or her pupils. This creates the correct classroom environment and gives the child the self-confidence to express solutions to mathematics. It also develops the communication and social interaction between teacher and pupil and ensures that problem solving and in general, the understanding of mathematics, is enhanced.

3.6 **Summary**

The aim of this chapter was firstly to explore the foundations of the curriculum reform and secondly to suggest the direction that the mathematics curriculum should take to address the problem of mathematics anxiety. Permeating throughout this chapter is the stress and emphasis on the teaching and learning aspect of the curriculum.

The comprehensive view of the curriculum allows for a more detailed analysis of all aspects of curriculum development and in particular aspects which pertain to process development rather than stipulating content and preconceived objectives.

Various approaches and views of the curriculum were discussed with the intention to provide a background to these orientations which were obviously selected for the value each has in giving attention to the mathematically anxious student.

The process approach emphasises the role of the teacher as a facilitator of knowledge and in helping each individual student to develop their own particular way of thinking about and understanding the procedures involved in mathematics. This is important to the mathematically anxious student who may have resorted to rote learning methods and feels particularly inadequate when these methods fail to help. The teacher-student relationship is emphasised and the understanding of the thought processes is seen as a critical aspect for alleviating feelings of anxiety in the mathematics classroom.

Cognitive psychology also emphasises the personal mental constructs of each individual and the move away from the behaviouristic approach in psychology has benefitted mathematics research. A greater interest in learning processes has led psychologists to turn their attention to mathematics to help them understand thinking in general. This move resembles that mentioned in Chapter Two where psychologists studying general anxiety and test anxiety found mathematics anxiety a useful domain to explore and research. In both instances the link between mathematics and psychology has meant an emphasis on teaching procedures and the developmental problems of the student.

An emphasis on the process approach and cognitive psychology could lead to mathematics learning being viewed as a set of procedures and a method of information processing which has to be taught. Thus, these ideas have to be tempered with more humanistic and socio-cultural interests.

The humanistic approach stresses the need for individual attention and the understanding of each person's unique genetic and environmental background. The construction of knowledge within each individual is shaped by that individual's background as well as by the teaching and learning process. This is closely linked to the child-centred approach which emphasises the need for a comprehensive view of the pupil which includes thought processes, emotional concerns and the socio-cultural environment of the pupil.

This comprehensive view of the individual requires an empathic approach by the teacher which enhances the teacher-pupil relationship and provides an atmosphere in the classroom conducive to confidence building and hence a reduction in anxiety provoking situations.

The sources of mathematics anxiety are also found in the affective and socio-cultural domains and therefore these elements become areas of concern for curriculum planning. The construction of knowledge is influenced by a number of mental aspects of human nature often referred to as the affective domain. Anxiety is one such variable and the occurrence of anxiety simultaneously with cognitive processes should influence teaching strategies.

The socio-cultural sources of mathematics anxiety have been explored in Section 2.2.2.1. The immediate problem faced by curriculum developers is the social attitude to change. Mathematics teaching must prepare students for a society that is rapidly changing in a new technological world. A learning of algorithms and rules will not prepare the student adequately for this new world. It is essential that pupils develop problem solving skills and are encouraged to use these skills in a non-threatening environment.

The developers of a new mathematics curriculum need to take cognisance of all the above ideas and incorporate them into a comprehensive plan for curriculum reform. The intention in this study is to describe those aspects of the curriculum planning process which influence the alleviation of mathematics anxiety.

All aspects of situation analysis have been discussed with an emphasis on the concerns of mathematics anxiety as an important variable in the teaching of mathematics and directing the answer to these concerns towards the constructivist approach. For this reason, of particular interest to this study are the psychological aspects of learning mathematics and the teaching methods of mathematics.

The psychology of learning mathematics provides some background to how each individual acquires knowledge and emphasises the need to provide a teaching base which will help to develop long term memory and an understanding of both procedures and concepts. Whilst the thinking processes involved are of central interest to psychologists, the mathematics teacher should be more concerned that students master basic skills and are able to apply their knowledge to new situations. The student must develop a self-confidence to use the mathematical concepts that have been learnt in the classroom.

In essence all the aspects discussed in this chapter revolve around the teaching of mathematics and in particular those aspects of teaching that influence the mathematics anxiety of the student. These aspects were categorised under the headings of classroom environment, understanding, self-confidence, communication, social interaction and problem solving. These factors are interrelated and all have an important impact on the child's mathematical development. It will become evident in Chapter Four that these factors are also of major concern to the constructivist theorist.

Mathematics is a unique subject and it was revealed in Section 2.2.2 that it has certain inherent features which cause anxiety. The manner in which mathematics is presented, the classroom environment, the characteristics for the teacher and the psychological and social make-up of the child all provide the arena in which mathematics anxiety manifests itself.

The empirical research described in Chapter Five is intended to give support to the research reviewed in the literature. In particular, the development of mathematics anxiety and how it affects the mathematics performance of students during the secondary school phase from standard six to standard ten. The importance of the teaching and learning process will once again be emphasised and the issues involving constructivism and the alleviation of mathematics anxiety will be re-examined in the light of the empirical research and the literature research.

A teaching philosophy which encompasses many of the ideals of the favoured curriculum theories discussed in this chapter, is the constructivist approach. Whilst incorporating many elements of other curriculum theories, constructivism provides a comprehensive approach to the teaching and learning of mathematics.

Constructivism is essentially a pupil-centred approach which emphasises the teaching process and in particular the development of a problem solving attitude.

In proposing the constructivist approach to the teaching of mathematics it is the intention of this study to describe in Chapter Four how this approach has evolved from the curriculum changes discussed in this chapter and how the affective domain and socio-cultural domain are both important elements of constructivist theory. Within these domains the development of each individual receives attention to provide a more humanistically acceptable preparation of the student emotionally, cognitively and socially. In the discussion of constructivism in Chapter Four it will become evident that this is a didactic approach which provides a teaching philosophy and a methodological focus which is comparable with the needs of the mathematically anxious child.

Finally, in Chapter Six, the components of the curriculum and the elements of situation analysis will be reviewed by integrating the ideas of constructivism, curriculum design and mathematics anxiety with the empirical research on mathematics anxiety.

In this way recommendations will be made for the teaching and learning of mathematics and the reduction of mathematics anxiety.

CHAPTER FOUR

THE CONSTRUCTIVIST APPROACH TO TEACHING AND LEARNING MATHEMATICS

4.1 Constructivism in perspective

Research into mathematics anxiety has revealed a wide concern for the teaching of mathematics (see 2.2.2.4) and a general agreement that mathematics anxiety affects mathematics performance (see 2.1.3.2). It is thus the contention of this thesis that unless special attention is given to building confidence in mathematics, very little progress will be made in developing mathematics performance.

The aspects of curriculum development emphasised in Chapter Three provide a background to a philosophy of teaching mathematics which could help to build confidence and alleviate mathematics anxiety. Whilst all the sources of mathematics anxiety do not necessarily stem from the classroom (see 2.2.2) this is one arena where many of the problems can be addressed.

In this chapter the concept of constructivism will be explained and thereafter a brief overview of this philosophy will be provided. The important aspects of teaching within a constructivist framework will be discussed. Finally reasons will be provided to explain why the constructivist approach is adopted as the philosophical basis for a curriculum reform which will address the problem of mathematics anxiety.

Constructivism is a theoretical perspective on how children learn mathematics. The constructivist view is described clearly by Ernest (1989:151) when he writes:

> *This is constructivism: the view that children construct their own knowledge of mathematics over a period of time in their own, unique ways, building on their pre-existing knowledge.*

Ernest (1989:151) goes on to explain that the emphasis of constructivism is in mental activity and the active construction of meaning from a multitude of experiences and social-interactions that children have in school and outside the school arena.

Constructivism adopts a cognitive position which requires teachers to understand how mathematical learning develops whilst at the same time providing teaching strategies which enable each individual to interact with others and to build on their experiences in developing their mathematical knowledge. It is a didactic approach which provides a philosophical view of the theory of knowledge whilst also recommending a strategy for teaching and learning.

Constructivism leads the teacher to think critically and imaginatively about the teaching-learning process. The premises of constructivism require the teacher to use certain criteria to judge the various choices of teaching method (Noddings 1990:18).

The constructivist approach could be viewed as a curriculum theory which emphasises the teaching and learning process. It has its foundations in a number of the curriculum ideas discussed in Section 3.3.2. These foundations become clear in this chapter as the roots of the constructivist approach will be traced to psychological studies and a concern for the socio-cultural environment. In addition, a humanistic and child-centred approach to curriculum development together with an emphasis on cognitive psychology and the thinking process all form part of the constructivist approach.

Moodley *et al* (1992:3) emphasise this multidimensional aspect of constructivism when stating that mathematics education has had the benefit of the thinking of mathematicians, philosophers, psychologists and educators in providing theoretical frameworks for the mathematics curriculum.

Steffe & Cobb (1983:84) emphasise the humanistic aspect of constructivism when they state:

> *The main goal of constructivist reformers is to humanize the mathematics education of children in their view of mathematics as a human activity.*

However, the constructivist approach encompasses more than just the humanistic ideals as it encompasses a more comprehensive view of the teaching of mathematics. As Noddings (1990:2) aptly exclaims:

> *Constructivism has almost become a battle cry for a reconsideration of our problem and our best road towards solution.*

The learning of mathematics is not a passive reception of facts but rather a complexity of thinking activities, communication of ideas, execution of procedures, reflecting on methods selected and exchanging of results. In other words, constructivism is each individual's process of thinking and developing knowledge.

Ernest (1989:6) describes how mathematics education in Britain is becoming more involved in the processes of learning mathematics. He views constructivism as the natural progression of research within cognitive psychology and mathematics teaching as the beneficiary of this research.

Ernest (1989:7) comments that:

> *Research in mathematics education continues to deliver new knowledge and exciting new perspectives from which to view and understand mathematics classroom teaching. Mathematics education is becoming increasingly accepted as an important area of knowledge in its own right* (Ernest 1989:7).

A constructivist approach may be viewed at two levels which are best understood as described by Kilpatrick (1987:2).

1. Knowledge is actively constructed by the cognizing subject, not passively received from the environment.

2. Coming to know is an adaptive process that organises one's experimential world; it does not discover an independent, pre-existing world outside the mind of the knower.

The first hypothesis is becoming generally accepted by mathematics educators as a useful and productive approach to reform in instructional methods. However, the second hypothesis is more problematic for teachers because of its total disregard for objectivity. This second level of hypothesis has been called radical constructivism and an explanation of this term is necessary at this time.

4.1.1 **Radical constructivism**

Radical constructivism is a theoretical conviction that knowledge cannot be simply transferred from parent to child or teacher to student but has to be actively built up by each learner in his or her own mind (Von Glasersfeld, 1991:xiii). The radical constructivist has taken seriously the works of cognitive psychologists, in particular Piaget. They believe that the results of our cognitive efforts help us cope in the world of our experience, rather than the traditional view that an objective representation of the world is presented by the teacher or parent and is accepted by the student.

Von Glasersfeld (1991:xv) explains that the term "radical" is used because this is a break from the traditional theory of knowledge and it has profound consequences for parents, teachers and researchers. The objectives now become embedded in generating particular ways of acting and thinking in children and students.

This radical constructivism is more orientated towards the philosophical ideals of constructivism than towards any pragmatic theory of teaching methodology. Schoenfeld (1992:290) criticises this philosophical orientation as presenting a total disregard for content matter for the idealistic views of non-objectivism. In criticising Von Glasersfeld's viewpoint, Schoenfeld asks the question:

> *How and what does one teach when the core of instruction, the subject matter, is no longer postulated to exist as an objective entity?* (Schoenfeld 1992:290).

However, Zietsman (1994:564) points out that radical constructivism is simply an epistemological position postulated by people such as Von Glasersfeld (1993) and she views this as "ground zero" of a constructivist approach. She argues that an epistemology does not require the philosopher to stipulate teaching strategies but rather lay the base line for educators to build a theory of teaching methodology.

Candy (1991:262) concurs with this explanation and says that:

> *It is rare to find theorists of learning and teaching who state their epistemologies explicitly.*

The theory of knowledge usually infers a formulation of teaching and learning methodologies based on a particular epistemological position.

Zietsman (1994:559) feels that constructivism may be suffering because the community want a clear cut, fully developed constructivist teaching "package" to be delivered. This is a valid argument and it is for this reason that this study concentrates on a constructivist teaching approach whilst accepting that the epistemological basis of this approach may be termed radical constructivism.

Von Glasersfeld (1993) himself is quoted in Zietsman (1994:569) as stating:

> *A constructivist attitude changes, or ought to change the attitudes of the teacher radically. And among those changes it has to be considered that a constructivist theory of knowing cannot tell the teacher what to do but it can tell teachers an awful lot of things that are not to be done, that are bound to be counter-productive.*

Thus, Zietsman (1994:566) points out that to proceed as a teacher the following aspects of the epistemology of constructivism determine what should be done in the classroom.

1. The teacher's view of the learner.
2. The teacher's view of how to develop instruction.
3. The teacher's view of how the classroom should be organised.

Constructivism, therefore, provides the "ground zero" or baseline for a constructivist teaching approach that emphasises a process-orientated approach (see 3.2.4) which is learner-centred. This view of constructivism is social constructivism.

4.1.2 <u>Social constructivism</u>

Social constructivism views mathematics as a human activity involving problem solving and problem posing, and hence is an activity which is accessible to all (Glencross, 1991:92). Penchaliah (1994:367) points out that social constructivism is an adoption of Vygotsky's view of the importance of learning within the context of social interaction and the crucial nature of human discourse.

It is clear that social constructivists accept the philosophy of constructivism but provide the pragmatic guidelines for teaching methodology. Niewoudt (1991:165) summarises this view of social constructivism and mathematics teaching and provides three critical guidelines.

1. Teachers and pupils are active meaning makers who continually give contextually based meanings to each others words and actions in the ongoing interaction between them.

2. The teaching process is one of negotiation of ideas rather than by the imposition of facts.

3. The goal of the teacher is to facilitate learning or teach how to learn rather than learning itself.

This social constructivist view of mathematics offers a number of common links with the aspects of teaching that are concerns of mathematics anxiety (see 1.1.3 and 3.4.1). These aspects are classroom environment, understanding, self-confidence, communication, social interaction and problem solving and will be addressed fully in Section 4.3 as the critical factors in the constructivist view of a child's mathematical development.

Stoker (1991:173) emphasises social interaction as the basis of children constructing their own meaning and describes the teaching process as follows:

> *The constructivist classroom is where children are involved not only in invention and investigation but in social interaction involving explanation, negotiation, sharing of ideas and focused discussion.*

Murray & Human (1990:347) agree with this emphasis on social interaction and view social constructivism as a "didactical contract" between teacher and pupils which governs overt behaviour and attitudes. This "didactical contract" requires pupils to solve problems independently and to be able to explain clearly how they went about it. Murray & Human (1990:348) describe the terms of the "didactical contract" as follows:

1. The objective of the learner is not to conform to any particular rules but rather to produce the information required by the task in the most effective way at his disposal.

2. The learner develops a strong desire not to be shown by the teacher.

3. Learners need to express themselves verbally or in writing and may use the teacher as consultant and adjudicator.

4. Communication is not a solo effort. It requires a certain adherence to the conventions operating in a community (e.g. they may write 100 and 3 instead of 103, but the convention of society is 103 and they must use this to be understood).

5. The teacher does not express personal preferences but rather clarifies why certain phrases or notations are ineffective as a means of communication. The use of correct communication is not to phase the teacher but rather to be clearly understood.

6. The pupils become self-reliant or achieve intellectual autonomy with respect to the execution of tasks and the communication of methods used to perform tasks.

Murray & Human (1990:347) contend that self-reliance is the primary goal of constructivist teaching and in fact believe that it is more than a primary goal because it is a pre-requisite for effective learning by construction. They report that one of the outcomes of the introduction of the socio-constructivist approach in primary schools in South Africa is this development of self-reliance which they describe as:

> *An absolute pre-requisite for attainment of the rate of progress, level of understanding, problem solving ability, level of thinking about numbers and computations, lack of anxiety and personal satisfaction.*

Thus, the social constructivist paradigm provides the basis of a teaching methodology which not only develops understanding and skills in mathematics but also addresses the affective domain in terms of self-concept in providing self-confidence and an alleviation of mathematics anxiety.

Social constructivism provides the tenets of a methodological approach to the teaching of mathematics that emphasises many of the important concerns of mathematics anxiety researchers. In Section 2.2.2 the sources of mathematics anxiety were categorised as socio-cultural, emotive, cognitive and educational. Social constructivism has its roots in all four of these categories and provides a pragmatic methodology which takes cognisance of the interrelationship of these issues as they manifest themselves in the classroom.

Whilst constructivism will be discussed in some detail in this chapter it should be stated from the outset that the purpose of this study is to introduce and promote the constructivist view that knowledge is actively constructed and not passively received. In addition, the socio-constructivist viewpoint is favoured as the theoretical basis for a better understanding of how to promote constructivism and how this approach to teaching would benefit the mathematically anxious child.

4.2 **Why constructivism?**

The question most people may ask is why is there a sudden need for constructivism. The answers to this question are evident when one considers the needs of present day pupils. Pupils that are being prepared for the twenty first century.

In the previous chapter it became clear that the traditional approach to curriculum design has become obsolete and a more comprehensive view of curriculum planning is now common place. Situational analysis has provided researchers with areas in which research can provide guidelines for improvement. A more humanistic and pupil-centred approach to curriculum planning is evident and teaching techniques have now become the central concern of mathematics researchers.

4.2.1 **The evolution of constructivism**

Constructivism has its roots in several learning theories, the most prominent of which is Piaget's developmental theory. Piaget used the term *reflective abstraction* which is described by Ginsberg & Opper (1988:217) as a reflection in two ways.

> *The first consists of a projection or reflection of actions into a higher level, and the second consists of a reflection upon and reorganization, or reworking of both the projected and previous action into a new and broader understanding.*

The idea of reflection is a cornerstone of the constructivist philosophy as theorists believe that only when teachers allow their pupils to reflect upon their actions will they learn to understand and assimilate their knowledge. Piaget was less interested in studying the contents of the child's thought than the basic organisation underlying it. Piaget insists that the learner must actively participate in the work in order to develop a *construction* of reality rather than merely accepting a *copy* of reality (Dean 1982:87).

Piaget's stages of development have provided a foundation for many curriculum planners. However, it is his ideas of how one comes to know anything and the relationship between the individual and the objective world that has relevance to constructivism theory.

Lerman (1989:213) comments that much concern and disquiet has been expressed in recent years with the rigidity, appropriateness and applicability of the Piagetian stages of development. This concern is most relevant because a fixation with stages of development and a stereotype approach to teaching at different levels can detract from the real value of recognising how pupils construct knowledge and allowing them the freedom to reflect this knowledge in the classroom and other social arenas.

Another renown cognitive psychologist, Kelly, believed that a child's development was influenced more by social constructs. Kelly (1955:242) saw human development in terms of the persons individual ways of interpreting the world around him. According to Kelly's theory of human development, people analyse events in terms of similarities and differences according to their own "personal constructs".

Once again this idea of each individual having his or her own personal constructs is central to constructivist theory. Whilst Kelly was more interested in a personality theory his ideas are noted by mathematics educators. Mathematics is one of the many concepts which require a person to build up his or her own understanding. The idea that this understanding is built up from within one's own constructs suggests a need for an approach to teaching which allows for the full participation of each individual.

Cognitive psychologists have provided an early insight into how people learn mathematics because they observe that:

> *learning mathematics reflects a good deal about intellectual development* (Dean, 1982:87).

Skemp's work is probably most relevant to researchers because he is a mathematician and an educational psychologist. Skemp (1971:54) expounds the merits of reflective thinking and how each individual needs to be made aware of the value that reflection has on understanding what one is doing. Kelly's "constructs" are termed "schemas" by Skemp (1971:40). These schemas are used by each individual to integrate existing knowledge and assimilate new knowledge. Skemp (1971:40) explains that by reflection a child is considering his or her current set of schema to set up new higher order schemas.

The cognitive psychologists have had a profound effect on recent thinking in mathematical education. Greer & Mulhern (1989:24) claim that:

> *Interest of cognitive psychologists in mathematical thinking and education, and of mathematics educators in cognitive psychology, is currently intense.*

Behaviourism has lost ground to cognitive development theory and even more rigid mathematics educators will have to take note of the new awareness of mathematics as a human activity set in cultural, social and historical contexts.

> *There is a reformation of the major concerns of the philosophy of mathematics away from the logical and towards the phenomenological-historical-experimental. This takes mathematics 'down from the sky' so to speak, and says, in effect, we are thinking these thoughts, we are writing these symbols, we are doing these mathematical things, and as a result of our activity, the consequences to us are such and such* (Davis & Hersh, 1986:304).

In effect, what Davis & Hersch are saying is that it is not enough to simply analyse mathematics education reform in terms of a cognitive theory. The theory itself also suggests a philosophy of consequences, outcomes or objectives which are ideally sought after by the educationalist.

4.2.2 **Beyond the cognitive view**

Constructivism goes beyond a purely cognitive view of development. Cognitive psychology tends to de-emphasise affect and socio-cultural factors and the context in which knowledge is acquired. The logical link between the psychology of learning and the practice of instruction is now being forged.

> *The problem seemed to be in the lack of consideration of other aspects of human behaviour, of interaction with other people and with the environment, of the influence of the history of the person, or even the culture, and the lack of consideration of the special problems and issues confronting an animate organism that must service as both an individual and as a species so that intellectual functioning might perhaps be placed in a proper perspective* (Norman, 1981:266).

In general, constructivism embraces a more balanced perspective of intellectual functioning which may have had its foundations in cognitive psychology but is now more concerned with the happenings in the classroom. Greer & Mulhern (1989:24) comment that:

> *Research on learning has moved from the laboratory to the classroom; among the many facets of such research are:*
>
> 1. *Teaching experiments, in which idea and theories are put directly to the test by basing actual instruction on them and evaluating the outcomes.*
>
> 2. *Analysis of the classroom as a social system. What are the rules (mostly implicit) which organise relations between pupils, teacher and subject matter?*
>
> 3. *Study of teachers themselves - not only their style of teaching, but also their beliefs about the nature of instruction, of children's learning and of the subject being taught.*

In Kwa-Zulu Natal, South Africa, this type of thinking amongst educationalists is already evident as far as a commitment to constructivist learning in mathematics is concerned. This theory was introduced experimentally into several schools and the outcomes proved to be positive. Teachers who were briefed on the theory and who adapted to this particular teaching style were also very enthusiastic about the results. The constructivist approach was then introduced to all primary schools and at present has reached the Standard Five level of senior primary mathematics teaching.

Thus educational research in general is moving towards a more humanistic approach of pupil-centred concerns. There is no longer a laboratory attitude towards the study of the learning process. In addition there is a new emphasis on the social context of learning in the classroom and a concern that the:

> *Day-to-day rituals and interaction that take place in mathematics classes define what it is to do mathematics* (Schoenfeld, 1987:37).

In other words teachers have to provide the classroom atmosphere which facilitates the correct interpretation of learning mathematics. This is no easy task as the teacher is not only responsible for organising the content material but also ensuring that each individual receives the necessary attention to provide a positive, non-threatening environment to express their mathematical thoughts.

Constructivism is a process of teaching which addresses all the needs of the modern day pupil. Before analysing the various facets of constructivist teaching and learning it is essential that one considers some of the needs of the present pupil who will be joining the work force in the twenty first century.

4.2.3 <u>The need for constructivism</u>

The need for constructivism has arisen from many research findings and theoretical postulations. In Britain, the "Cockcroft Report" (Cockcroft 1982) made strong recommendations for change in the teaching of mathematics. In the USA, the Commission on Standards for School Mathematics (1987) provided a comprehensive and unique approach to curriculum reform which emphasised instructional method. In South Africa, there is a general recognition that present teaching methods were not providing the type of mathematical thinking required outside school (N.E.C.C. Mathematics Commission, 1993).

The "Cockcroft Report" (Cockcroft 1982) was perhaps the forerunner in recommending comprehensive reform in the United Kingdom. The recommendation of W.H. Cockcroft and his team were widely publicised and mathematics educators all over the world were able to draw parallels with the situation in their own countries.

A number of themes arose from the Cockcroft Report which are themes adopted by the constructivist theorists (see 4.3) and also by those concerned with mathematics anxiety (see 2.3). The following aspects are the needs for mathematics education which the Cockcroft Committee emphasises throughout its report.

1. A need for less emphasis on algorithms and a rejection of a narrow concentration on computational skills.

2. A need for changes in classroom practice.

3. A need for teachers to relate mathematics more to the real world.

4. A need to revise content of the mathematics syllabus to reflect developments in technology and industry.

5. A need for curriculum reform with an emphasis to "bottom-up" reconstruction, i.e. reform starts with teachers in the classroom providing initial direction.

6. A need for reform of examinations and assessment methods.

All these aspects will be discussed in Section 4.3 as the mathematical development needs of the child. However, to continue on the theme for needs of constructivism it is interesting to note some parallels expressed by the Commission on Standards for School Mathematics (1989). In summary the curriculum and evaluation standards for school mathematics produced by the National Council for Teachers of Mathematics provides very similar recommendations to the Cockcroft Report.

1. A need for instruction to persistently emphasise "doing" rather than "knowing that".

2. A need for greater focus on problem solving skills and open-ended questioning.

3. A need for basic skills in mathematics to include more than algorithms and computational skills.

4. The standards represent a "bottom-up" recommendation for curriculum reform because they were compiled by teachers.

5. A need for assessment to include a wider range of measures than conventional testing.

6. A provision for changes in classroom instruction. The standards provide examples for each level of instruction.

The KwaZulu Natal Education Department's Subject Committee for Mathematics (1991) formulated principles which also echo a number of important needs listed above.

1. The need to emphasise the value and usefulness of mathematics.

2. A need to emphasise the importance of mathematics as an essential element of communication in the modern society.

3. The pupil learning mathematics is conceptualised as an active mathematical thinker who tries to construct meaning of what he is doing on the basis of personal experience and who is developing his way of thinking as his experience broadens, always building on the knowledge which he has already constructed.

4. The need for the curriculum to provide mathematical processes by which they are actively and productively involved in learning.

5. The need for problem solving to be the central focus of teaching and learning mathematics. Not only is the ability to solve problems a major reason for studying mathematics, but problem solving provides a context for learning and doing mathematics.

Glatthorn (1988:9) summarizes the needs of a renewed mathematics curriculum for the twenty first century as follows:

1. Understand the language, notation and deductive nature of mathematics.

2. Understand the basic concepts and skills of algebra, geometry and functions.

3. Develop problem solving skills and use those skills to solve mathematical problems, including real-life problems.

4. Use the calculator and computer in solving problems.

5. Communicate mathematically by learning how to use mathematical symbols and representations.

6. Reason mathematically, understanding the nature and limitations of statistical reasoning.

7. Understand the essential mathematical processes of generalizing or abstracting, hypothesizing or conjecturing, proving and applying.

8. Know how to estimate and make approximations and be able to judge the reasonableness of a result.

The range of recommendations listed above express some theoretical postulations and, especially in the case of Glatthorn (1988:9), some very specific tasks for mathematics educators. In analysing the need for constructivism one must be aware of needs expressed universally and decide whether or not a constructivist approach satisfies these needs (see 4.3). In this thesis the added task of providing a curriculum recommendation which would address the problems of the mathematically anxious is also of central concern.

The next task is then to highlight the specific areas which have been listed in this section and have a prominent place in constructivist theory. It is generally agreed that the mathematical development of a present day pupil needs to be reviewed. As this is the case it is important that mathematics educators review their ideas on the learning and teaching of mathematics. A constructivist approach provides the essential ingredients in the areas which have been prominent throughout this section. What follows is a closer investigation of the essential tenets of constructivism discussed under the following headings:

1. A constructivist view of mathematical development
2. Reflection and constructivism
3. Representation and constructivism

4. Implementing a constructivist approach in the classroom
5. Constructivism and mathematics anxiety

4.3 <u>A constructivist view of mathematical development</u>

Clements & Battista (1990:34) summarise the constructivist view of learning as follows:

1. Knowledge is actively created by the child, not passively received from the environment.

2. Children create new mathematical knowledge by reflecting on their physical and mental actions.

3. Ideas are constructed or made meaningful when children integrate them into their existing structures of knowledge.

4. There is no one true reality, only individual interpretations of the world. (The radical view)

5. Interpretations are shaped by experience and social interactions. Thus, learning mathematics is a process of adapting to and organising one's quantitive world, not discovering pre-existing ideas imposed by others.

6. Learning is a social process in which children grow into the intellectual life of those around them and mathematical ideas and truths, both in use and in meaning, are co-operatively established by the members of a culture.

The above tenets of the constructivist view of learning have a number of implications for teachers of mathematics which need to be clarified.

The old algorithms of mathematics no longer form the cornerstone of mathematics teaching. Rules, regulations and rigidity are contrary to the needs of a flexible, creative thinker. Mathematics teaching must adapt to the times of a new technological age in industry and private enterprise.

The question thus arises, what do we need to teach pupils? Constructivist answers to this question adopt the stance that the adult can never predict what a child will learn and that adults should not presume that they necessarily understand our way of reasoning. Children will only learn what they understand from a situation using their own points of reference. As mentioned before the constructivist approach could be described as a humanistic, pupil-centred approach which calls for a problem-centred teaching methodology (see 4.1) to develop the mathematical power of each pupil. Mathematical power may be defined as the confidence in each individual in his or her use of mathematics.

The needs of the mathematically anxious child were categorised under headings in Section 1.1.3 and this theme was further developed as implications for the teaching of mathematics in Section 2.3 and as critical variables of a mathematics curriculum design in Section 3.4.1.

For convenience and easier cross-reference these needs may also be categorised as essential aspects of the constructivist approach to the teaching of mathematics. In this way it becomes clear that the constructivist approach to mathematics development incorporates all the needs of the mathematically anxious student. In essence, the constructivist approach emphasises the following needs for the mathematical development of the child.

1. Classroom environment
2. Understanding
3. Communication
4. Social interaction
5. Intellectual autonomy and self-confidence
6. Problem-solving skills

4.3.1 <u>Classroom environment</u>

As stated above it is the intention of this thesis to describe a constructivist approach to teaching mathematics with an equal emphasis on classroom atmosphere and classroom activities. The obvious reason for this approach is the concern of the author for the affective variable of mathematics anxiety. However, it is also the conviction that the constructivist theory should not be totally engulfed in problem-centred activities without considering the socio-cultural, emotive and affective elements which compose the classroom atmosphere. Teachers have to ensure that the following aspects of their classroom environment are considered:

1. Non-threatening
2. Social-interaction/communication

A constructivist approach by the teacher requires uninhibited responses from the pupil. It is important that teachers are equipped to break the cycle of poor attitudes and anxieties and are eager and able to provide their pupils with positive experiences in the learning of mathematics.

It is necessary for all teachers to develop a sense of empathy for their pupils. Comments such as "you should know this" or "this is a very easy section" should be avoided. Mathematically anxious pupils are especially sensitive to this insinuated criticism and are unlikely to respond positively in fear of being found lacking in the task at hand.

The mathematics teacher also needs to de-emphasise the rigidity of right and wrongness in mathematics. The idea of pupils reflecting their methods to the class and teacher, helps alleviate this rigidity if the teacher carefully interprets each students response without a negative reaction.

Without overstressing the point it is sometimes better to acknowledge that some mathematical thinking is difficult and requires more time and thought. However, when difficulty is acknowledged it should be communicated with a faith in the learner's ability to master the task.

An insensitive teacher can be detrimental to the constructivist ideals in the classroom. The following should be avoided:

1. Making condescending remarks.
2. Using mathematics as a punishment.
3. Refusing to answer legitimate questions.
4. Refusing assistance when necessary.
5. Creating humiliating experiences for pupils.

6. Setting unreasonable tasks.

It is essential that pupils are provided with the supportive climate in which taking risks is perceived as acceptable. It should be emphasised that although some mathematics can be purely mechanical and computational, it is more important to develop inventive approaches from within the pupils' own constructs. Educated guessing, estimation, creative and unique methods and perseverance should all be supported in the classroom. Positive reinforcement should be used regularly to encourage pupils to investigate their personal inventive ideas.

The constructivist approach does not only emphasise an improved mathematical knowledge but a more comprehensive development of the child as a productive human being. It is worth discussing each of the above aspects of mathematical development and emphasise their importance in the teaching approach.

4.3.2 **Understanding**

Understanding mathematics must be the objective of any theory of knowledge. Constructivism is no different but obviously the ideas and theories of the constructivists differ somewhat from other theories. Putman, Lampert & Peterson (1989:89) summarised what it is to know and understand mathematics as follows:

1. Understanding as representation. This means internalizing symbols and systems for representing mathematical ideas and being able to use them fluently.

2. Understanding as knowledge structure. The view that each individual has the capacity to construct or acquire appropriate knowledge structures.

3. Understanding as connections among types of knowledge. The connections between conceptual and procedural knowledge and the connections between formal (in-school) and informal (out-of-school) knowledge.

4. Learning as the active constructions of knowledge. Active reorganisation of one's cognitive structures and integrating new information with existing structures.

5. Understanding as situated cognition. All knowledge is considered

to be interactively situated in physical and social contexts.

This summary of understanding contains the essential elements emphasised by leading researchers and psychologists. Kelly (1955:242) was one of the earliest and most renown researchers to postulate a personal construct theory. The idea that people build up their own understanding which acquires a personal meaning has become a cornerstone of constructivist theory. In addition, Kelly was mainly interested in inter-personal relationships which also shape each individual's understanding and relates to the social-interaction aspect of constructivist theory.

Ginsburg & Opper (1988:217) and Entwhistle (1983:167) describe Piaget's theory of cognitive development in similar terms to Phillips (1973:19) who states that:

> *Piaget conceives of intellectual development as a continual process of organisation and reorganisation of structures, each new organisation integrating the previous one into itself.*

Whilst the concept of learning by developing one's own constructs is fundamental to constructivist theory it cannot be accepted as such a rigid and orderly process as predicted by Kelly and especially Piaget. The idea of compartmentalising stages of understanding may or may not be accepted. This is not an issue in constructivism. All teachers will be aware that their pupils do reach certain levels of development and then continue to the next, but for the constructivist a teacher must recognise the understanding of each individual. Constructivism requires an active role from the teacher and participation and co-operation from the pupil.

Cockcroft (1981:69) comments that:

> *Because understanding is an internal state of mind which has to be achieved individually by each pupil, it cannot be observed directly by the teacher.*

Constructivism is a pupil-centred approach which aims at providing each individual with "mathematics power". Mathematics power for the individual means that a person has the experience and understanding to participate constructively in society.

Defining understanding is not as important as providing understanding and this provision is evident in the classroom and in the social-interaction opportunities of the classroom.

Davis et al (1990:3) describe the constructivist view of learning when they state that:

> *It is assumed that learners have to construct their own knowledge - individually and collectively. Each learner has a tool kit of conceptions and skills with which he or she must construct knowledge to solve problems presented by the environment. The role of the community - other learners and teachers - is to provide the settings, pose the challenges, and offer the support that will encourage mathematical construction.*

Understanding mathematics is not a case of analysing cognitive processes but rather ensuring the correct development of knowledge through problem-centred and socio-interactive activity.

A significant factor in the increase and retention of mathematics anxiety is the amount of rote learning taking place in mathematics. This source of mathematics anxiety was discussed in Chapter Two but has specific relevance to how pupils come to truly understand mathematics.

A constructivist approach to understanding supports a learning environment which provides a variety of experiences which allow the pupils to develop their problem solving skills and abstract underlying rules and theory for themselves. Memorization that follows from this insight will be more clearly understood and retained (see 4.3.6).

4.3.3 <u>Communication</u>

Communication is an essential ingredient for the classroom and in everyday life. It is also necessary if one is to learn mathematics with understanding and meaning.

To learn to communicate mathematically implies that one understands the medium of communication, i.e. the mathematical domain.

To think or talk about mathematics is part of the constructivist strategy because it requires students to express their ideas in some mathematical way. There are five common ways in which mathematics ideas are represented: spoken or written language, concrete objects, pictures, real-life situations and written symbols (Cooney & Hirsch, 1990:32). All these representations are useful and may be used when the context is most appropriate.

When pupils are given the opportunity to talk about their mathematical understanding, problems of genuine communication arise. By learning to cope with these problems and by dealing with the mathematical task at hand, provides the opportunities for learning to understand mathematics.

In a problem-centred environment the teacher is a facilitator helping pupils to clarify their explanations and assisting them to verbalize their thinking and encourage them to present alternative solutions. In order for children to feel confident in showing their mathematical thinking they must actively attempt to communicate with each other and with the teacher. Successful communication allows the child to negotiate his or her meanings and it depends on all members of the class expressing genuine respect and support for one another's ideas (Commission on Standards for School Mathematics, 1989:29).

Verbalising and communicating a possible solution to a problem ensures that one is at ease with the solution and meaning one has ascribed to the problem. Presenting solution methods, inventions and insights provides the ideal setting for students to learn from each other.

Wheatley (1991:16) believes that communication is an essential part of problem-centred learning. He believes there are three components which ensure problem-centred learning: tasks, co-operative groups and sharing. The teacher needs to communicate the task or ask students to frame their own tasks. The organisation of groups should stimulate discussion amongst students and create opportunities to challenge ideas and perhaps reconceptualize their thoughts. Sharing provides the following:

> *A forum for students to construct explanations of their reasoning. In the process of telling others how they thought about a problem, students elaborate and refine their thinking and deepen their understanding* (Wheatley, 1991:19).

Co-operative learning in groups and the sharing of ideas with the whole class allow the teacher to be the facilitator and not the authority figure. Intellectual autonomy is bestowed on the student and he or she must converse mathematically with other students. Noddings & Shore (1984:112) believe that discussion transforms intuitions into public products and this public discussion provides the learning environment for constructive understanding.

In essence the above theorists provide a description of the constructivist approach which assumes a dynamic view of the classroom environment in which pupils are actively engaged in developing their mathematics by exploration and discussion. The act of communicating clarifies thinking and forces students to engage in doing mathematics and understanding what they are doing.

4.3.4 Social interaction

The classroom environment, under the conditions recommended by constructivists, becomes a hive of important social interaction. This social interaction provides a facility for teachers to understand their students' mathematical knowledge. Teachers become aware of how their students think mathematically, how they express mathematical ideas and how they interpret the mathematical expressions of others (Commission on Standards for School Mathematics, 1989:214).

Communication is a social activity and the teacher needs to be aware of the climate that is set in the classroom.

> *When children learn mathematics in school, they do so in a classroom*

> *where certain standards of conduct are established either explicitly or implicitly. These standards, or norms, influence the way children interact with the teacher and with each other, which in turn influences both what mathematics the children learn and how they learn it* (Cooney & Hirsch, 1990:12).

In the constructivist approach the teacher provides ample opportunities for discussion and social interaction. This may take the form of pupils being encouraged to work co-operatively or to listen to one another's explanations. In this classroom environment their are two types of learning activities taking place. One is learning to solve the mathematical problem and the other is learning to work productively with one's peers.

When a child is prepared to verbalize his or her mathematical meaning of a concept it is often an indication that they understand the concept fully themselves. Once pupils accept a social co-operation, unique learning opportunities arise. These include opportunities for verbalizing their thinking, explaining and justifying their solutions and asking for clarifications (Cooney & Hirsch, 1990:19).

Cobb, Yackel & Wood (1992:17) state that:

> *The constructivist's challenge is to explain how students construct their mathematical ways of knowing as they interact with others in the course of their mathematical acculturation.*

The important issue is that mathematics must not be viewed as a prestructured environment but rather a dynamic learning environment in which teachers and students mutually influence and adapt to each other's mathematical activity as they interact in instructional situations.

The importance of social interaction is at the forefront of constructivist theory and rightly so. Learning mathematics begins at home and need not be any different from other learning. Children begin constructing this personal view of the world and mathematics is an essential part of this world. Bauersfeld (1980:25) says that:

> *Teaching and learning mathematics is realized through human interaction.*

It is a kind of mutual influencing, an interdependence of the actions of both teacher and student on many levels. It is not a unilateral sender-receiver relation. The student's reconstruction of meaning is a construction via social negotiation about what is meant and about which performance of meaning gets the teacher's (or the peer's) sanction.

This sharing of ideas and having one's own ideas accepted by others is a mirror of real life learning. Social interaction in the classroom allows for this inter-subjectivity and possible differences of opinion are discussed and agreements are reached when concepts are mutually agreed to and understood by all concerned. Gergen (1982:270) summarises the constructivist viewpoint when he says:

Knowledge is not something people possess in their heads, but rather something people do together.

The classroom situation must therefore reflect this dynamic view of how people acquire knowledge. Teachers must give ample opportunities for their students to express their points of view both in group settings and in the classroom as a whole. Gadanidis (1994:93) believes that a constructivist teacher's emphasis should be on creating learning environments that help students create good schemas of mathematics understanding. The dynamics of the classroom should be a microcosm of society and reflect the interaction commonplace in social discourse.

A mutual trust is built up between teacher and student in this type of classroom environment and the students learn a lot more than just mathematics. They develop beliefs and attitudes about mathematics which provide a self-confidence in their mathematical ability and a positive form of motivation. Most importantly, perhaps, is that the constructivist approach aims at providing the scholar with intellectual autonomy which is needed in everyday life.

Whilst acknowledging individual input and performance in the classroom the teacher must be fully aware that constructivism also takes account of mathematical learning as an interactive constructive activity.

Opportunities to construct mathematical knowledge will often arise from the following social interaction:

1. Attempts to resolve conflicting points of view between two or more pupils.

2. Attempts to reconstruct and verbalize a mathematical idea or solution to a second party.

3. To construct a consensual domain within which to co-ordinate mathematical activity with that of others (Cobb *et al*, 1991:6).

The social interaction perspective of constructivism is seen as the catalyst for autonomous individual cognitive development (Cobb *et al*, 1991:6).

Students need to actively participate in the classroom community's discussions and negotiations. The teacher is part of this community and will facilitate individual learning by helping students share and mutually construct mathematical interpretations and understandings.

Positive student attitudes can be developed from a student's sense of being part of a mutual support system which can encourage curiosity, experimentation and intuitive thinking. Cobb *et al* (1991:6) emphasise three important factors pertaining to the constructivist's approach to the classroom atmosphere.

1. Mathematics at school is typically associated with regulations or arbitrary rules and conventions. To change this, a development of inquiry mathematics needs to replace traditional teaching. The classroom must provide frequent opportunity for pupils to discuss, critique, explain and when necessary justify their interpretation and solutions.

2. The teacher's role must not be one where he or she cues students until they can act as though they have learned what the teacher had in mind. The teacher should rather act in a role of initiating and guiding mathematical negotiations by:

   (a) highlighting conflicts between alternative interpretations or solutions;

   (b) helping students develop productive small-group collaborative relationships;

(c) facilitating mathematical dialogue between students;

(d) implicitly legitimizing selected aspects of contributions to a discussion in light of their fruitfulness for further mathematical constructives;

(e) redescribing student's explanations in more sophisticated terms that are nonetheless comprehensible to students;

(f) guiding the development of interpretations when particular representational systems are established.

3. Classroom social norms are crucial to the constitution of an inquiry mathematics tradition. Social norms that enable pupils to engage in small-group work without constant teacher monitoring is essential to the collaborative learning approach. This small-group work must include the following:

    (a) perseverance in solving personally challenging problems;

    (b) explaining personal solutions to others;

    (c) listening to and trying to make sense of another pupil's explanation;

    (d) attempting to achieve consensus about an answer or interpretations.

Schoenfeld (1987:205) refers to the classroom atmosphere required as creating a microcosm of mathematical culture. It could also be argued that this social interaction in the classroom is in fact a microcosm of the future real world that the students will eventually encounter.

Cobb *et al* (1991:8) view the product of the whole constructive process as follows:

> *In the process, students come to view mathematics as an activity in which they are obliged to resolve problematic situations by constructing personally meaningful/justifiable solutions as they actively contribute to the interactive constitution of an inquiry mathematics tradition.*

Cobb *et al* (1991:8) believe that the strength of the constructivist theory lies more in the social-interaction perspective. The teachers form an essential ingredient of this social microcosm and it is essential that they have reason and motivation to want to reorganise their pedagogical practice.

It is a fundamental fact that without the support of the teacher the constructivist approach will never be a success. Teachers need to realise the pro-active nature of their role in initiating and guiding the re-negotiation of classroom social norms before they begin to use the instructional activities proposed by constructivists.

Brousseau (1984:112) describes the typical mathematics teacher's dilemma as follows:

> *The more explicit I am about the behaviour I wish my students to display, the more likely it is that they will display that behaviour without recourse to the understanding which the behaviour is meant to indicate: that is, the more likely they will take the form for the substance.*

In other words teachers are continuously under pressure to fulfil their obligations to completing a syllabus and will often be tempted to revert to behaviour which leads to the excessive teaching of algorithms and less towards inquiry mathematics.

A belief by teachers in the term socio-constructivism will help them to develop a teaching style compatible with the constructivist theory. The classroom provides the social interaction needed for teachers to learn as well as their pupils as they re-negotiate both social norms and mathematical meanings and practices. Learning is an ongoing, long term process and the classroom experiences are a crucial source of pedagogical problems which teachers learn to analyse and help to resolve. This process is beneficial to the teacher as he or she is continually reorganizing the knowledge and beliefs about learning and teaching.

The classroom atmosphere thus becomes a learning environment which is as risk-free as possible with pupils willing to be active in the learning process and teachers attempting to initiate and guide the development of an inquiry mathematics tradition in their classrooms.

4.3.5 <u>**Intellectual autonomy and self-confidence**</u>

The constructivist approach is essentially a reaction to a teaching context which emphasises drill and repetition. This traditional technique of teachers ensuring that their students retain the knowledge of given algorithms is tedious, monotonous and has an extremely harmful effect on their pupils' confidence and intellectual autonomy.

Students need to be encouraged to use their own methods to solve problems and provide answers with their own personal explanations. Mathematical drills can rob students of the autonomy of their own methods of solving problems as well as robbing them of the confidence they need to function as people who can do mathematics independently, both in the school context and later in adult life.

Lerman (1988:412) describes a reaction that many mathematics teachers experience when he outlines a typical reaction by a student when faced with a new problem. This response is, I haven't learnt the method for that so I can't do it. This demonstrates the students reliance on the teacher and total lack of autonomy.

Intellectual autonomy and self-confidence are developed by adopting a constructivist approach to classroom teaching. The teacher is not a possessor of knowledge who conveys that knowledge to students who are viewed as the receptors of knowledge. Lerman (1988:412) describes the teacher as:

> *A more experienced mathematician than the students, guiding them in the development of processes, way of thought and techniques appropriate to doing mathematics.*

Lerman (1988:413) continues his description of the teaching and learning environment in mathematics by using cooking as an analogy. This simple analogy provides food for thought when one considers the similarities with a modern approach to learning strategies. Lerman (1988:413) describes the learning to cook situation as follows:

> *I am not discouraged from trying any recipe when cooking, perhaps because never having been taught to cook, I am not aware of what is*

difficult or what should be beyond my experience. When I have failures I learn from them and I have of course also learned from watching or questioning others, drawing on their experience.

This learning sequence is common to many real life situations and yet it is often rejected in the mathematics classroom. Pupils need to be reminded that this is how they learn and that knowledge does not come wrapped in neatly organised parcels.

Supporters of a more rigid approach to teaching mathematics may argue that this gives a student confidence as they follow set rules. This may be true when mathematics is always presented in a given way and when manufactured problems never differ from those drilled in the classroom. However, once a variety of examples and problems are encountered the student will struggle and thus lose confidence. This variety of problems is inevitable in everyday life and it is essential that students are equipped with the ability and confidence to tackle new problems each day.

The anxiety of a mathematics student may be temporarily alleviated by presenting mathematics as a repetitive set of procedures but this will only lead to greater anxiety at a later stage. Elementary mathematics may be presented as a series of routines which can be fairly easily memorised. However, the number of routines to be learnt eventually becomes overbearing and these routines are difficult to adapt to other problems based on the same mathematical ideas. When the apparent success of early years is no longer evident the pupil becomes increasingly anxious as the learnt methods become less effective (see 3.4.1.2).

For the constructivist teacher, doing the right thing is not enough to be competent. One must know what one is doing and why it is an acceptable method. This is the road to competence in mathematics and a development of self-confidence. The problem of lack of confidence often only becomes evident in later years of schooling. During the earlier years a student memorises what he or she has to do and the marks appear satisfactory. When in secondary school the work load increases and the nature of the problems are such that memorization is no longer adequate, the trusted method is of very limited use and thus the self-confidence level drops dramatically.

Constructivist teachers believe that this syndrome must not be allowed to begin. Students should be encouraged to develop their mathematical autonomy from an early age. This can be achieved by presenting mathematics as an exciting adventure of experiments, exploration and creativity in which everyone can participate. The teacher influences the attitudes of his or her students by providing the correct learning environment.

Teaching understanding and problem solving skills in a collaborative environment in which cognitive and social considerations are understood is the platform required for building a confidence and autonomy for each student. This self-confidence and intellectual autonomy is essential for their positive progression in the work-place and in general adult life.

4.3.6 **Problem solving skills**

Problem solving skills from the constructivists perspective is not purely a strategy on how to approach specific problems. It is better described by Schmalz (1989:685) who refers to the idea as a problem solving attitude.

Problem solving is not just a case of doing an exercise of word problems in the classroom. Cockcroft (1982:94) specifically warns of the dangers of a strategy type of approach to problem solving and he presents the constructivist viewpoint on this matter quite succinctly when he states:

> *When young children first come to school, much of their mathematics is "doing". They explore the mathematical situations which they encounter - perhaps sorting objects into different categories or fitting shapes together - and come to their own conclusions. At this stage their mathematical thinking may reach a high level of independence. As they grow older this independence thinking needs to continue; it should not give way to a method of learning which is based on the assimilation of received mathematical knowledge and whose set truth is "this is the way I was told to do it."*

The Commission on Standards for School Mathematics (1989:214), Cockcroft (1982:94) and Schmalz (1989:686) all refer to various aspects of a problem solving attitude which are important to the constructivist theory.

These aspects all refer to a student's individual mathematics disposition. Mathematics educators will know that the following aspects are critical to the teaching of mathematics.

1. Confidence in using mathematics as a tool to communicate, to reason and to solve problems.

2. Flexibility in exploring mathematical ideas and trying alternative methods in the solution of a problem.

3. Willingness to persevere at a mathematical task.

4. Interest, curiosity and inventiveness regarding mathematics.

5. Tendency to monitor and reflect upon their own thinking and performance. (Schmalz, 1989:686)

This approach to problem solving is clearly a plea to the teacher to provide the correct teaching style and a suitable classroom atmosphere. It is not another set of rules for solving problems but rather an attitude or mind set which encourages an open-minded approach to problems.

Behaviouristic influences have caused school learning to become rule orientated and a memorising of facts and procedures. Wheatley (1991:15) states that:

> *Favourable conditions for learning exist when a person is faced with a task for which no known procedure is available. That is, when the learner finds herself in a problematic situation.*

At a very simple level it is important that teachers allow their pupils to explore ways to add $15+49$. To a young pupil this is a task for which no set procedure has been taught. This type of approach could be used through to the upper levels of secondary school by posing questions on how to discover a formula for adding terms of a number sequence.

Wheatley (1991:15) discounts claims that a set of facts and skills must be routinely practised before problem solving can take place. He believes that children who face the problems described above will develop their own meaningful procedures and,

> *In the process build meanings which provide the foundation for rapid advancement in mathematics learning* (Cobb & Wheatley, 1988:26).

Mathematics taught from this problem-centred perspective will have considerable benefits for students. However, the problem of persuading students to use their own initiative must be addressed from an early age and continued throughout schooling. It is all too true that in the majority of classrooms today students are more ego involved than task orientated (Wheatley, 1991:15). Students in a direct instruction classroom environment use their skills to imitate the teacher's procedures to ensure that they please the teacher and obtain good marks.

In contrast, a problem-centred environment provides more intrinsic motivations which they will value in later years. Solving problems becomes a natural phenomenon allowing students to be task orientated and make use of their natural instincts to focus on learning for its own sake.

In South Africa an examination orientated society and an overfull syllabus mitigate against these idealistic classroom situations (see 3.3.3.3). Whilst accepting its importance one must be careful not to overemphasise the problem solving component of the constructivist approach as this may be an area where teachers find it difficult to match practice with theory. For this reason the attention in this thesis is rather biased towards the classroom atmosphere, the social aspects of constructivism and the importance of communication. However, the need to improve problem solving skills remains an essential goal that must be realised through a curriculum approach which takes cognisance of the need for allocating teachers more time to provide problem solving opportunities for their classes.

4.4 **Reflection and constructivism**

The idea of reflective thinking is considered as one of the pillars of constructivism.

Schmalz (1989:685) says that an essential component of a student's problem solving attitude is the tendency to monitor and reflect upon their own thinking and performance.

Computational strategies vary considerably amongst pupils and are only revealed by individual reflection. Reflecting on routine procedures can reveal various different approaches which may all be acceptable.

A key to teachers understanding the reflective process of their students may be an in-service course or part of teacher training which involves teachers or prospective teachers in their own reflective situations. Duckworth (1984:2) describes a reflection and constructing exercise in which a group of in-service teachers were engaged as learners in subject areas with which they were less comfortable. The intent was to help teachers pay attention to themselves as learners. Duckworth (1984:4) reported that:

> *The active involvement of these teachers as in a learning situation with peers helped them reflect on their own understandings and let them be more aware of the complexities involved in the learning process.*

This type of activity assists teachers to understand the key elements that they need to bring to the classroom situation. In the constructivist approach the two key elements are social interaction and reflection.

Teachers will be encouraged to talk about their own experiences, how they react to new problem solving situations and how they learn mathematics. In this way students will learn to communicate their thoughts and reflect on how mathematics is done.

> *In Mathematics the reflective process, wherein a construct becomes the object of scrutiny itself is essential* (Davis *et al*, 1990:109).

This reflective process requires the use of mathematics as a language to describe some mathematical action. The very act of a student formulating an expression of his or her views promotes reflection which often leads to revision and a better understanding of a concept.

Reflection will help students review their solutions and modify their position if necessary. Students formulate their communicative acts in order to be understood. When these ideas are not shared, discrepancies in individual interpretations become apparent. Negotiation then takes place and the students modify their individual interpretations and understanding (Cobb, Yackel & Wood, 1992:17).

Listening to a student's thoughts also has great benefit for the teacher because he or she gains immediate insight into the understanding of the child. By adopting the role of a listener the teacher can identify faulty reasoning and thus facilitate in guiding the student in reviewing his or her viewpoint and modifying their position.

Teachers will discover that in order to teach well, they need to be aware of their students thinking. A didactic model of the constructivist approach suggests a strategy of listening to students thinking aloud and guiding students to concentrate on the task at hand. It also means reassuring students that their reflections are acceptable, that some things are right, some are correctable and that their thinking is an acceptable process.

4.5 **Representation and constructivism**

A representational approach to teaching implies that teachers represent their knowledge with learned responses and instructional material which is familiar to them. The pupils are then expected to use these representations to develop their own knowledge.

This approach is favoured by teachers because it provides an instructional tool with which they are familiar. However, it assumes that specific mathematical meanings are inherent in external representations. Teachers may feel that using blocks or an abacus is an inventive way of introducing students to certain concepts such as "sequences and series", however, not all pupils will be able to use this representation to construct the correct internal representations. The teacher therefore ends up explaining the relationship and instructing the child on how to make sense of the material.

In essence this method is a variation of the "transmission of knowledge" or "absorption of knowledge" approach to teaching and learning. The pupil is a passive recipient of the representations which are familiar to the teacher and it is assumed that pupils will inevitably construct the correct internal representation. This view of learning contradicts the beliefs of constructivist theory for several reasons.

1. Emphasis is on the teachers interpretation of constructional materials.

2. The teacher is viewed as the expert who provides mathematical interpretations for the child.

3. Qualitative differences in pupils' understandings of concepts are not considered.

4. It ignores the belief that mathematical meanings are developed socially and culturally.

5. Mathematical meaning is formalised without consideration to the prior knowledge of the child.

6. Mathematical structures are taught and then methods of application follow.

7. Mathematical knowledge at school becomes separated from what the child knows in other settings.

8. Individual and group activity is ignored.

Cobb, Yackel & Wood (1992:2) report that investigations have shown that mathematical meaning is not inherent in external representations but rather that meanings given to these representations are a product of students' interpretive activity.

Whilst not wishing to labour over this question of representation it is important for the teacher to recognise the fact that attributing his or her interpretations of a representation is not the only way it may be perceived. This is an important tenet of all methods of teaching which purport to be constructivist.

Dearden (1967:145) describes the problem clearly when he states:

> *When a teacher presents a child with some apparatus or materials ..... he (or she) typically has in mind some one particular conception of what he (or she) presents in this way. But then the incredible assumption seems to be made that the teacher's conception of the situation somehow confers a special uniqueness on it such that the children must also quite inevitably conceive of it in this way.*

Dearden is questioning the teaching approach which assumes the teacher's mathematical construction is the only possible interpretation and the fact that a teacher using this approach may ignore the valuable input and interpretations of their students.

Clearly this representational model of learning is in stark contrast to the constructivist theory which emphasises the pupil's interpretations of the material. Whilst using instructional material to represent a mathematical idea may be viewed as innovative and inventive teaching, it is merely an adaptation of the traditional view of education.

The teacher expects these visual representations to substantiate a rigid principle based on the passive transmission or absorption of knowledge. The teacher plays the role of the expert and the pupil's unique interpretive activity is neglected. To the constructivist this is not the required model for teaching and learning mathematics.

4.6 <u>Implementing a constructivist approach in the mathematics classroom</u>

The previous sections (4.3 to 4.5) of this chapter have illustrated that constructivism embraces many key issues for mathematics teachers and learners. These issues all need to be addressed in the classroom if a constructivist approach is to be implemented at secondary school level. The emphasis in this thesis will also be to explain how the constructivist approach in the classroom will provide an environment suitable for pupils who may develop mathematics anxiety (see 4.7.4).

Firstly let us consider the teacher's role in the classroom with a view to constructivist theory. Constructivism is referred to as a problem-centred approach by Olivier in NECC Mathematics Commission report (1993:35). Whilst problem-centred does describe an important aspect of the constructivist approach it is not a comprehensive description. Constructivism to the teacher must be more than just a problem solving exercise.

In assessing the role of the teacher it is essential to consider all the important aspects of constructivism. In addition, a problem-centred approach alone will not address the problems of a mathematically anxious pupil. It has been shown in this chapter that important aspects of constructivism involve a number of concepts (see 4.3). The classroom situation must provide the pupil with all the ingredients proposed by a constructivist approach. In short the teacher will provide the classroom atmosphere and the classroom activities which best assist the mathematical development of the child (see 4.3).

The adoption of a constructivist approach may present a complicated view for the teaching of mathematics and there is a real risk that unless practical examples of the implementation of constructivism in the classroom is forthcoming, teachers may reject this approach offhand. Whilst a comprehensive text on teaching through constructivism is not the task of this study, it is deemed necessary to identify and describe the specific methodological elements that could complement teaching strategies in the classroom. Extra classroom activities need to be added to the secondary school mathematics programme and the syllabus based topics need to be introduced in a way that adopts the tenets of mathematical development through constructivism.

4.6.1 **Extra classroom activities in the secondary school**

The fundamental purpose of the constructivist approach is the recognition of the pupils' knowledge and experiences which they bring with them to lessons. Pupils should be allowed to use their experiences and existing knowledge to build further knowledge.

Steffe (1990:167) is concerned that teachers are traditionally textbook bound and find it very difficult to change their teaching strategies:

> *Teachers who are mathematically inactive usually present mathematics as static, dualistic (either right or wrong), and as consisting of routine procedures.*

Sections of mathematics are taught as a sequence of steps and this set procedure will yield the correct answer.

Whilst the need for procedural knowledge need not be argued, it is important that teaching does not start and end with procedures. Constructivism requires that teachers provide for more than procedural knowledge. Mathematics learning is based more on experience, intuition and insight.

Byers (1983:23) feels that teachers are providing their students with a false impression of mathematics as a subject:

> *What the student is exposed to is a formal sanitized version of the subject.*

Byers believes that any inquiry in mathematical thinking is rather firmly brushed aside in preference to pre-determined methods. This static belief in formal procedures is passed on from generation to generation as students themselves become teachers.

Constructivism is a reaction to such beliefs in that it adopts the stance that teachers should view mathematics learning as a human activity and mathematical meaning is constructed as a result of such activity. The theoretical position of constructivism has been fully discussed in this chapter. What needs more discussion is the type of classroom activity best suited for a constructivist approach. Teaching in the secondary school provides a challenge for the mathematics teacher. The restrictions of examinations and an overfull syllabus often causes teachers to resort to "chalk and talk" methods. It is not possible to provide an outline of how each topic could be presented with the constructivist philosophy in mind but some topics will be selected to illustrate the suggested teaching methodology.

In addition to adopting a constructivist approach to the classroom activities of the required syllabus, there are certain areas of mathematical activities that are often neglected in the secondary school. By integrating these areas into the normal syllabus, the mathematics teacher will be able to encourage explorative ideas and stimulate discussion.

In essence, the recommendation of this thesis is for classroom activities which will provide the student with opportunities to explore ideas by interacting with the teacher and with fellow pupils. Before providing constructivistically based ideas on how topics in the formal syllabus may be introduced let us consider the three areas that Murray & Human (1990:347) suggest should be an integral part of the constructivist teaching approach to mathematics and which enhance mathematical thinking. These are:

1. Providing a sense for numbers.
2. Dealing with novel problems.
3. Allowing pupils to reflect their methods.

4.6.1.1 **Providing a sense for numbers**

There was a time when "mental arithmetic" was a part of the secondary school syllabus and in fact in South Africa a "mental arithmetic" examination was written at the end of the standard eight year (year ten of formal schooling).

It is the contention of this thesis that this type of arithmetic should be re-introduced at secondary school level because it provides a facility for pupils to construct their own methods and develop their own personal number concepts.

A sense for numbers means more than learning facts and algorithms. Teachers need to provide their pupils with information in such a way that associations and patterns are organised and linked to other facts to help structure meaning.

Number sense thus refers to the development of a conceptual undertaking of mathematics. Reys (1992:3) states that:

Number sense refers to an intuitive feeling for numbers and their various uses and interpretations; an appreciation for various levels of accuracy when figuring; the ability to detect arithmetical errors; and a common sense approach to using numbers.

In essence, number sense envelops all the necessary understanding processes required for mathematical development, and as such must permeate all aspects of mathematics teaching and learning.

Pupils with number sense take advantage of their own understanding of the relationships between numbers and operations to find the most efficient method of tackling a task. The following examples illustrate a high degree of number sense.

Example 1

$$4\frac{1}{3} + 1\frac{1}{4} + 1\frac{2}{3}$$

taught procedure at primary school level

$$6\frac{4 + 3 + 8}{12}$$

$$6\frac{15}{12}$$

$$7\frac{3}{12}$$

$$7\frac{1}{4}$$

$$\dashrightarrow$$

required display of number sense at secondary school level

$$(4\frac{1}{3} + 1\frac{2}{3}) + 1\frac{1}{4}$$

$$6 + 1\frac{1}{4}$$

$$7\frac{1}{4}$$

$$\dashrightarrow$$

Example 2

$$\frac{5 \times 48}{7} + \frac{2 \times 48}{7}$$

taught procedure at primary school level
$$\frac{240}{7} + \frac{96}{7}$$
$$= \frac{336}{7}$$
$$= 48$$

required display of number sense at secondary school level

$$\frac{7 \times 48}{7}$$
$$= 48$$

In addition to a display of the insight revealed in the above two examples, pupils with number sense will display a keen sense of approximation and estimation and recognise unreasonable results for calculations. These tasks are not considered as purely primary school arithmetic but rather as the ongoing development of number sense throughout school. In the new Interim Core Syllabus Standard 6 - 10 for South African Schools (Department of Education, 1995:2) one of the stated specific aims of mathematics education is:

> *To develop number sense and computational capabilities and to judge the reasonableness of results by estimation.*

Reys (1992:5) describes number sense as:

> *Both the ability of the learner to make logical connections between new information and previously acquired knowledge and the drive within the learner to make forming these connections a priority.*

Markovitz & Sowder (1994:4) were also concerned that the rigid methods of teaching numbers in primary school will ultimately have a negative effect on secondary school pupils. Their research involved intervention in the instruction of seventh graders (Standard Five). The recommendation from the research by Markovitz & Sowder (1994:24) is that mental computation should be based on inventing and exploring strategies and that examples should be unfamiliar and novel. Whilst the examples shown above will provide the student with alternative thought patterns the old methods have already become entrenched. New teaching strategies are better with new material.

Sadly a traditional form of teaching procedures has led to a student society that does not make this type of learning a priority. Too often the reality in the classroom is that students are geared to mastering rules and algorithms, which are often poorly understood.

The teacher's task is to provide the classroom activity necessary to promote number sense. It is essential for the constructivist approach that this begins in primary school and continues through to secondary school. Exploring each pupil's method of computation often reveals points of discussion amongst a class on the value of each method and an agreement on which may be considered to be the most efficient. This process can permeate all mathematics teaching as in secondary school, number patterns, number sequences and many of the arithmetic tasks developed earlier are required. For example, factorising a trinomial such as $x^2 + 6x - 72$ requires the student to consider factors of 72 with a difference of 6. Secondary school students with a poorly developed number sense tend to spend far too much time exploring the possible factors. It is the duty of the teacher to ensure that developing number sense becomes a central theme in all their teaching. Thornton & Tucker (1989:21) stress that:

> *Number sense develops over time. This development is best nurtured if the focus is consistent, day by day, and occurs frequently within each mathematics lesson.*

The teacher must encourage exploration, discussion and mathematical thinking by selecting appropriate problems and activities. These problems and activities will vary for each standard but Reys (1992:11) describes the common characteristics a teacher should identify when setting tasks for number sense. The characteristics of these tasks are:

1. They encourage students to think about what they are doing and to share their thoughts with others.

2. They promote creativity and investigation and allow for many answers and solution strategies.

3. They help students to know when it is appropriate to estimate or to produce an exact answer and when it is appropriate to compute mentally, on paper, or with a calculator.

4. They help students see the regularity of mathematics and the connection between mathematics and the real world.

5. They convey the idea of mathematics as an exciting dynamic discovery of ideas and relationships.

This development of number sense will alleviate a growing mathematical anxiety throughout the school years as the student will become more comfortable with the work he or she is doing.

4.6.1.2 **Dealing with novel problems**

A constructivist approach requires that teachers provide opportunities for investigation and that group activities and social interaction in the classroom need to be encouraged. These activities require the teacher to be innovative and creative in his or her approach to mathematics.

The student in the secondary school is becoming more inquisitive and probing and more arithmetically aware and active. A rigid and traditional approach to teaching which is purely syllabus bound and based on an explain-practice-memorize technique can kill any enthusiasm for mathematics. The opportunity has to be grasped to develop the inquisitive mind.

Mathematics must have meaning and the classroom activities must provide opportunities for exploration. Teachers need to be inventive in their approach and using their personal skills to help students develop what Schmalz (1989:685) refers to as a problem solving attitude. As explained in Section 4.3.6, the development of a problem solving attitude is a central tenet of the constructivist approach.

To provide unique opportunities to solve problems, the teacher needs to constantly introduce novel problems to the class at secondary school level. These novel problems could be tackled by groups within the class and a discussion of methods and results should be encouraged. This type of activity need not be bound to syllabus topics and could be introduced on a weekly or daily basis.

It is the contention of this thesis that a re-introduction of "mental arithmetic" to develop number skills together with an introduction of the solving of novel problems on a regular basis should form part of the constructivist teacher's strategy. These classroom opportunities will serve to establish an attitude which will allow students to be more confident when dealing with number manipulation and problem solving in the context of the formal syllabus.

Books such as "The Guiness Book of Records" could form a basis for exploratory mathematical tasks which are process-orientated and will help develop the characteristics listed by Reys (1992:11) in the previous section. "The Guiness Book of Records" provides a wealth of material for investigational purposes. Heights, distances, volumes, mass, age, etc., all provide ideas for the use of numbers and investigational skills as well as approximation and estimation. For example, the record for the number of crumpets eaten could lead to an investigation of the volume of these crumpets. This volume could be estimated by stacking the crumpets and calculating the approximate volume of the cylindrical shape.

Another example could involve investigating how many McDonald's hamburgers are sold each day by considering the number of outlets, the speed of service and the estimated demand.

In yet another example, the area of the world's largest pizza could be calculated and a comparison could be made with the number of normal size pizzas that could be taken from the same area as the large pizza.

All the above examples provide excellent opportunities for number awareness, approximation and estimation and an investigational approach. These elements provide a sense of enjoyment in mathematics, a building of confidence and a competence in knowing that they are using their own personal number skills to investigate the task at hand.

It is important that pupils learn mathematics in the context of problem situations which facilitate appreciation of the true meanings and purposes of mathematical ideas and procedures.

There is a wealth of examples in the numerous mathematics olympiads that are produced to provide the teacher with excellent material to give their students. These examples often require a different approach from the traditional procedures and therefore give the students an opportunity to use their exploratory skills. They also often reveal the underlying tension as pupils become frustrated because learnt rules and methods can no longer be depended upon. Examples of these type of questions could be as follows:

<u>Example 1</u>

Each of the dates in the following sequence conform to a particular pattern

23 March 1969

14 May 1970

7 October 1970

Discover the pattern and then write down the next date in the sequence.

Pupils of all ages will enjoy playing with numbers to discover the answer. Some excellent methods and ideas will be revealed and in fact an alternative solution which can be well substantiated may be found. Pupils with a good sense for numbers will probably solve the puzzle.

$23 \times 3 = 69$

$14 \times 5 = 70$

$7 \times 10 = 70$

There are no further suitable factors of 70, none for 71, therefore 72 is analysed to find suitable factors,

2 x 36; 4 x 18; 6 x 12; 3 x 24; 8 x 9.

Only the combinations 6 x 12 and 8 x 9 or 9 x 8 are suitable. Therefore the answer is 9 August 1972.

Example 2

Find the next two numbers in the number sequence,

6; 24; 60; 120; 210; ...

$1 \times 2 \times 3 = 6$

$2 \times 3 \times 4 = 24$

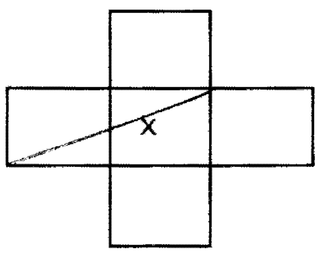
$3 \times 4 \times 5 = 60$

$4 \times 5 \times 6 = 120$

$5 \times 6 \times 7 = 210$

The pattern looks easy but it normally takes some time for pupils to discover. It is particularly interesting to note that pupils who have been taught sequences and series formally have difficulty discovering other patterns than geometric or arithmetic sequences.

Example 3



A cross is composed of 5 equal squares. If x = 10cm, find the total area of the cross.

The use of Pythagoras' theorem and area produces some interesting methods as pupils explore the possible solution.

These examples all provide the constructivist idea of a problem-centred approach to mathematical thinking. The teacher can lead the students to accept their own role as one of a responsibility to acquire knowledge by developing their own learning process. Computational structures are not authoritative prescriptions provided by teachers but rather logical constructions of personal thinking.

It is interesting to note that whilst mathematics olympiads are fairly well supported in South Africa, the entry numbers are disappointing. This may be a reflection on the teacher's willingness to experiment or the lack of time available due to an overfull syllabus. Whatever the reason, the fact remains that many schools do not enter the competitions and some leading schools do not encourage their students to participate in these problem-centred activities.

The objectives of a problem-centred approach to teaching is not to merely provide problems to solve but to facilitate the process in which these problems are approached. The emphasis is on problem solving ability but with understanding and social interaction and communication in mathematics. In addition, the development of a computational dexterity becomes a reality as pupils gain in confidence to use their own mathematical skills.

A problem-centred environment is not one in which the teacher provides strategies or rules for doing problems but rather one in which students develop a confidence to provide their own strategies. This mathematical ability is facilitated by the teacher in a classroom where small group activity provides the socio-interaction which enhances learning. Students learn from this interaction by:

1. Co-operating to complete a problem activity
2. Agreeing on answers and solution methods.
3. Explaining their solutions.
4. Listening to the solutions of others.

5. Persevering to figure out problems for themselves.

In essence, the last mentioned ability will identify the type of pupil with the tenacity necessary to do mathematics. Too often problem solving is just too much hard work and it is easier to depend on others to provide the answers.

4.6.1.3 <u>Allowing pupils to reflect their methods</u>

To verbalise what one is doing ensures one is examining facts and attempting to understand. It is true of any situation that an individual will gain a better understanding by verbalising what he or she understands from that particular situation. The teacher can never predict what a child will learn and can never assume that the child understands his or her thinking. Children can only learn that which they understand from the situation. It is, therefore, important that time is allowed during classroom activities for children to express their ideas and methods.

The idea of reflection was discussed earlier in this chapter (see 4.4). However, for the purpose of classroom activity it is important to examine the type of reflective activity which the teacher can use in the classroom situation.

Reflection is not a new concept as it has been used in research by a number of mathematical researchers. In England the Assessment of Performance Unit (APU) of the Department of Education and Science conduct a number of surveys. One of these surveys included pupils being required to perform practical mathematical tasks and then describing to a facilitator the method being used (Foxman *et al*, 1982:11).

It is interesting to note that the survey described above received positive support from the testers mainly for the reflective elements of the testing sequence. Foxman *et al* (1982:51) say that:

> *Testers welcomed the opportunity to work with pupils from different schools and remarked that through individual interviews they had gained a useful insight into pupils' thinking and the problems encountered.*

In the APU secondary survey report on the 1980 research, areas, shapes and volumes formed the main theme of the practical tests. Foxman *et al* (1982:14) describe how one triangular shape is given to the pupil and this is used to compare with the areas of other given shapes, i.e. larger triangles, squares, rectangles, parallelograms. The variety of methods used is fascinating but more important is that pupils methods revealed their mistakes. This task used as a classroom group activity would lead to excellent group discussion and healthy social activity to produce a consensus of the best method.

Rees & Barr (1984:11) are more explicit about their pupils' responses and have recorded them for readers to assess. They make the point that listening and diagnosing methods improved the competence and confidence of students and teachers. Rees & Barr (1984:14) discovered from their research that:

> *There is little doubt that most students attempt to remember routes which they think they have been taught and which they apply with little understanding.*

A description of two seemingly similar tasks reveals the need for observation and reflection to understand the pupil's method.

Rees & Barr (1984:153) compare the tasks of the equations

$$\frac{2}{x} = \frac{1}{3} \quad \text{and} \quad \frac{x}{8} = \frac{3}{12}$$

For the first example the pupil responds fairly quickly and almost intuitively produces the answer, e.g. two over something equals one third - must be six because half of six is three and in fractions the bigger number is lower than the small number.

The answer to the second task is far less confident or competent although the pupil described by Rees & Barr (1984:153) does reveal a certain degree of number sense. The pupil's response is described as follows:

> *Pupil:* *Well ... 8 goes into ... that ... well ... (laughs) ... sorry ... you times that by ... you've added 4 to that ... ohh! ... hold it I'm stuck! ... ahh! ... (laughs) ... right! so ... that! ... you've added 4, 'cause 8 goes ... 4 and 8 ... I mean 4 goes into 8 so ... so ...*

so ... it goes ... it goes ... mmmmmm ... so 4 goes three times into that ... ooooh.

Teacher: You're doing well. Keep trying.

Pupil: So it goes 2 into 8 so I think it will be 2.

The response to this task is worth examining very closely as it reveals many factors related to mathematical performance.

1. Lack of confidence, anxiety in performing the task, revealed by the nervous laughs.

2. A number sense which requires adaptation from the previous easier yet similar task.

3. The perseverance to continue and try different methods.

4. The value of encouragement from a facilitator to restore confidence.

5. The value of reflection for the facilitator to recognise what the pupil is doing.

Maher & Alston (1990:147) describe similar situations in their research on constructivism in the classroom. They report that the whole process of problem solving activities being reflected in the classroom led to increased enthusiasm by teachers and pupils.

> *Teachers soon discover that children are interested in the activities and are naturally motivated by the creative possibilities of constructing their own models to fill the requirements of each problem* (Maher & Alston, 1990:161).

The benefit of reflection in the classroom is not only to the pupil. The Teacher can analyse the finer detail of their pupils' mathematical thinking and thus select activities which will help clarify misconceptions (see 4.4).

4.6.2 <u>Teaching the formal secondary school syllabus</u>

It is not possible to provide detailed lesson plans for the constructivist teacher in the secondary school. Indeed this is not desirable as one of the major tenets of constructivism is the belief in each individual's unique ability to construct knowledge. This belief must extend to a belief that each teacher has the ability and desire to construct his or her own ideas of how each topic may be introduced.

However, it is important that the constructivist theory does not become divorced from the practical need for students to master the mathematical topics that are required to be taught as stipulated by the Interim Core Syllabus (Department of Education, 1995). Whatever value a teacher may award the constructivist theory it will only be enhanced if the theory provides results in practice.

In the eyes of the constructivist the classroom is a microcosm of society and mathematics learning takes place in a similar way to all general knowledge. In other words people acquire knowledge by their interaction in society and by developing new constructs through negotiation and communication with their fellow human beings.

Thus the teacher must be motivated to reorganise their teaching strategies to accept the idea of socio-constructivism because the classroom is part of the individual's social world. Socio-constructivism asserts that there is a body of mathematical knowledge that is in existence and the challenge for the teacher is to find ways for pupils to discover it for themselves.

When introducing the various mathematical topics at secondary school level the teacher needs to attempt to adhere to certain basic principles which are embodied in the constructivist theory. Whilst some topics may not readily lend themselves to these learning principles, it is the teacher's task to provide the correct environment wherever possible. According to Goldin (1990:31) the basic constructivist principles are:

1. Mathematics is constructed uniquely by each student and not merely transmitted by the teacher.

2. Mathematics is not an independent body of knowledge which is abstract from everyday life.

3. Mathematical learning occurs most effectively through discovery, problem solving and application.

4. The creation of learning environments should allow for exploration and problem solving.

5. Methods of assessment should include interviews and small group case studies.

6. Lesson plans should reflect how learners acquire knowledge and how they may investigate problems.

To provide direction of how these principles could be utilised in practice let us consider a few mathematical topics in the present secondary school syllabus.

4.6.2.1 **Ratio and proportion** (Standard Six)

The introduction of this topic could involve the use of model cars, model aeroplanes, doll houses, etc. The class could work in groups and explore the ratio of the model to that of the original article.

A visit to a car factory or the engineering faculty at university may provide other concepts of how ratios are utilized to investigate the design of prototype cars, bridges, etc.

Group work could also involve geometric elements by cutting out triangles and investigating the ratio of their sides.

Pupils may now have grasped the concept of ratio and be able to provide numerical examples of interesting concepts in their own life. For example, ratio between their own weight and height and perhaps the rest of their family.

4.6.2.2 **Trigonometry** (Standard Eight)

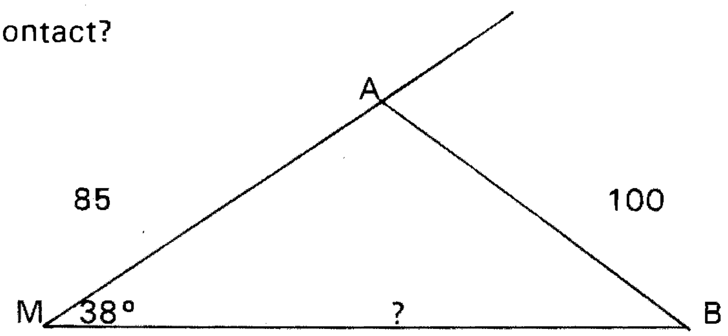
This topic provides opportunities for students to further their investigations of angles and ratios.

Students should be encouraged to build a "trigonometry board" which would consist of a unit circle centred on a Cartesian plane with an arm fixed at the centre of the circle which is able to rotate freely.

Students would then be free to investigate concepts of angles and ratios by measuring on the prepared board. Discussion in groups and guided investigation by the teacher could lead to the revelation of the basic trigonometric ratios.

Trigonometry is a topic which can be related to problems in everyday life and thus provide an incentive for students to discover the power of trigonometry.

The simple idea of calculating the height of a building after measuring a distance from the foot of the building and then an angle of elevation to the top of the building makes trigonometry relevant. Pupils will soon discover that trigonometry gives them a new found concept to integrate with their geometry knowledge to help solve previously unsolvable problems such as:
Two police cars travel along roads that are at an angle of 38° to each other. The one stops after 85km and the other continues to stay in radio contact. If their radios have a range of 100km, how far could the other police car travel before the two cars lost radio contact?



The first task for pupils would be a discussion, in groups, of how this problem could be illustrated diagrammatically. Once this has been achieved the relevant trigonometry techniques could be discussed and a final agreement to the correct solution path would be negotiated.

4.6.2.3 **Sequences and series** (Standard Ten)

This topic also lends itself to investigative techniques and exploratory introductions.

Students could be asked immediately to form their own number patterns and challenge others in their group to discover how they formed their number pattern. A homework task could require students to explore areas where number patterns may be useful or are evident in everyday life. The results should reveal some simple examples of bank savings, mortgage repayments, weight increase or loss, height increase, etc. These examples could then be supplemented by more directed problems introduced by the teacher. The investigation of receiving a set increase in salary each month compared to an exponential increase will stimulate healthy discussion.

The use of the "Guinness Book of Records" could also provide a wealth of questions to be explored. For example, one could take the height of the tallest man in the world and pose the question that if he was 0,6 metres tall at birth and his height increased by five percent each year, how old was he when he reached his maximum height?

A similar exercise could be constructed using the example of the total number of pancakes eaten by the world record holder. If he eats 20 in the first minute and this decreased by 2 each minute thereafter, how long would it take for him to reach the record?

Sequences and series also provides students with an added concept which enhances their arithmetic skills and subsequently provides them with a confidence to do mathematics and demonstrate and explain to others how it is done. The previously impossible task of adding the first 1000 counting numbers rapidly in their head or even on a calculator is now made simple when they discover for themselves the formula $Sn = \frac{n(n + 1)}{2}$

As mentioned at the beginning of Section 4.6.2, it is not the task of this thesis to provide ideas for the introduction of every topic in the syllabus. The three topics chosen do lend themselves to an investigational approach but then it is not difficult to adopt this approach to many topics in the current syllabus.

4.6.2.4 **General constructivist strategies**

In the first analysis it is the teacher that sets the classroom strategies and it will inevitably be the teacher's philosophy of how knowledge is constructed that influence the mathematical activities in the classroom.

Schoenfeld (1992:290) maintains that a teacher's belief structure determines the nature of the classroom environment and hence determines learning outcomes. Teachers who adopt a constructivist approach will de-emphasise forms of the instructional process that feature factual and procedural knowledge and seek an instructional methodology that emphasises deeper understanding.

The teacher in a constructivist environment is a critical mediator in this instructional process, ensuring that hard work and effort are suitably deployed and rewarded. The teacher ensures maximum task involvement by getting students to collaborate with one another in groups.

The focus of all classroom activities in a constructivist learning environment is on sense making i.e. constructivist teaching fosters the view that mathematics activity is meaningful. All the topics in the syllabus should provide opportunities for classroom practices which foster task orientation and enhance beliefs that success in mathematics depends on attempts to make sense of subject matter.

Opportunities for students to construct mathematical knowledge occur as they interact with both their teacher and their fellow students. Classroom interaction should be fostered by arranging group problem solving sessions as well as whole class discussions whenever possible. It is in this environment of social interaction that knowledge is constructed as it facilitates the development of the obligation of making sense of things. In this environment, each student gains a better understanding through the expectancy of having to explain and justify how they solve a problem.

Constructivism serves to empower students. It induces through the idea of discovery and investigations that mathematicians are not divine individuals. It gives the teacher the opportunity to interact on a more realistic level with the class and enables the teacher to expose the mathematical genius in each and everyone of the students.

A constructivist approach to classroom activities may appear time consuming but if the concepts are well developed at an early stage there will be little need to reinforce learning at a later stage. In addition, these methods restore confidence and a new faith is developed in the student to become task-orientated and mathematically competent. These attributes developed by a constructivist approach provide the central reason why constructivism serves the interests of the mathematically anxious child.

4.7 Constructivism and mathematics anxiety

The writing and research on mathematics anxiety is not as prolific now as it was in the 1970s and early 1980s. One reason for this may be that opportunities for new research have not been evident. However, it is more likely that with the interest in a humanising of mathematics instruction, those concerns of mathematics anxiety researchers are being addressed. Teaching styles, parent contact, socio-cultural factors are all aspects of a constructivist approach to mathematics education. A constructivist approach will fulfil the needs of those concerned with mathematics anxiety in the teaching of mathematics.

A constructivist approach provides the necessary structures to address the varied sources of mathematics anxiety and stem the perpetuation of the problem.

The term "socio-constructivism" as used by Olivier (NECC Mathematics Commission 1993) emphasises the concerns which were discussed in Section 2.2.2.1, i.e. that mathematics anxiety has its roots in the socio-cultural aspects pertaining to each individual. Socio-constructivism is not an alternative approach but rather a more explicit title for the constructivists views of education.

Mathematics education does not take place in a vacuum nor does it start with individuals with a "clean plate". Rather, it takes place in a social context and much of the knowledge gained by children is in an informal context.

> *We have to investigate our subjects' perceptions, purposes, premises and ways of working things out if we are to understand their behaviour. Even at the contextual level, as we try to understand the effects of physical and cultural environments on people, we have to look at their purposive interaction with those environments. We no longer believe that people are simply caused to behave in certain ways by an environment that is entirely external to and independent of their cognitive processes* (Noddings, 1990:14).

The previous chapter has served to describe the move towards this philosophy. The traditional view of the curriculum is no longer accepted. Ernest (1989:4) says that

> *The move towards new styles of teaching echoes a worldwide recognition that to fulfil personal and societal needs the goals of mathematics education must include the development of higher level thinking skills, particularly problem solving.*

The acknowledgement of a need for skill and understanding or procedural and conceptual knowledge and a leaning towards understanding the cognitive processes has led to a constructivist following amongst mathematics educators and researchers. The constructivist approach acknowledges the importance of content discussion in curriculum design but the emphasis is on the teaching and learning aspects of the curriculum. In particular humanistic, pupil-centred teaching techniques are proposed by constructivists.

In Section 2.2.2 the sources of mathematics anxiety were categorised under the headings socio-cultural, emotive, cognitive and educational factors. It is no coincidence that the modernistic approaches to the teaching of mathematics tend to address the problems of mathematics education under the same headings. The constructivist approach is no exception.

Ernest (1989:151) emphasises the importance of social, cognitive and educational factors as the concerns of constructivist theory. Ernest explains that addressing these concerns means allowing students to use their pre-existing knowledge in a classroom atmosphere which allows for participation and provides a non-threatening environment.

Constructivism can serve the needs of the mathematically anxious pupils but it is important that all those involved with a child's development recognise the important forces of this new approach. Teachers, parents and indeed the wider community need some knowledge of the ideals of constructivism. To assist with this understanding and to provide the emphasis on how constructivism assists the mathematically anxious child, it is worth considering the sources of mathematics anxiety once again.

4.7.1 **Socio-cultural factors and constructivism**

Constructivists recognise the importance of society and culture and the impact they have on the development of mathematical knowledge of each individual. Simpson (NECC Mathematics Commission, 1993) says that:

> *If one accepts the notion of a real world impacting on human beings, who in turn react thereto, and that this reaction/action changes that real world, one could proceed to conclude that the development of mathematics is a socially motivated process.*

Schoenfeld (1992:290) also makes the point that:

> *There is a real world or objective reality out there and that the (sometimes flavoured) representations of it that we build serve as the basis for our reasoning.*

Olivier (NECC Mathematics Commission, 1993) says that:

> *It is not our children who are not ready for school - it is our schools that are not ready for our children.*

Thus the impact of society and particularly the home environment is a fundamental starting point for constructivists. In fact researchers like Olivier prefer to use the term socio-constructivism to emphasise the importance of the social and cultural aspects of constructivist theory. However, what is rather disturbing is that very little reference is made to educating society to understand the implications of constructivism. After reading the literature referred to above, one gains the distinct impression that even constructivism becomes an objective approach. That the outcome of producing more competent mathematicians that are more flexible and may adjust more easily to the vigours of a changing world are the objectives of constructivists.

It is important for the psychological development of the pupil that one does not neglect involving parents in the process of mathematics education. Yackel *et al* (1990:15) describe learning through social interaction but then review this as purely social-interaction in the classroom environment.

In these early styles of constructivism in the primary schools, parents are already talking about the "new mathematics" and of course, this conjures up all types of negative connotations. Parents need to know that this is not a new mathematics but rather a unique and positive approach to the way mathematics is taught.

In Chapter Two, (see 2.2.2.1) the negative influence that parents have on causing mathematics anxiety in children was discussed. Without the support of the parents and an informed public, the socio-constructivist approach in the classroom will be undermined by negative connotations outside the classroom.

Donovan (1990:17) emphasises that:

> *As teachers, we need to understand better the people and social groups who influence what happens in our classrooms, including the established structures and constraints that help determine access to, and success in, schooling.*

Thus a fundamental task of the constructivist should be communication. Not only with the pupil but with parents and the whole school community. Each school serves a different community with different social and cultural beliefs and different social and cultural needs.

In a multicultural society such as the South African situation, parental involvement in education and especially mathematics education is an important factor. Clements

& Battista (1980:34) make the point that:

> *Mathematical ideas and truths, both in use and in meaning, are co-operatively established by the members of a culture. Thus, the constructivist classroom is seen as a culture in which students are involved not only in discovery and invention but in a social discourse involving explanation, negotiation, sharing and evaluation.*

The proposal of this thesis is that the socio-constructivist model is to be used to alleviate mathematics anxiety. The tenets of this model do serve to reduce anxiety by addressing the problems of lack of understanding and lack of task-orientation. The emphasis on classroom strategies that will develop deeper understanding and a task directed attitude were expounded in Section 4.3.2 of this chapter. Task avoidance and a rote learning approach are two aspects which were emphasised as the causes of mathematics anxiety in Section 2.2.2.4.

Thus, the constructivist approach which emphasises a co-operation between teacher and student and student and student is a social model serving the needs of the mathematically anxious child.

However, this co-operation has to extend to outside the mathematics classroom if one truly believes that constructivism is building on what is already learnt and if other agents causing mathematics anxiety are to be addressed. A number of strategies should be a priority as schools and teachers become more involved in constructivism. These strategies include:

1. Producing an "easy-to-read" information brochure describing the constructivist perspective. Not only the instructional activities but also the desired outcomes.

2. Holding information evenings whereby parents and other interested people can freely participate in discussion.

3. Providing opportunities for teachers from different schools to gather and work through a programme to develop their skills.

This type of intervention is necessary if one expects the teacher to create the correct atmosphere in the classroom. Surely social constructivism is intended to include the broader community and thus eliminate the unneeded negative forces that may develop outside the classroom. It is essential that the revival of the term "New Maths" does not become the battle cry for those parents and pupils who are eagerly seeking a new excuse for poor performance and a new way of defending anxiety.

4.7.2 **Emotive factors and constructivism**

Constructivism addresses the emotive factors of mathematics education by presenting a pupil-centred approach to teaching. In fact constructivism goes further than most pupil-centred approaches in that it suggests a more caring attitude. Davis *et al* (1990:191) stress that to accomplish the goals of constructivism we have to care deeply for the children. They are adamant that children who feel cared for are more likely to engage freely in the type of intellectual activity of problem solving that the constructivists propose.

Davis *et al* (1990:191) proceed to make the observation that when we open ourselves to caring relations, we learn to listen. This is an essential aspect for any teacher using constructivist ideas because there is a need for them to listen and understand each individuals thought processes.

Thus constructivism addresses the problems of connotation and myths discussed in Section 2.2.2.2. Mathematics is no longer mystified as described by Koggelman & Warren (1978:30). Mathematics becomes an individual's own thought processes and Davis *et al* (1990:188) believe in acquiring of an even deeper understanding of oneself and one's own modes of learning and thinking.

However, Davis *et al* (1990:188) acknowledge that there may be some misconceptions that arise through personal constructs being formed and it is here that there is a need for further research. Davis *et al* (1990:188) pose the following important questions:

1. *What misconceptions arise regularly in given topics and processes?*

2. *What kind of learner is most subject to a particular misconception?*
3. *What activities successfully challenge particular misconceptions?*
4. *What diagnostic techniques are especially effective in probing for various misconceptions?*

These questions are particularly relevant for the mathematically anxious child as it is the misconceptions (connotation and myths) associated with mathematics that cause emotive blockages for these pupils.

These connotations and myths were discussed fully in Section 2.2.2.2 and it is perhaps amongst these emotive issues that the answer can be found to Question One posed by Davis *et al.* In answer to Question Two, many misconceptions in the form of connotations and myths would be identified in the mathematically anxious student. The answers to Questions Three and Four lie in the constructivist approach which encourages the teacher to listen carefully to each student's mathematical thinking.

Reflective thinking by pupils and empathic listening by teachers will provide the relationship between teacher and pupil which will assist in alleviating anxiety. By paying attention to students' thinking, teachers are providing a less anxious environment. The teacher becomes less of an authoritative figure and more of a facilitator of knowledge assisting co-operative student constructors.

Once students have an understanding of how they learn and begin to reflect upon their knowledge, they become free of anxiety and more able to cope with the demands of the real world outside the classroom.

4.7.3 **Cognitive factors and constructivism**

It is within cognitive psychology that constructivism makes a major contribution. Ernest (1989:6) says that aspects of the mathematics curriculum are under expert scrutiny and that there is presently a critical examination of some of the sacred cows of mathematics teaching.

Some cognitive factors which may contribute to anxiety in mathematics were listed in Chapter Two and all these factors are addressed when a constructivist approach is adopted and the following items are given attention.

(a) The abstractness of mathematics is not disguised but rather highlighted from an early age. Davis *et al* (1990:189) says constructivism requires that we take seriously the verb 'to abstract'. This means that pupils learn not to start with formulae but rather with experiment.

(b) Students are continuously asked to contribute their own ideas to every lesson.

(c) The usefulness of mathematics is stressed. Students build on their own knowledge gained from a multiplicity of experiences and social interactions throughout life.

(d) The subject becomes more personalised with a pupil-centred approach which aims at learners creating their own mathematical knowledge.

(e) Assessment techniques become less rigid as each individual's contribution becomes important. Ernest (1989:176) says that once mathematics is seen for what it is - as much a cultural product as any other - a number of myths about mathematics can no longer be sustained. The myth of the objectivity of mathematics is exploded - scholars are forced to admit that it does not hold.

(f) The constructivist approach is a philosophy of teaching and as such does not become more difficult as children progress through school. On the contrary, pupils feel more relaxed and become more confident in presenting their own ideas.

(g) The use of language is another important aspect of constructivist thinking. Stoker (NECC Mathematics Commission, 1993) emphasises the socio-cultural aspects of constructivism and states that the emphasis on the cultural role of language as the link between culture and thought is what makes the socio-cultural model appropriate for mathematics education. The encouragement of interaction through language requires a better understanding from the pupils.

Constructivism aims at getting to know how a child reasons and what kinds of new constructions he is capable of when teachers encourage spontaneity. The cognitive processes of each individual are more important than the material being studied. The essential aim of the constructivist teacher is to provide their students with confidence in their own cognitive processes. The students develop "mathematical power" which provides an autonomy and a belief that they do not receive mathematical knowledge from their teachers but rather from their own explorations and thinking.

By developing an autonomy and a belief in one's ability, the problems of mathematics anxiety are alleviated. Students develop a self-confidence which enables them to become more task orientated and confident in their ability to solve problems. The constructivist approach also helps develop a self-understanding in each student as they are able to discuss their problems freely with the teacher and fellow students. This self-understanding may incorporate a knowledge of where their anxieties stem from and how best they are addressed in this new environment.

4.7.4 Educational factors and constructivism

A constructivist philosophy necessarily involves the entire school community. The school environment and the classroom must be able to provide the correct atmosphere for students to feel free to contribute their opinions and social and cultural observations. Construction and reconstruction must be considered paramount at the school.

The literature reveals that a number of researchers have found that unfortunately there is often evidence that well intentioned curriculum ideas are often not carried out by the teacher. The constructivist ideals are often neglected when it comes to classroom practice.

Edwards & Mercer (1987:125) found that well-intentioned teachers often engaged learners in specific activities in order to obtain specific results and Stoker (NECC Mathematics Commission, 1993) makes the observation that so-called constructivist-type classrooms do not make sufficiently problematic the authority of the teacher or the textbook.

Despite claims of being progressive teachers, Edwards & Mercer (1989:125) found that constructivist-style teachers depended on the use of power in the classroom.

Davis *et al* (1990:188) concede that constructivism does not offer pedagogical recipes or convenience. In fact, constructivism requires much hard work from the teacher as many familiar tools and many familiar attitudes, must be questioned, modified or just plain discarded (Davis *et al*, 1990:188).

The salient point which is being stressed by all these authors is that constructivism is only as good as the teacher and the educational institution. In Section 2.2.2.4 the same point was made regarding mathematics anxiety when Eisenberg (1991), Stodolsky (1985) and Greenwood (1984) all expressed concern that the main cause of anxiety is teaching methodology.

The school and the teacher provide the educational environment that must be for the benefit of the student. Constructivists as well as those people concerned with mathematics anxiety would agree on certain requirements to enhance understanding in the educational process and these requirements should be maintained in the classroom at all times.

The following list emphasises some of the requirements of mathematics educators when understanding is a pre-requisite for success.

1. Correct performance does not necessarily mean understanding. Teachers will need to investigate ways of thinking.

2. Merely showing a student the right way to do a problem is not always going to ensure understanding.

3. Teachers must not give students formulas and algorithms but rather the ability to think for themselves.

4. The school must provide investigational experiences and the teacher must equip the student with ideas for problem solving.

5. Problems must not be rote drill but rather genuine problems where one does not, at the outset, know how to find a solution.

These teaching requirements may seem overbearing for the teacher but Davis *et al* (1990:190) see the constructivist approach as a challenge to educationalists but a challenge which is well worthwhile. They state that:

> *Whatever else it may be, constructivism is not without consequences. Adopt a constructivist point of view and you will need to change your expectations of schools, of teachers, of content, of teacher education and of research methodologies* (Davis *et al*, 1990:191).

In the above quote, Davis *et al* mention four factors influencing reform in mathematics education that were discussed in Section 2.2.2.4 as part of educational or school factors which influence mathematics anxiety.

In essence, this study is concerned with the teaching of mathematics and whilst factors such as school expectations, content requirements, the education of teachers and research methodologies may all contribute to mathematics education, it is the didactic approach that remains the central theme of this thesis. After all considerations and categorizations of the sources of mathematics anxiety (see 2.2.2) and their links with the constructivist approach (see 4.7) it is the teaching environment which presents the most convenient arena to provide solutions to alleviating mathematics anxiety.

4.8 **Summary and synthesis**

In this chapter constructivism was introduced as a philosophy of learning as well as a methodology for teaching mathematics. In its philosophical form, constructivism appears to be radical but in a practical sense a more teaching orientated branch of constructivism is termed social constructivism.

The roots of constructivism are explored (see 4.2) and the need for a constructivist approach was established (see 4.2.3). The social constructivist approach provides the emphasis in this thesis as it is evident that the socio-constructivist view of mathematical development (see 4.3) stresses the same elements of teaching that were important considerations to mathematics anxiety researchers (see 1.1.3 and 2.3).

These factors were categorised under the headings, classroom environment, understanding, self-confidence, communication, social interaction and problem solving and were more fully explored in the context of mathematics teaching and mathematics anxiety in curriculum design (see 3.4).

Whilst not central themes of this study, other aspects of presentation of mathematics were discussed. The positive value of reflection as a component of constructive teaching was developed in Section 4.4 and the positive and negative considerations of representation in mathematics teaching were discussed in Section 4.5. In addition, examples were given of how the constructivist approach can be used to introduce formal syllabus topics and how extra classroom activities can help develop the minds of the pupils at secondary school level (see 4.6.1 and 4.6.2). In the final Section, 4.7, the sources of mathematics anxiety were linked to the constructivist approach emphasising areas of common concern such as socio-cultural conditions, emotions and in particular self-confidence, cognitive beliefs and attitudes and educational factors which are mainly teacher orientated.

Teaching methodology is emphasised and the tenets of the constructivist approach provide the structure for teachers to implement in the classroom. In addition, the constructivist theory has similar objectives to those concerned with teaching mathematics to alleviate mathematics anxiety.

The similarities are found in the emphasis on the teacher and teaching methodology in constructivism and mathematics anxiety research. Both areas involve a need for the re-education of teachers and a move towards a pupil-centred approach to teaching methods. A deeper understanding of the work and an empowerment of pupils to feel confident in what they do are all part of the teaching challenge.

Battista & Clements (1990:34) state two major goals of constructivism as:

1. Students should develop mathematical structures that are more complex, abstract and powerful than the ones they currently possess so that they are increasingly capable of solving a wide variety of meaningful problems.

2. Students should become autonomous and self-motivated in their mathematical activity. Such students believe that mathematics is a way of thinking about problems.

A third major goal is implicit in constructivist theory and that is the development of self-confidence in one's ability to do mathematics. This self-confidence necessarily implies a lower anxiety as students are expected to communicate their ideas in a social context in the classroom. The student is guided towards an independence in mathematics which gives him the confidence to discuss methods and control and create his own mathematical destiny.

Constructivism is a philosophical approach to the acquisition of knowledge. This becomes the synthesis of the ideals expressed on mathematics anxiety, curriculum design and in particular the teaching process which best addresses the requirements of these issues. The purpose of this synthesis should be for the benefit of the pupils and thus the final goal should be to produce students who possess mathematical autonomy and confidence (see 6.5).

For this reason the summary and synthesis of the tenets of constructivism should be categorised in terms of the required teaching methodology as it was described in Section 4.3. Not only does this categorisation include all the constructivist expectations, it also coincides with the required attention areas of concern for mathematics anxiety. In the final analysis it is the six critical variables first identified in Section 1.1.3 that provide the centre of concern for this study which emphasises teaching methodology. These critical variables of classroom environment, understanding, self-confidence, communication, social interaction and problem solving serve as a theme for this study as they categorise the most important aspects of a constructivist approach and a concern for a mathematics anxiety approach to the teaching of mathematics.

The following chapter provides details of empirical research on the affective variable of mathematics anxiety which debilitates the outcome of the constructivist ideals of student autonomy and confidence in doing mathematics (see 2.1.3).

Mathematics anxiety as an affective variable is a central theme of this thesis. However, it is pointless to study a psychological aspect of curriculum planning without providing a broader view of curriculum and a synthesis that suggests a recommended remedial path and a required outcome.

The previous chapters have established the importance of mathematics anxiety as a variable in curriculum design and how a constructivist approach to teaching could provide a remedy. The empirical research in Chapter Five is intended to provide support to literature claims that mathematics anxiety is widespread (see 2.1.3.1) and that it is debilitating (see 2.1.3.2).

It is a proposal of this study to establish a comprehensive approach to the remediation of mathematics anxiety which includes curriculum planning and instructional techniques. For this reason components of a systems approach to curriculum design are analysed and critical aspects of teaching and learning are emphasised.

Teaching methods will be categorised once again in terms of classroom environment, understanding, self-confidence, communication, social interaction and problem solving. This will provide a convenient cross reference to Sections 1.1.3, 2.3, 3.4.1 and 4.3 but will also include supporting evidence from the empirical research in Chapter Five to formulate recommendations for the teaching and learning of mathematics.

CHAPTER FIVE
EMPIRICAL RESEARCH

5.1 Introduction

The research outlined in this chapter describes a longitudinal case study of a group of pupils from their standard five year through their entire secondary school career. In addition, an investigation into the mathematics anxiety of first year trainee teachers at a College of Education was also undertaken.

The purpose of the longitudinal case study and the research into teachers in training is to provide evidence to support the arguments of this thesis (see 1.5.4). Whilst generalisations are not appropriate from case studies, the important issues from this research emphasise a number of aspects of mathematics anxiety which have provided the theme for this thesis. The case study, therefore, is intended to provide support for the following aspects:

1. To establish that mathematics anxiety does exist at secondary school level and is measurable.

2. To establish that mathematics anxiety has a negative effect on mathematics performance.

3. To investigate any change in mathematics anxiety during the secondary school years.

4. To investigate the progress in mathematics achievement during the secondary school years.

5. To investigate perceptions and attitudes which are a source of mathematics anxiety.

6. To investigate critical elements of teaching methodology that affect mathematics.

These aspects will be discussed again in Chapter Six when the central elements of this thesis are synthesised together with the conclusions made from the results of the literature research and the empirical research.

In this chapter a brief synopsis of the empirical part of the research will be provided before a more detailed description is given of the specific pre-planning, the instruments, the procedures, the data organisation, the statistical results and the analysis of these results.

The longitudinal study consisted of several aspects.

1. The entire group of Standard 5 pupils at a local primary school were asked to complete a mathematics anxiety rating scale (see Appendix 1). They were also given a mathematics test (see Appendix 2) to record the level of their mathematics performance at this stage.
2. The mathematics performance and English performance was monitored by recording the final results for each subject throughout the five years of secondary school culminating in the Natal Senior Certificate examination programme which included four control tests as well as other means of classroom assessment (see Table 5; 5.6.4.5).
3. During September of the Standard 10 year the students writing the Natal Senior Certificate examinations were asked to complete another mathematics anxiety rating scale (see Appendix 4). In addition, a further questionnaire (see Appendix 5) was designed to provide a more specific insight into the problems connected to mathematics anxiety. The intention here was to provide more evidence for teachers of specific points of concern.
4. A parallel study was carried out on teachers at a College of Education in Natal. These student teachers were asked to complete a mathematics anxiety rating scale (see Appendix 6) and also provide details of their background and aspirations as mathematics teachers. The intention of this research was to provide empirical support to the literature claims made in Section 2.2.2.1 that teachers perpetuate mathematics anxiety.

5.2 <u>The composition of the groups used for empirical research</u>

5.2.1 <u>Composition of the school group</u>

The school survey was composed of 53 boys and 56 girls, the entire Standard 5 group at a Durban primary school were selected to complete the initial set of tests. Of this group nine papers were spoilt and not used. 48 boys and 52 girls completed the tests successfully and the data from these tests was used.

This group cannot be taken as representative of the population and general references will not be conclusive. However, for a longitudinal case study over six years it was believed to be more expedient, efficient and convenient to monitor the progress of this group from their Standard 5 year at school. At this stage, however, it must be stated that the follow-up study at Standard 10 level proved to be somewhat disappointing, although significant in a sense, due to a significant number of students not being able to complete the mathematics anxiety rating scale and questionnaire at this stage. This was due to several reasons listed below:

1.	No longer taking mathematics as a subject	(21)
2.	Failed a year at school	(5)
3.	Discontinued schooling	(7)
4.	Left Durban area	(3)
5.	Left South Africa	(4)
6.	Not traced	(5)

(See Appendix 7)

The fact that 45 of the 100 students were not tested at the Standard 10 stage detracts from the significance of the data collected. However, it is interesting to note that 33 of the non-participants (i.e. 33 of the 45) had either left school, failed or discontinued with mathematics. It was also significant that in the correlation of the original 100 pupils, mathematics anxiety and Standard 5 performance was measured as -0,39. However, excluding the 45 the correlation between mathematics anxiety and mathematics performance using the same data at Standard 5 level was -0,22. These figures indicate that the group of 45 made a significant difference to the negative correlation between mathematics anxiety and mathematics performance.

Thus the anxiety ratings of the group of 45 showed a high negative correlation with their mathematics performance. Poor performance coupled with high anxiety is, therefore, a significant factor contributing to why 33 of the group of 45 did not reach Standard 10 level mathematics.

At the Standard 10 stage of testing the 55 participants was made up of 29 males and 26 females from an original 48 males and 52 females. The reduction in the numbers of females was due mainly to the fact that they were no longer doing mathematics and therefore preferred not to complete the questionnaires as many of the items described situations no longer relevant to them.

5.2.2 <u>Composition of the College of Education group</u>

The composition of the College of Education survey consisted of 148 first year student teachers of which 31 were male and 117 female. 33 (17 male, 16 female) had elected a special major course in mathematics. This course involved doing a higher level of mathematics over four years and generally was viewed by the College as being training to teach up to and including standard 7 pupils. 115 (14 males, 101 females) had elected to do the general course or were compelled to do a general course in mathematics as this is a necessary requirement for students. This course is covered over three years and is aimed at preparing the student to teach mathematics up to and including standard 5.

5.3 <u>Pre-planning</u>

The Mathematics Anxiety Rating Scales for the Standard Five year, the Standard Ten year and the student teachers were prepared by heeding the advice of Richardson & Woolfolk (Sarason, 1980:274). Items were carefully selected to maintain the reliability, stability and validity of the original MARS test as explained in Chapter Two. However, some items were not appropriate for the South African situation and some items needed to be reworded to suit the South African context. The correlation of items used in the three adapted mathematics anxiety rating scales and those in the original MARS test which Richardson & Suinn (1972:551) developed, is shown in Appendix 8.

It can be seen that the local tests were adaptations of the MARS with the exception of 3 items in the Standard 10 mathematics anxiety rating scales.

A test-retest reliability was conducted on all three instruments with a two week break between testing. The test-retest reliability coefficients are indicated below:

Standard 5 MARS	0,82	(n = 80)
Standard 10 MARS	0,78	(n = 150)
Student teacher MARS	0,84	(n = 20)

The students used for the test-retest reliability were in a similar environment to the target groups in all three instances. In the case of the Standard 5 and Standard 10 groups pupils from another Durban school were used and for the student teachers another College of Education was used for the pre-planning.

The questionnaire (Appendix 5) for the Standard 10 stage was prepared to include key attitude and anxiety elements that have been discussed in the previous chapters of this thesis. The questionnaire (Appendix 5) for the Standard 10 stage was prepared for the specific purpose of investigating learner characteristics which were implicit in the literature study of the sources of mathematics anxiety. In particular, connotations, myths and attitudes related to the needs, abilities, perceptions and experiences of the students were explored by means of this questionnaire. Once again a test-retest reliability coefficient was calculated after a group of 150 Standard 10 pupils from another Durban school were asked to complete the questionnaire twice with a 2 week break between the first and second testing. The test-retest coefficient was calculated at 0,72 (n = 150).

Huysamen (1983:42) recommends that content validity should be evaluated on logical grounds by experts in the field involved. He sees this task as one of judging the following two aspects.

1. Do the items adequately represent the tasks as defined by the test constructor?
2. Do the written items represent the execution of these tasks?

For the purpose of confirming content validity, Professor M. Thurlow, Dean of the Faculty of Education at the University of Natal, Durban was consulted. After careful scrutiny and an insistence that this researcher justifies each item he was able to agree that the above two requirements had been met.

The Primary mathematics test was designed to test the mathematical performance of the students at the Standard 5 level. This test would represent an independent assessment of mathematics ability and also provided the necessary common test for all the students at this level. A rating scale graded from 'Very Easy' to 'Very Difficult' on a five point scale (see Appendix 3) was attached to the Primary mathematics test. Pupils were able to indicate the perceived degree of difficulty of each of the 25 questions they had answered.

The standardised Primary School test was designed to contain mathematical questions appropriate for Standard 5 students but care was taken to provide questions which would test insight and not purely mechanical processes. For example, $0,4 \times 0,4 = 0,16$ may not provide an indication of an understanding of the place value in the multiplication of decimals. However, $0,3 \times 0,3 = 0,09$ requires a clearer understanding of this concept.

Arrangements were made to record the mathematics and English marks throughout each pupil's secondary school career.

A more precise description of the instruments used is provided in the next section.

5.4 Instruments

A copy of each instrument used has been included in the Appendix. A total of five instruments were used to provide the data for this study.

The three mathematics anxiety rating scales used in Standard 5, Standard 10 and at the College of Education were all similar and adapted from the MARS test as indicated in Appendix 8. However, more items were added at Standard 10 and the College of Education level because these items were able to include questions involving calculators, geometry, formula, computers, etc. The College of Education instrument also differed in that it included personal questions to provide background knowledge of each student (see Appendix 6).

The attitude questionnaire used at the Standard 10 level was constructed by this researcher to probe elements of the literature study on mathematics anxiety. Questions therefore pertain to elements of mathematics myths, mathematics teaching, social influences and cognitive procedures. The intention of this questionnaire was to provide empirical evidence to support the sources of mathematics anxiety which were discussed in Section 2.2.2.

5.4.1 **Instruments used at Standard Five level** (Administered, March 1985)

1. Standard Five Mathematics Anxiety Rating Scale - a Likert-type scale consisting of 22 items with a required response ranging from "Not at all" (1 point) to "Very much" (5 points), (see Appendix 1).
2. Standard Five standardised mathematics test - a 25-question test designed to provide some insight into the mathematics ability of each pupil, (see Appendix 2) together with an Easy/Difficult rating scale (see Appendix 3).

5.4.2 **Instruments used at Standard Ten level** (Administered, September 1990)

1. Standard Ten Mathematics Anxiety Rating Scale - a Likert-type scale consisting of 30 items requiring responses from "Not at all" (1 point) to "Very much (5 points), (see Appendix 4).
2. An attitude questionnaire
   Consisting of 40 items relating to anxiety based situations. A Likert scale of responses ranging from "Very true" (4 points) to "Not at all true" (1 point), (see Appendix 5).

5.4.3 **Instrument used at College of Education** (Administered, April 1985)

A mathematics anxiety rating scale - a Likert-type scale consisting of 30 items requiring responses from "Not at all (1 point) to "Very much" (5 points). Personal details of mathematics achievement and the level of mathematics that the student could feel confident teaching were also included, (see Appendix 6).

5.5 <u>Procedures and administration of tests</u>

All tests were administered by the researcher.

In 1985 the target group of Standard Five pupils were given the Mathematics Anxiety Rating Scale, designed for this age group, to complete. They were asked to respond to twenty-two (22) items describing situations which could possibly arouse anxiety.

The Standard Five group was then given the Primary Mathematics Test which consisted of 25 questions involving mathematical calculations appropriate for testing the ability of students at this level. No time limit was set for this test.

Finally the pupils were asked to assess the degree of difficulty of each of the questions they had just completed. The four mathematics teachers at this level at the school were also asked to complete the degree of difficulty assessment for each question.

From 1985 to 1990 the final examination marks in mathematics and English for each pupil was recorded.

In 1990 two further tests were administered towards the end of the year (September). Firstly, the group were once again asked to complete a Mathematics Anxiety Rating Scale which, this time, consisted of thirty (30) responses to various anxiety arousing situations. The extra items were added to include questions which now involved calculators, geometry, formula, computers, etc., (see Appendix 5).

The students were then asked to respond to a set of items on a questionnaire consisting of forty items on mathematics anxiety related issues.

The latest I.Q. score for each participant was also recorded and the final matric result for mathematics and English was recorded.

A parallel study on student teachers in the 1985 year was carried out at a local College of Education. 148 student teachers were asked to complete a mathematics anxiety rating scale consisting of 30 responses and provide certain personal details.

5.6 Analysis of data

The data was obtained from the five instruments described earlier in this chapter. For convenience, this data has been organised in the various phases of the study. The College of Education survey, the Standard 5 research and the Standard 10 research is detailed separately at this stage. Later a more general discussion will be included.

5.6.1 Data Organisation

A. The Standard Five Mathematics Anxiety Rating Scale responses were scored as follows:

Not at all	1)	
A little	2)	
A fair amount	3)	total of 22 responses
Much	4)	
Very much	5)	

A total score was calculated for each participant.
Possible range was 22 (low) to 110 (high anxiety rating), (see Appendix 1).

B. The Primary Mathematics Test consisting of 25 questions (see Appendix 2) was scored as 1 mark for each question. Percentages were then calculated for each candidate.

The degree of difficulty assessment for each question was rated from 1 for "Very easy" to 5 for "Very difficult", (see Appendix 3).

C. The Standard Ten Mathematics Anxiety Rating Scale responses (see Appendix 4) were scored as follows:

Not at all	1)	
A little	2)	
A fair amount	3)	total of 30 responses
Much	4)	
Very Much	5)	

A total score was calculated for each respondent.
Possible range 30 (low anxiety) to 150 (high anxiety).

D. The Standard Ten questionnaire (see Appendix 5) was rated as follows:

Very true	4)
Sort of true	3)
Not very true	2)
Not at all true	1)

The mean score was calculated for each of the 40 responses.

E. The College of Education mathematics anxiety rating scale (see Appendix 6) was rated as follows:

Not at all	1)	
A little	2)	
A fair amount	3)	A total of 30 responses
Much	4)	
Very much	5)	

A total score was calculated for each respondent. Possible range 30 (low anxiety) to 150 (high anxiety). The school mathematics achievement of each student was recorded as well as the school level that each student felt confident to teach.

A full description of the procedures involved at the College of Education, at Standard 5 level and at the Standard 10 level are now provided.

5.6.2 <u>The College of Education survey</u>

Mr Mike Keeley (Personal Interview, 1985), acting Head of the Mathematics Department of Edgewood College at the time, expressed concern about the general standard of mathematics of all students and recognises the fact that for many of the teachers who will be teaching primary school children their first mathematics lessons, are in fact uncertain of their own mathematical ability. With the grateful help of the Edgewood staff, the mathematics anxiety rating scale (Appendix 6) was administered to all first year students. As all first year students are required to select one of two courses in mathematics, a short questionnaire was added to the mathematics anxiety rating scale. These first year trainee teachers consisted of two main groups:

1. Those specialising in mathematics;
2. Those doing a general course in mathematics.

The special course in mathematics is the more advanced and is aimed at training teachers who would teach mathematics to Phase 3 in a secondary school (i.e. to Standard 7). The general course is taken over three years preparing students to teach mathematics at primary school level. They would then specialise in subjects of their choice in their fourth year. These students would probably teach mathematics in the primary school. It is worth noting that students who had obtained the best matriculation results were not necessarily those enrolled in the special major mathematics course. This information was gathered from the five questions included in the first part of the mathematics anxiety rating scale, (see Appendix 6).

5.6.2.1 **The aims of the survey rating scales**

1. To construct and use a mathematics anxiety scale which may provide a basis from which to develop a mathematics anxiety scale suitable for college students in South Africa.
2. To discover the level of mathematics anxiety amongst first year trainee teachers at a College of Education.
3. To investigate the level of personal school mathematics achievement amongst trainee teachers at a College of Education.
4. To investigate the highest level to which each student is confident to teach.
5. To provide empirical support to literature claims that teachers perpetuate mathematics anxiety (see 2.2.2.1) by transferring these anxieties to their pupils.

5.6.2.2 **The questionnaire**

This was designed to indicate firstly some general information about each student and secondly to measure their degree of mathematics anxiety (see Appendix 6). The students were not required to provide their names but there were five questions in the first section requiring some details of mathematics achievement and other personal details. The second section consisted of thirty items on a mathematics anxiety rating scale. Once again the full MARS test was considered to be too long and items which were considered relevant to this population were chosen (see 5.3).

5.6.2.3 **The sample**

The questionnaire was administered to 148 first year students at Edgewood College of Education. Details of these 148 students are given below:

31 Males	
117 Females	
33 Special Major Course	(17 Males, 16 Females)
115 General Course	(14 Males, 101 Females)

16 did not take mathematics to Standard 10.
91 obtained a standard grade mathematics pass.
41 obtained a higher grade mathematics pass.

5.6.2.4 **The results**

1. The information gained from the 5 questions on personal details which preceded the mathematics anxiety rating scale revealed that 37 students rated themselves competent to teach only to Standard 4, whilst 111 students rated themselves competent to teach to a level higher than Standard 4. Thus 75% of the students regard themselves as being competent to teach to Standard 5 and higher. A complete breakdown of this information reveals the data depicted in Table 1:

   Considering the mathematics results of many of these students it is rather surprising that so many students feel confident enough to teach to high school pupils. There was no relationship between this perceived level of confidence and the students matriculation mathematics results (i.e. questions 4 and 5; see Appendix 6). In fact some students who had not done well at school felt they could teach a higher level than many who had done well at school.

TABLE 1: Confidence levels of students

Highest Standard that student felt confident to teach	No. of Students
None	5
1	11
2	3
3	7
4	11
5	39
6	7
7	31
8	22
9	3
10	9

2. With the confidence indicated in the first set of data, one would expect the mathematics anxiety levels to be relatively low. However, contrary to this, the anxiety levels were high. The 30 item test yielded a mean of 70,66 and a standard deviation of 17,64. (Possible high score 150 and low score 30). These figures reveal a higher level of mathematics anxiety than that reported by Suinn et al (1972) on the 98 item MARS Test i.e. 187,3 mean and 55,5 standard deviation. (Possible high score 490 and low score 98).

3. The five items which were most anxiety provoking were found to be in this order:
   (a) Waiting to get the results of a mathematics test in which you expected to do badly.

   (b) Receiving your final mathematics results in the post.

   (c) Studying for a mathematics examination.

   (d) Not having the formula needed to solve a particular problem.

(e) Being called upon unexpectedly to recite in a mathematics class.

The five items which measured the least anxiety were found to be in this order:

(a) Entering a mathematics class.

(b) Listening to a lecture in a mathematics class.

(c) Working with a calculator.

(d) Watching a lecturer do mathematics on the board.

(e) Discussing a mathematics problem with someone in your class who does well at mathematics.

The test situation is evident as a high anxiety provoking factor. However, for these student it appears that the results of the testing and the studying for the test cause more anxiety than the test itself. Although the passive classroom situations and the lecturer figure are not rated high anxiety provoking items, the data revealed that the judgement of the lecturer or fellow students is a crucial factor influencing anxiety.

4. Using the students' past matriculation mathematics result as a guide to the level of mathematics competence, the analysis of anxiety scores depicted in Table 2 was obtained.

The pattern of results here is such as would be expected and although a true statistical test of correlation cannot be performed, these results indicate that the level of anxiety is higher for those students whose mathematics performance is poorer.

TABLE 2 Anxiety Scores

	No. of Students	Mean Anxiety Score	Standard Deviation
Students who had not taken mathematics to Standard 10	16	91,75	17,88
Students with a standard grade mathematics pass of D symbol or below	44	73,25	14,5
Students with a standard grade mathematics pass of C symbol or above	46	69,06	15,31
Students with a higher grade mathematics pass of D symbol or below	30	65,53	13,9
Students with a higher grade mathematics pass of C symbol or higher	12	49,25	10,05

5.6.2.5 **Discussion**

1. The reason that perceived teaching competence level does not relate to the mathematics anxiety rating could be because students tended to write down the level they were intending to teach rather than what they truly felt competent to teach. It should be noted here that many of the students who were enrolled for the special major course of mathematics were not the top mathematics matriculants in the first year group.
2. The level of mathematics anxiety does appear to relate to past mathematics achievement with the non-matriculant mathematics students proving to be the most anxious and the top mathematics matriculants showing least anxiety. This would seem to confirm that the anxiety questionnaire did indeed measure anxiety although more data would need to be gathered to

confirm this.

3. The points raised at the beginning of this limited survey may be answered as follows:
   (a) Mathematics anxiety scales can be constructed by adopting the MARS items to suit students at a South African College of Education.
   (b) The level of mathematics anxiety of first year trainee teachers at Edgewood College of Education is generally very high.
   (c) The present level of mathematics anxiety appears to be negatively correlated with past mathematics achievements, (i.e. higher anxiety levels correlated with lower mathematical achievement).
   (d) The level of perceived teaching competence is related more to the course selection (i.e. Junior Primary, Senior Primary, Specialising) than to any anxiety rating or mathematics achievement.
4. Once again one must be warned against generalising from these results. The intention of this survey was not to obtain any type of concrete evidence but more to illustrate general trends in mathematics anxiety measurement and to use these ideas in the South African situation. This type of testing could form the basis of a more comprehensive study which could indicate long-term requirements in a solution to mathematics anxiety. In addition the survey reveals a need for concern in teacher training and the quality and competence of teachers being trained for the early secondary school classes. A full set of results of this study can be found in Appendix 9.

5.6.3 **Research at Standard 5 level**

The procedures described here take the form of a case study as a select group of Standard 5 pupils all attending the same school were the sample used for this survey. Whilst the Standard 5 level of procedures forms a study in its own right, the intention was to monitor the progress of the students for the remainder of their school career.

As a case study the results of this survey may not be used for generalisation and are not necessarily comprehensive and conclusive. However, the results and the research material used provide a basis for relevant discussion on the topic of mathematics performance and/or mathematics anxiety.

The statistical results of this research are intended to give support to the literature claims made in Sections 2.1.3.1 and 2.1.3.2 and stated in Section 5.1. It is intended that this study will give added substance to the research provided in these sections which shows that mathematics anxiety exists and has a negative effect on mathematics performance.

5.6.3.1 **The aims**

1. To gain information on the suitability of administering a mathematics anxiety questionnaire to primary school pupils.
2. To investigate the mathematics performance of the pupils in Standard 5.
3. To investigate the childrens' perception of the degree of difficulty of various mathematics topics.
4. To investigate teachers' perception of the degree of difficulty of various mathematics topics.
5. To investigate whether or not there would be a significant negative correlation between scores on the mathematics test and scores on the mathematics anxiety rating scale.

A description of the testing procedure is outlined below.

5.6.3.2 **The tests**

1. A mathematics anxiety rating scale was constructed for this survey (see Appendix 1). This scale consisted of 22 items relating to mathematical incidents and was adapted from the MARS test according to the criteria described earlier in this section.
2. A primary mathematics test was constructed and consisted of 25 questions (see Appendix 2).
3. A Rating Scale (on a five point system ranging from very easy to very difficult) was attached to the Primary Mathematics Test (see Appendix 3). Pupils could indicate the perceived degree of difficulty of each of the 25

questions they had just answered.

5.6.3.3 **The sample**

56 girls and 53 boys in Standard 5 at a Durban Primary School. The pupils had been streamed in an A, B, C and D class and the mathematics teachers of these classes assisted in the experiment.

All 109 of the pupils completed the Primary Mathematics Test and the Degree of Difficulty Scale. However, 9 of the Mathematics Anxiety Scales were incomplete and were not used in the results. These 9 included 6 where responses to questions were not given and 3 where respondents provided 2 or more responses to each question.

5.6.3.4 **The method**

1. The four mathematics teachers of the classes at this primary school were asked to complete the "Degree of Difficulty" rating scale to indicate how their pupils would cope with the questions.
2. The teachers then asked their pupils to complete the mathematics anxiety rating scale. No time limit was set.
3. The pupils were then required to complete the Primary Mathematics Test. No time limit was set.
4. Finally the pupils were asked to assess the degree of difficulty of each of the questions they had just completed.

5.6.3.5 **The results**

1. A full set of results of the mathematics anxiety rating, the results of the primary mathematics test compared with the results of the schools end of first term results and the results of the degree of difficulty of each question in the primary mathematics test can be found in Appendix 10. It was clear from the mathematics anxiety scale that many of the pupils were highly anxious about certain situations. The following items were rated most anxiety provoking by the pupils:

1. Waiting to get a mathematics test returned in which you expected to do badly.

2. Thinking about a mathematics test one hour before writing.

3. Writing a mathematics test.

4. Thinking about a mathematics test one day before.

5. Studying for a mathematics test.

The least provoking situations were as follows:

1. Watching a teacher doing mathematics on the board.

2. Playing cards where numbers are involved.

3. Sitting in your mathematics class.

4. Checking your change after buying several items at a shop.

5. Watching someone work with a calculator.

These results indicate that anxiety at this early age is probably strongly test-related. Active classroom issues (i.e. answering questions or explaining a problem) were next in line whilst passive classroom issues (i.e. sitting in class, watching a teacher do mathematics) and every-day life events revealed little anxiety.

After discussion of these issues with the students it was evident that many of the items normally included on the MARS questionnaire are not suitable for pupils of 11 and 12 years of age. Some of the reasons for this are:

(a) Calculators and computers are not a necessity or a threat at this early age. They are not expected to be competent with these instruments at this time in their life.

(b) Shopping problems are not of great consequence because they do not do the shopping and are hardly ever involved in purchasing many articles at a time.

(c) Calculating G.S.T. is not anxiety provoking because they do not buy expensive articles and when spending money on a 30 cent sweet approximate value of G.S.T. is all that is necessary.

(d) Adults who are not confident of their arithmetical ability feel more threatened in public situations (such as shops) where their mathematical inadequacies may be detected.

2. From a possible high score (on the mathematics anxiety scale) of 110 and low score of 22 (22 items rated 1 to 5) the mean score was 49 and the Standard Deviation 15,9. The correlation between mathematics anxiety and performance on the Primary Mathematics Test was -0,39, (level of significance 0,05%). The results are similar to the normative data published by Suinn *et al* (1972). On a sample of 119 State University students the mean was 187,3 and the Standard Deviation 55,5, (using the 98 item MARS with possible high score of 490 and low of 98) whilst the correlation between MARS and Performance was recorded as -0,35.

However, contrary to the findings of researchers with university students this primary school survey revealed a similar level of anxiety pattern amongst girls and boys:

48 boys	Mean 48	Standard Deviation 12,1
52 girls	Mean 50	Standard Deviation 11,1

The Pearson Product Moment Correlations between anxiety ratings and Performance on the Primary Test were -0,32 for girls and -0,49 for boys, (level of significance 0,05%). It would appear that in this group the boys' anxiety affected their performance more than the girls. However, in both groups a significant negative correlation was found between anxiety and performance.

3. The Primary Mathematics Test (see Appendix 2) revealed some significant facts mainly pertaining to the questions attempting to reveal understanding or insight. Questions 8 and 11 appear to be the same and yet proved to distinguish between those who have learnt rules and those who understand decimals fully. Question 8 was 0,4 x 0,4 and this yielded 41% success rate. Question 11 was 0,3 x 0,3 and the success rate here was only 3%.

This result is significant because only the pupil who truly understands the place value of the decimal would arrive at the correct answer for question 11, whereas the answer to question 8 is likely to be correct even if the pupil does not understand place value. This is because the answer to 0,3 x 0,3 = 0,09 requires a zero after the decimal comma whereas 0,4 x 0,4 does not require such an adjustment.

Another significant factor which was revealed by these questions was the fact that 40% of the pupils who rated Question 8 easy or very easy, got it wrong. Whilst 77% of the pupils who rated Question 11 easy or very easy, got it wrong. In addition, of the four teachers asked to complete the degree of difficulty scale, one teacher rated both Question 8 and 11 very easy whilst two teachers rated them as easy.

This aspect is significant when one considers the emphasis given to understanding, communication, social interaction and reflection in the constructivist approach to teaching (see 4.3). The pupils do not fully understand place value in the multiplication of decimals and are making an obvious error. The teachers themselves are unable to identify questions which would reveal any lack of understanding. The constructivist classroom (as described in 4.4) takes cognisance of this fact by emphasising the need for teachers to listen to pupils' solutions and for the pupils themselves to reflect on how they reached such a solution.

Similar results were revealed by other questions such as

$\frac{2}{x} = \frac{1}{3}$ (79% correct) and $\frac{x}{8} = \frac{3}{12}$ (45% correct)

The idea of ratios has to be understood to do the second question whilst the first one could be arrived at by doubling 3 or by knowing equivalent fractions. Once again the path that a pupil takes to reach the solution is all important for the teacher to provide assistance.

Adding decimals is not necessarily understood by pupils who can add 298,78; 72,36 and 13,89 (77% correct). Question 17 requires adding decimals with a different number of digits after the comma, 16,36; 1,9 and 243,075. This question yielded only a 59% success rate, whilst all four teachers regarded it as either easy or very easy and 33% of the pupils who regarded it as easy or very easy, got it wrong.

The questions which were related to problems which may be encountered in everyday life revealed significant results. Questions 16 and 22 required the relatively simple arithmetic processes of $2 \div 8$ and $100 \times 0{,}52$ respectively. However, only 45% (question 16) and 14% (question 22) of the students were able to give the correct answers. In contrast questions 1 and 23 required similar arithmetic knowledge of decimals ($37 \div 100$) and division ($816 \div 8$) and 69% (question 1) and 66% (question 23) of the students were able to answer these two questions. This appears to indicate that the link between classroom mathematics and the practical application of the various processes is often lost.

Here again, one of the reasons why constructivism is proposed in this thesis is because it is an approach which addresses the call for mathematics to relate to real-life problems (see 4.6.1.2). The constructivist approach to curriculum is more than a problem solving approach, it is rather a development of a problem solving attitude. Glatthorn (1988:9) emphasises that a renewed mathematics curriculum for the twenty first century must include a development of problem solving skills to solve mathematical problems which include real-life problems.

The average score on the primary school test was 51% whilst a recent end-of-term test written by all the pupils in the target group yielded an average of 66%. In view of the fact that the primary test was constructed to identify true understanding of the concepts tested and included problems related to real-life, it was deemed more appropriate to use these results rather than the regular test results provided by the teachers. Hence, the correlations with mathematics anxiety were measured with the primary test scores.

5.6.3.6 **Discussion**

A full set of the results of this survey together with copies of the survey material are provided in the appendix, (see Appendix 1, 2, 3 and 10). After studying these results the following answers are given to the questions posed at the beginning of this research (see 5.6.3.1).

1. The mathematics anxiety questionnaire could be adapted to suit primary school children. The results indicate that items relating more to test situations are the most anxiety provoking at this age. The high negative correlation of mathematics anxiety scores and mathematics performance indicates that the mathematics anxiety scale did indeed measure mathematics anxiety.
2. The insight-type of question does identify those pupils who do not fully understand a topic.
3. The pupils have difficulty assessing the degree of difficulty of an insight-type of question (i.e. a question which reveals understanding of a concept).
4. The teachers have difficulty assessing the degree of difficulty of questions which are able to reveal the contextual understanding of the pupil.
5. The results reveal a negative correlation between mathematics anxiety rating scores and mathematics performance of the Primary Test.

Other significant findings were as follows:

1. Marks obtained on the Primary Mathematics Test differed considerably from those that the pupils had recently received in the school's end-of-first term test, (see Appendix 10). This suggests that the added element of insight-type questions is a distinguishing feature when mathematics performance is assessed at this level of schooling.
2. Pupils show signs of anxiety at an early age and this anxiety has a negative effect on their performance.
3. There appears to be a need for more of practical applications of classroom mathematics to everyday life problems.

In closing the discussion it should be mentioned that these findings may not be taken as conclusive evidence nor should they be generalised. However, the ideas incorporated in this small survey served as a stimulus for the larger longitudinal study which follows.

5.6.4 <u>Research at the Standard 10 level</u>

After monitoring the results of the students over the next five years a further study was undertaken once the majority of students had reached the Standard 10 level. As mentioned in Section 5.1, a number of students were not included in the case study at this level due mainly to the fact that they had left school or were no longer doing mathematics. This group provide interesting data in themselves in that their anxiety levels were generally higher than average. However, the exclusion of their testing at the Standard 10 level would appear to have diminished the significance of the Standard 10 anxiety ratings and the answers to the questionnaire.

5.6.4.1 <u>The aims</u>

1. To construct and use a mathematics anxiety rating scale which may provide a basis from which to develop a suitable instrument for the South African situation.
2. To investigate the level of mathematics anxiety of the same students who were tested at the Standard 5 level.
3. To construct and administer an attitude questionnaire which investigates learner characteristics pertinent to anxiety provoking situations.
4. To collect data pertaining to the performance in both English and mathematics from Standard 5 through to Standard 10.

5.6.4.2 <u>The instruments</u>

The instruments used were fully described in Section 5.4 but are listed here once again:

1. Mathematics anxiety rating scale to provide a measure of the level of mathematics anxiety at the Standard 10 level (see Appendix 4).

2. A questionnaire of 40 items to investigate learner characteristics and attitudes in specific situations (see Appendix 5).

5.6.4.3 **The sample**

The sample eventually consisted of 55 of the original 100 students who were tested in Standard 5. The reasons for the reduction in numbers is explained fully in Section 5.2, and is listed in Appendix 7. The 55 students comprised 29 Males and 26 Females.

5.6.4.4 **The results from the questionnaire data**

The results of the testing at the Standard 10 level need to be divided into two distinct categories:

1. The questionnaire which was used to attempt to identify certain aspects of mathematics anxiety which may or may not influence the students attitude and performance in the subject.
2. The final results of the longitudinal study of students having been monitored over a six year period. The full details of this data collection can be found in Table 5 which is described fully in Section 5.6.4.5.

The first survey questionnaire forms a basis for discussion on relevant issues relating to mathematics anxiety which could influence the teaching situation.

The second collection of data provides a basis for a more scientific analysis of data. This will include the hypothesis testing of parametric data.

The data from both these procedures is explained in the presentation that follows.

The mean scores of the responses to each question in the Standard 10 questionnaire are listed in Table 3. Responses were graded on a 4-point scale ranging from 1 for "Not at all true" to 4 for "Very true".

The questionnaire consisted of 40 questions relating to anxiety provoking situations, symbol 'A', and the myths associated with mathematics that may cause anxiety, symbol 'M'. The two types of questions were assigned randomly. A high mean score indicates that the majority of students responded "True" or "Very true" to the item.

The motivation for the construction of this questionnaire is provided in Section 5.3. The questions are compiled to probe the claims made in the literature of the sources of mathematics anxiety (see 2.2.2) and in particular aspects described under the heading of "emotive factors" (see 2.2.2.2) and detailing connotations and myths commonly linked to mathematics.

The significance of the mean scores listed in Table 3 are discussed under learner characteristics in Section 6.3.8. These learner characteristics can be categorised under certain headings which will require attention during the situation analysis stage of curriculum development (see 6.3.8).

a) Needs of the learner
b) Ability of the learner
c) Perceptions of the learner
d) Experiences of the learner
e) Aspirations of the learner

Attention to these learner characteristics is intended to provide support for a more humanistic and learner-centred approach to the teaching of mathematics.

TABLE 3: Mean scores of responses to questions

ITEMS		MEAN
1.	I get anxious when confronted with a mathematics problem.	2,42A
2.	I understand mathematics but my anxiety has a negative effect on my performance in class.	1,99A
3.	I understand mathematics but I make too many careless errors.	3,29A
4.	My teachers cause me to be anxious about mathematics.	1,80A
5.	My parents cause me to be anxious about mathematics.	1,67A
6.	I get anxious when confronted with a mathematics test.	2,53A
7.	I get anxious when writing a mathematics examination.	2,92A
8.	I feel more anxious when writing a mathematics examination.	2,51A
9.	The mathematics I learn at school is mostly facts and methods that have to be monitored.	2,42M
10.	Mathematics ability is a gift that only some people have.	2,82M
11.	In mathematics something is either right or it is wrong.	3,15M
12.	In mathematics you can be creative and discover things by yourself.	2,40M
13.	Mathematics problems can be done correctly in only one way.	1,56M
14.	Real mathematics problems can be solved by common sense instead of the rules you learn at school.	2,52M
15.	To solve mathematics problems you have to be taught the right procedures or you cannot do anything.	2,80M
16.	The best way to do mathematics is to memorise all the formulae.	2,69M
17.	The best way to do mathematics is to use your own initiative.	2,90M
18.	The best way to do mathematics is to persevere till you hit on the right procedure.	3,31M
19.	The best way to do mathematics is to ask someone to help you find the right procedure.	2,64M

ITEMS		MEAN
20.	The best way to do mathematics is to practice different procedures.	3,49M
21.	Over the years I have become more confident in my ability to do mathematics.	2,84A
22.	I have coped with school mathematics but will never be confident in my ability to do mathematics.	2,38A
23.	Mathematics has become less fun than it was at first.	2,62A
24.	Mathematics has never been a subject I enjoyed.	2,31M
25.	Mathematics is done by working intensively until the problem is solved.	3,31M
26.	Mathematicians do problems quickly in their heads.	2,49M
27.	There is a magic key to doing mathematics.	2,20M
28.	My mathematics ability is related to the mathematics ability of my parents.	1,69M
29.	Some people have a mathematics mind and others do not.	3,00M
30.	Mathematics is not a creative subject.	2,73M
31.	Mathematics requires logic, not intuition.	3,16M
32.	I can do mathematics but I need more time.	2,95A
33.	The questions in mathematics are always difficult to understand.	2,47A
34.	It is always important in mathematics to get the answer exactly right.	3,11M
35.	I am never sure how to start a mathematics problem.	2,38A
36.	Calculators have made mathematics more difficult.	1,64A
37.	I am more confident about mathematics when I am able to discuss problems with my class mates.	3,10A
38.	I am more confident about mathematics when I am able to discuss problems with my teacher.	3,24A
39.	Boys are better than girls at mathematics.	2,15M
40.	I do not feel comfortable asking a question in a mathematics class.	1,93A

5.6.4.5 **The results from tests, examinations and the Mathematics Anxiety Rating Scales data**

The data collection from the Standard 10 mathematics anxiety rating scale is presented together with the results of the Standard 5 tests and the details of the longitudinal collection of data in table form (see Table 5).

The details provided on this table consist of the following:

Column

1	Latest I.Q. Score.
2	Percentage obtained on Primary Mathematics Test (Control Test).
3-14	English and Mathematics marks from Standard 5 to Standard 10.
15	Anxiety Rating for Standard 5 (maximum 110).
16	Anxiety Rating for Standard 10 (maximum 150).
17-18	The percentage for Anxiety Rating in Standard 5 and Standard 10 was calculated for comparison.

Notes relevant to Table 1:

1. Only the 55 pupils who had continued with mathematics to Standard 10 level were included in the final data analysis.

2. Where candidates had changed to a Standard Grade course, marks were converted back to Higher Grade to allow for standardised comparison. The recognised factor or Standard Grade mark multiplied by three over four was used.

3. To standardise the matriculation mark, symbols were converted to average percentages as indicated in Table 4.

The standardised method used conforms to the University norms for the allocation of points to Matric symbols for University entrance.

TABLE 4: Conversion of symbols to percentages

HG	SG	LG OR FUNC	AVERAGE	PERCENTAGE
A			90	100 - 80
B			75	79 - 70
C			65	69 - 60
D	A		55	59 - 50
E	B		45	49 - 40
F	C		37	39 - 33
FF	D	A	32	33 - 30
G	E	B	27	29 - 25
GG	F	C	22	25 - 20
H	G	D	10	19 - 0
Below this			5	

When results of applicants are compared by universities to consider the potential of each prospective student there is a need to construct a method which compares results of matriculation symbols gained on the various levels of higher grade, standard grade, lower grade and functional grade. Points are then allocated for each subject and applicants with the highest number of points will gain entrance.

This point system is not relevant to this study but it does provide an idea of how the perceived academic value of, say an A symbol at higher grade for mathematics compares with an A symbol at standard grade for mathematics. For this reason the table above was used to provide a standard comparison of the matric results of the students in the target group.

TABLE 5: Data from the longitudinal study

SCHOOL CODE	M/F	IQ	STD5 CONT TEST %	1985 Std.5 Engm %	Mark Math %	1986 Std.6 Engm %	Mark Math %	1987 Std.7 Engm %	Mark Math %	1988 Std.8 Engm %	Mark Math %	1989 Std.9 Engm %	Mark Math %	1990 Std10 Engm %	Mark Math %	ANXIETY RATING Std5 110	Std10 150	ANXIETY % Std5	Std10
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1 N	M	132	68	70	77	63	71	69	70	64	73	64	77	55	90	43	56	39	37
2 N	M	136	80	81	71	75	70	80	63	73	53	74	51	75	55	52	93	47	62
3 N	M	129	76	64	81	72	85	75	88	71	85	72	82	65	90	57	58	52	39
4 N	M	110	52	54	44	50	55	52	48	53	32	50	40	45	45	67	87	61	58
5 N	M	111	40	61	48	59	67	60	70	65	74	60	63	55	55	57	94	52	63
6 N	M	122	52	80	57	66	52	74	46	69	38	68	34	65	37	48	56	44	37
7 N	M	95	28	53	35	52	47	50	40	49	43	46	41	55	37	58	116	53	77
8 N	M	125	84	76	85	74	84	76	86	71	85	70	76	65	55	36	63	33	42
9 N	M	135	84	83	85	84	89	82	89	81	83	78	84	75	90	33	72	30	48
10 N	M	117	60	75	79	72	77	70	83	69	86	74	85	75	90	50	84	45	56
11 N	M	90	40	50	50	46	47	50	65	51	63	48	69	75	65	42	65	38	43
12 N	M	88	40	62	55	52	66	55	54	56	45	48	42	55	37	63	77	57	51
13 N	M	122	52	69	58	66	79	69	69	60	45	55	35	55	32	31	53	28	35
14 N	M	127	76	56	63	53	76	55	65	53	84	56	67	55	65	33	43	30	29
15 N	M	103	48	53	43	48	48	47	41	46	30	34	25	45	22	45	70	41	47
16 N	M	118	68	77	72	66	76	70	70	44	68	64	68	55	55	42	58	38	39
17 N	M	107	48	55	56	55	80	54	80	56	70	53	60	55	32	37	100	34	67
18 N	M	97	64	53	46	53	39	58	47	60	60	51	47	65	45	42	87	38	58
19 N	M	128	52	69	72	67	76	76	63	70	66	69	55	55	55	31	67	28	45
20 N	M	113	44	66	51	63	54	63	59	57	62	55	37	65	45	28	68	25	45
21 N	M	114	44	67	59	59	75	58	63	56	40	51	34	55	55	38	61	35	41
22 N	M	114	64	69	61	67	74	75	61	75	56	74	61	27	5	44	55	40	37
23 N	M	105	44	56	55	43	64	58	57	57	41	56	57	55	27	55	94	50	63
24 N	M	114	48	63	77	67	78	64	67	59	70	60	51	55	37	31	64	28	43
25 K	M	111	44	61	63	62	67	62	64	61	70	63	64	55	65	36	56	33	37
26 K	M	107	64	52	52	56	44	50	39	48	31	53	40	65	45	43	75	39	50
27 D	M	117	64	62	64	50	56	49	78	46	83	49	60	65	65	33	73	30	49
28 C	M	102	32	41	41	43	44	44	38	34	34	33	21	27	32	56	100	51	67
29 C	M	101	56	49	62	45	53	38	41	48	42	29	26	27	27	58	40	53	27
30 NG	F	117	64	74	63	64	72	60	63	62	45	57	42	75	55	63	96	57	64
31 NG	F	118	68	85	84	86	85	70	79	71	66	77	72	90	65	38	61	35	41
32 NG	F	128	36	75	63	71	72	62	58	65	43	66	70	75	55	49	81	45	54
33 NG	F	114	40	76	68	71	72	67	70	70	46	66	27	75	32	48	74	44	49
34 NG	F	122	44	76	65	69	61	62	65	62	31	60	47	65	45	42	86	38	57
35 NG	F	117	56	75	69	67	68	55	58	52	39	57	34	55	32	49	96	45	64
36 NG	F	119	44	64	61	57	66	53	65	58	59	58	66	65	65	48	80	44	53
37 NG	F	130	68	79	83	84	83	68	75	66	73	76	59	90	55	36	70	33	47
38 NG	F	99	44	72	46	64	57	61	64	64	40	62	49	45	27	60	61	55	41
39 NG	F	101	44	55	49	49	48	48	47	49	18	48	33	65	55	46	71	42	47
40 NG	F	127	64	74	69	67	74	65	82	70	63	68	62	65	37	45	116	41	77
41 NG	F	114	60	81	72	71	86	74	78	70	60	62	53	65	37	76	87	69	58
42 NG	F	120	72	78	82	81	85	74	71	70	60	68	50	75	55	54	85	49	57
43 NG	F	123	68	82	76	71	76	75	67	73	48	80	62	75	55	42	113	38	75
44 NG	F	111	56	69	68	71	72	64	72	63	62	58	54	75	75	52	90	47	60
45 NG	F	116	52	78	70	70	72	61	81	76	74	70	77	65	55	57	79	52	53
46 NG	F	127	48	59	57	44	69	47	52	46	23	41	37	45	32	42	53	38	35
47 NG	F	139	76	79	76	71	72	67	66	62	40	62	34	65	32	53	59	48	39
48 NG	F	113	40	64	56	72	66	60	70	64	51	70	59	65	55	61	73	55	49
49 NG	F	110	60	83	83	79	83	63	84	68	71	65	68	65	55	57	63	52	42
50 NG	F	117	52	66	57	71	68	59	60	65	61	66	62	65	65	51	65	46	43
51 NG	F	107	28	55	51	66	46	68	52	49	57	52	55	55	55	62	50	56	33
52 NG	F	123	64	74	70	67	77	69	71	70	49	65	53	65	45	50	58	45	39
53 DA	F	116	28	54	57	62	63	42	54	51	42	46	27	27	22	47	104	43	69
54 GC	F	118	44	62	54	69	69	69	67	71	64	62	59	55	32	46	76	42	51
55 GC	F	129	72	84	72	79	86	72	59	74	56	72	42	75	65	40	67	36	45

5.6.4.6 **Discussion**

At this stage it may be appropriate to make some observations concerning the initial aims of the Standard 10 survey which were stated in Section 5.6.4.1. These are general observations which will be substantiated by statistical procedures in Section 5.7.

1. The mathematics anxiety rating scale appeared to be appropriate but the strength of the statistical data, used for the hypothesis testing, was diminished due to the reduction in the number of students tested.

2. For many students, the anxiety levels were similar or higher at Standard 10 level when compared with those at Standard 5 level. (See Table 5 columns 17 and 18.) 37 of the 55 increased their anxiety score. Average anxiety score increased from 42,5 to 48,9.

3. The questionnaire proved to be valuable in identifying certain aspects related to mathematics anxiety which may provide teachers with an indication of how their students think.

4. The data collected on mathematics and English performance was used to investigate significant comparisons of progress in each subject. English was used as a "control subject" so that the trend in mathematics performance over the five years of secondary schooling could be compared with the trend in performance with another subject.

The mathematics anxiety rating scale used revealed that the items which attracted the highest anxiety rating response all described active participation and evaluative situations whilst the less anxiety-provoking items all described passive and non-evaluative situations (see 5.6.3.5).

The five highest anxiety-provoking situations were:

1. Waiting to get a mathematics test returned in which you expected to do badly.

2. Not having the formula needed to solve a mathematics problem.

3. Being faced with a mathematics problem unlike one that you have done before.

4. Thinking about a mathematics test one hour before.

5. Writing a mathematics test.

The least anxiety-provoking items were found to be:

1. Watching someone work with a calculator.

2. Playing cards where numbers are involved.

3. Working with a calculator.

4. Sitting in a mathematics class.

5. Checking your change after buying several items at a shop.

The mean scores for each item of the Standard 10 questionnaire are listed next to the items in Section 5.6.4.4. and each of these items should be discussed separately. An average score of over 2 reveals a high percentage of respondents believe that the description is either "true" or "very true". The following observations regarding the Standard 10 questionnaire are important.

1. High levels of mathematics anxiety are revealed by the responses to questions 1, 3, 6, 7, 8.

2. The mathematics anxiety levels are high because the items used all describe active and evaluative situations.

3. Teachers and parents are not perceived to be a major source of mathematics anxiety (see items 4, 5 and 28).

4. A greater confidence in mathematics ability is indicated in item 21 but not confirmed by the responses to item 22.

5. A number of mathematics myths are perceived to be "true" or "very true". Most particular, the idea that mathematics is a subject for the select few, is strongly believed (see items 10 and 29).

6. Other beliefs in mathematics myths are confirmed in items 16, 25, 30, 31, 34 and 39. The highest average score was 3,11 for item 34 which states that, "It is always important in math to get the answer exactly right."

A need for a more open approach to mathematics and more active participation by students is indicated by the responses to items 37 and 38. Students would obviously prefer more time to discuss their problems in mathematics with the teacher and their class group.

A more detailed discussion and synthesis of results will be conducted after the statistical data has been presented.

5.7 **Statistical procedures**

This case study includes many facets which will allow for interesting discussions later. However, it is essential that the formal statistical procedures of the longitudinal study are presented at this stage.

The longitudinal study involved a sample of 100 students originally in Standard 5 at a local school. The follow-up study at the Standard 10 level was restricted to 55 of the original students. The drop in numbers was mainly due to a number of students not continuing with mathematics as a subject or leaving school.

It must be stated from the outset that this study met with many of the difficulties related to any longitudinal study. In particular it was difficult to trace subjects even though they originated in one local community school. It was also difficult to maintain the co-operation of all the students. It was especially difficult to communicate with those students who had moved away and those who no longer pursued the same mathematical course. The expected emphasis on the mathematics anxiety at Standard 10 level was diminished because of the fact that many of the highly anxious students were excluded from the study by the time they had reached Standard 10. Being able to study mathematics at different levels (e.g. standard grade; functional mathematics) may also reduce anxiety.

5.7.1 **Hypothesis testing**

The following hypothesis were tested using the data collected as described in this chapter. A significance level within 0,05% was expected before consideration was given to reject the null hypothesis.

A. Null Hypothesis (H_{0A}) Mathematics anxiety has no affect on mathematics performance.

   Alternative Hypothesis (H_{1A}) There exists a negative relationship between mathematics anxiety and mathematics performance.

B. Null Hypothesis (H_{0B}) Mathematics anxiety does not continue to affect mathematics performance over the high school years.
Alternative Hypothesis (H_{1B}) Mathematics anxiety continues to have a negative effect on mathematics performance over the high school years.

C. Null Hypothesis (H_{0C}) Mathematics performance and English performance are equally predictable from prior performance.
Alternative Hypothesis (H_{1C}) Mathematics performance is less predictable than English performance when prior performance is considered.

D. Null Hypothesis (H_{0D}) The mathematics anxiety level does not affect failure in mathematics or discontinuance of mathematics by students.
Alternative Hypothesis (H_{1D}) High mathematics anxiety ratings are positively related to the failure rate and drop-out rate in mathematics.

Hypothesis D was only added at the final stage because it became obvious that data related to hypothesis A and B had become skewed because the final group tested had experienced better success with mathematics.

5.7.2 **Statistical measurements**

A number of correlations were calculated using the data in Table 5. The various data was found to conform to a normal distribution and therefore the Pearson Product Moment Correlations for parametric data were calculated.

5.7.2.1 **Correlations**

The correlations and significance levels are all presented in Tables 6 to 12. In each case the Pearson Product Moment Correlation co-efficient was calculated.

TABLE 6

	CORRELATION	SIGNIFICANCE LEVEL	N
ANXIETY STD.5 LEVEL vs ANXIETY STD.10 LEVEL	0,26	0,05	55

TABLE 7

	ANXIETY STD.5 LEVEL		
MATHEMATICS PERFORMANCE	CORRELATION	SIGNIFICANCE LEVEL	N
Mathematics 5	-0,22	0,10	55
Mathematics 6	-0,16	0,23	55
Mathematics 7	-0,16	0,23	55
Mathematics 8	-0,28	0,03	55
Mathematics 9	-0,12	0,34	55
Mathematics 10	-0,14	0,29	55

TABLE 8

	ANXIETY STD.10 LEVEL		
MATHEMATICS PERFORMANCE	CORRELATION	SIGNIFICANCE LEVEL	N
Mathematics 5	-0,18	0,30	55
Mathematics 6	-0,09	0,19	55
Mathematics 7	-0,03	0,49	55
Mathematics 8	-0,14	0,80	55
Mathematics 9	-0,09	0,30	55
Mathematics 10	-0,13	0,52	55

TABLE 9

I.Q. vs MATHEMATICS PERFORMANCE		I.Q. vs ENGLISH PERFORMANCE
Std.5	0,66	0,63
Std.6	0,64	0,60
Std.7	0,47	0,60
Std.8	0,29	0,51
Std.9	0,31	0,61
Std.10	0,33	0,34

Significance level 0,05
N = 55

TABLE 10

MATHEMATICS PERFORMANCE AT EACH LEVEL						
MATHEMATICS PERFORMANCE	STD.5	STD.6	STD.7	STD.8	STD.9	STD.10
Std.5	1	0,82	0,74	0,54	0,52	0,46
Std.6	0,82	1	0,78	0,51	0,49	0,31
Std.7	0,74	0,78	1	0,75	0,73	0,48
Std.8	0,54	0,51	0,75	1	0,79	0,59
Std.9	0,52	0,50	0,73	0,79	1	0,67
Std.10	0,46	0,31	0,48	0,59	0,67	1

Significance level 0,05
N = 55

TABLE 11

ENGLISH PERFORMANCE AT EACH LEVEL						
ENGLISH PERFORMANCE	STD.5	STD.6	STD.7	STD.8	STD.9	STD.10
Std.5	1	0,85	0,77	0,76	0,81	0,61
Std.6	0,85	1	0,79	0,78	0,85	0,60
Std.7	0,77	0,79	1	0,81	0,86	0,50
Std.8	0,76	0,78	0,81	1	0,87	0,51
Std.9	0,81	0,85	0,86	0,87	1	0,62
Std.10	0,61	0,60	0,50	0,51	0,62	1

Significance level 0,05
N = 55

TABLE 12

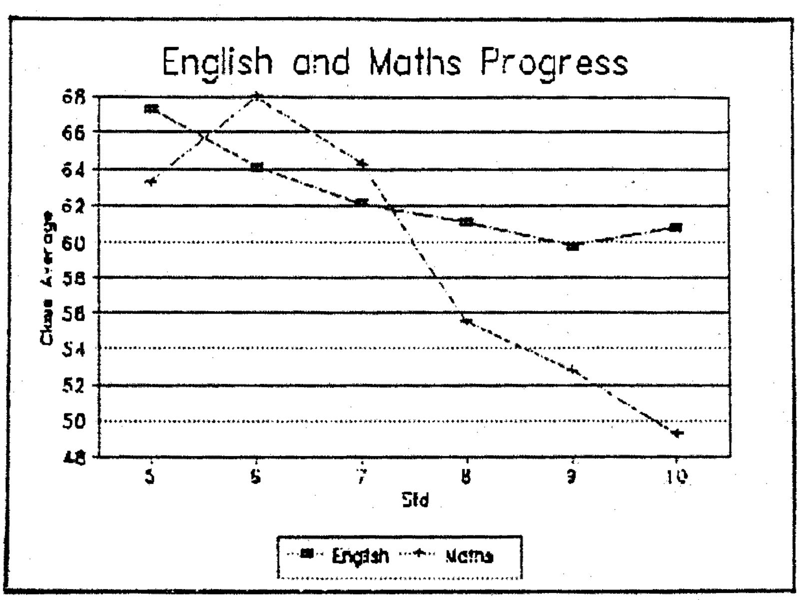
MATHEMATICS / ENGLISH PERFORMANCE AT EACH LEVEL						
	STD.5	STD.6	STD.7	STD.8	STD.9	STD.10
Std.5	0,76	0,75	0,65	0,62	0,70	0,51
Std.6	0,71	0,70	0,64	0,66	0,68	0,38
Std.7	0,61	0,63	0,59	0,63	0,67	0,49
Std.8	0,27	0,42	0,46	0,43	0,50	0,33
Std.9	0,34	0,42	0,49	0,50	0,64	0,42
Std.10	0,26	0,35	0,33	0,28	0,42	0,58

Significance level 0,05
N = 55

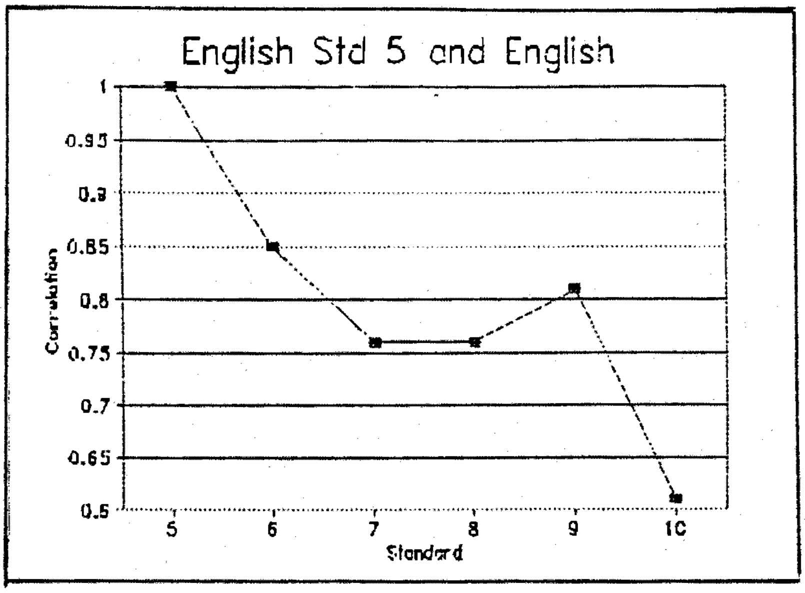
5.7.2.2 **Graphical presentations**

The following graphical representations were developed from the statistical data collected.

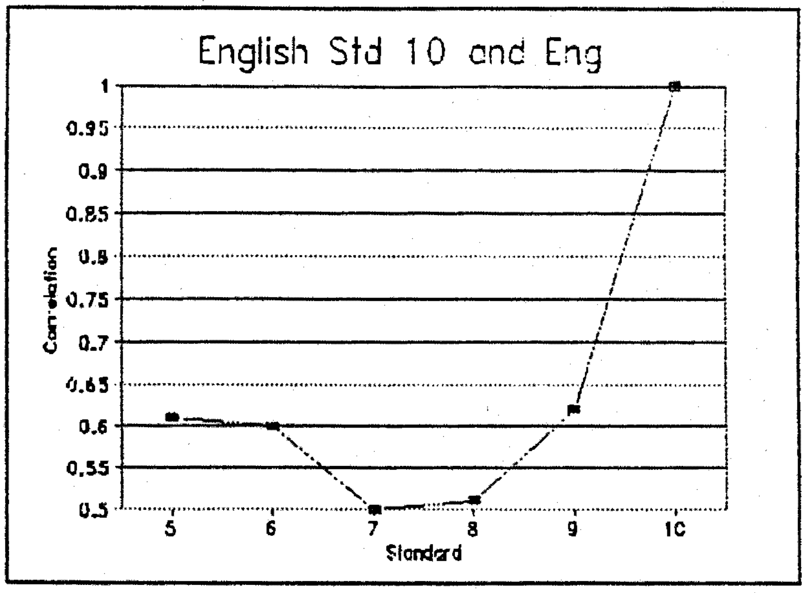
Graph 1 The average Mathematics Performance at each level is compared with the average English Performance at each level.



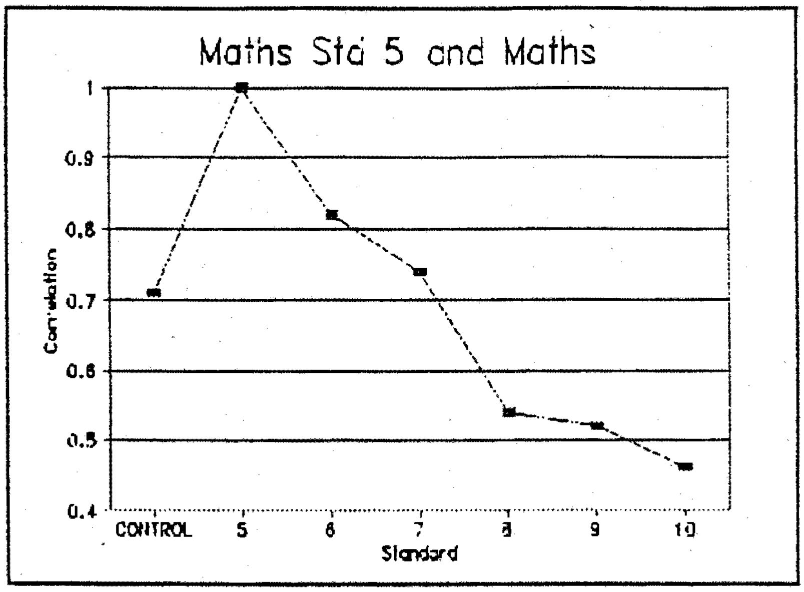
Graph 2 The English Performance at Standard 5 level had a positive correlation with the English Performance at the various other levels.



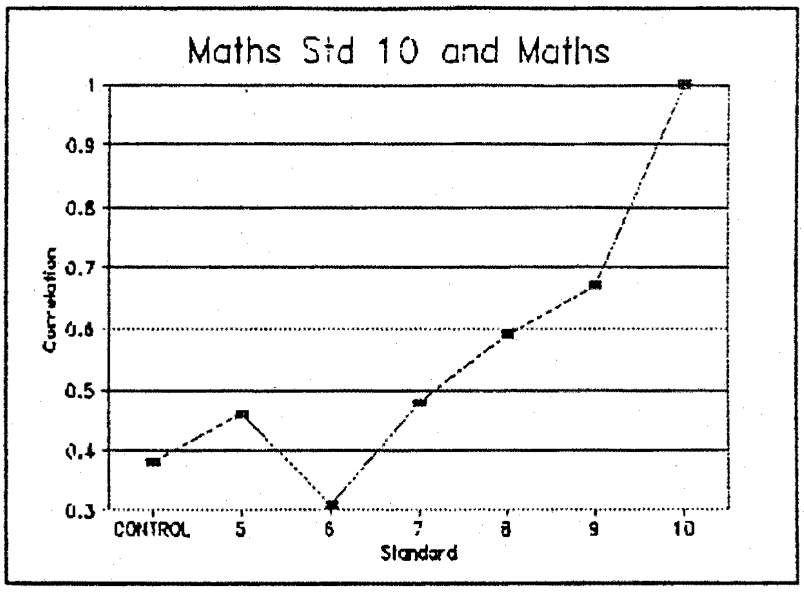
Graph 3 The English Performance at Standard 10 level had a positive correlation with the English Performance at various other levels.



Graph 4 The Mathematics Performance at Standard 5 level had a positive correlation with Mathematics Performance at other levels.

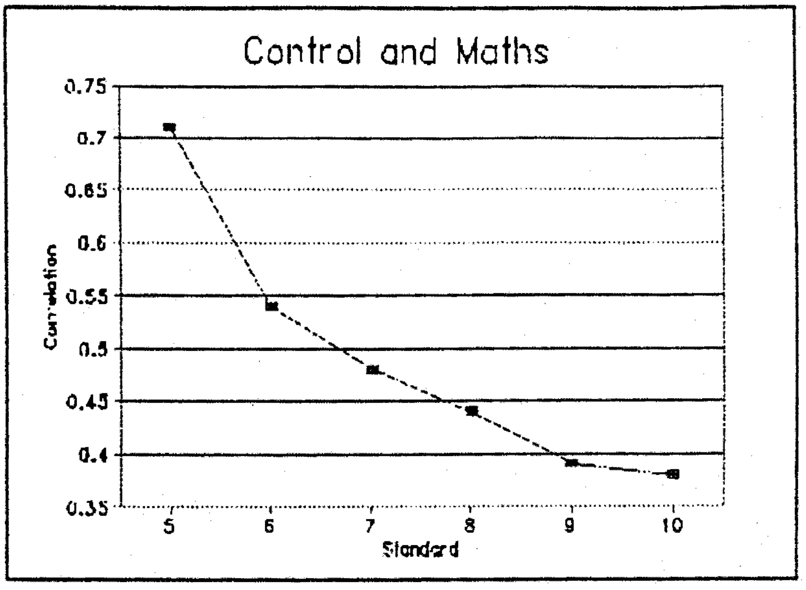


Graph 5 The Mathematics Performance at Standard 10 level had a positive correlation with Mathematics Performance at other levels.



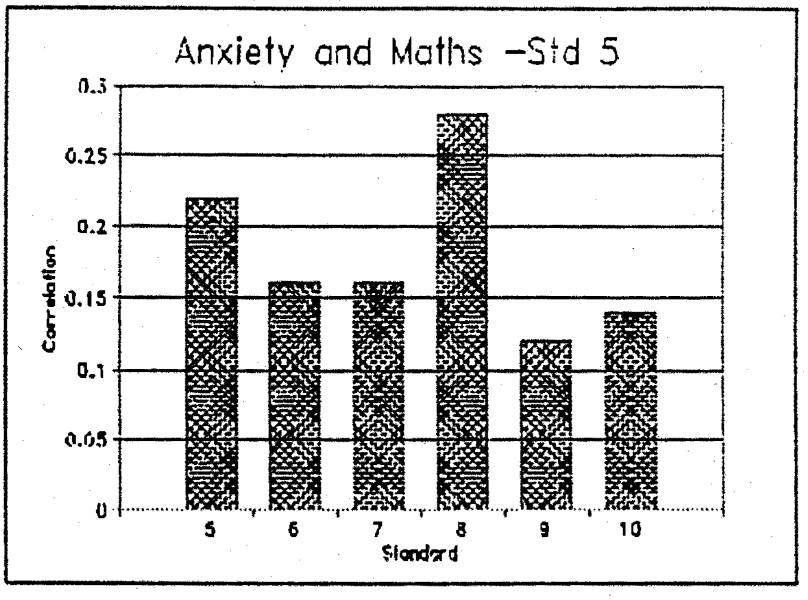
Graph 6

The Control Test written at Standard 5 level had a positive correlation to the Mathematics Performance at various levels.

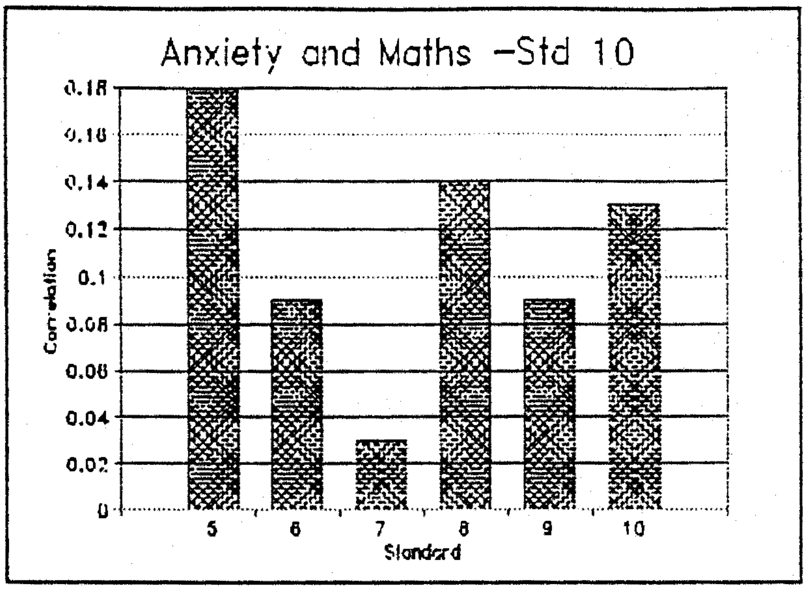


Graph 7

Anxiety ratings at Standard 5 level had a negative correlation with Mathematics Performance at various levels.



Graph 8 Anxiety ratings at Standard 10 level had a negative correlation with Mathematics Performance at various levels.



5.7.2.3 **The null hypothesis**

It should be noted that as a case study research,the generalisation of results is not appropriate. Therefore, the purpose of rejecting the null hypothesis as proposed from A to D would not have provided conclusive empirical evidence to support the central ideas of mathematics anxiety and mathematics performance expressed in these hypotheses.

As mentioned in Section 5.7 the longitudinal study presented too many difficulties in investigating the required hypotheses. The main reason for this was the fact that most of the students who had experienced high anxiety levels at Standard 5 level were no longer included in the final survey at Standard 10 level. As a result it can be seen by the correlations presented above that although a negative correlation existed between mathematics anxiety and mathematics performance it was not within the required significance level.

Hypothesis A

The null hypothesis may be rejected on the evidence of the Standard 5 survey. The alternative hypothesis was accepted at this level from the statistical evidence of a negative correlation between mathematics anxiety and mathematics performance of -0,39 [significance level 0,05; N = 100] (see 5.6.3.5). The null hypothesis may not be rejected at the Standard 10 level. Whilst a correlation of -0,13 was found this was not within the pre-required significance level [significance level 0,52; N = 55] (see Table 8).

Hypothesis B

The null hypothesis may not be rejected. Although negative correlations between mathematics anxiety and mathematics performance existed throughout the five secondary school years, the statistical data was not significant enough to reject the null hypothesis (see Table 7 and 8). However, there is evidence that anxiety levels increase over the years. 37 of the 55 students showed an increase in their anxiety scores from Standard 5 to Standard 10 and the average anxiety score increased from 42,5 to 48,9.

Hypothesis C

There is evidence to reject the null hypothesis. The positive correlations between Standard 5 mathematics performance and subsequent years decreases more drastically than that of English performance (see Tables 10 and 11).

The Standard 5 level performance in both English and mathematics becomes less of a predictor of future success as the students progress through school. However, the mathematics performance at Standard 5 level is a less effective predictor than the English performance at Standard 5 level.

Hypothesis D

There is evidence to reject the null hypothesis. The correlation between mathematics anxiety, Standard 5 level, and mathematics performance at Standard 5 level was found to be -0,39 when N=100 (significance level 0,05). [See section 5.6.3.5]. However, the correlation between mathematics anxiety, Standard 5 level and mathematics performance at Standard 5 level was found to be -0,22 when N=55 (significance level 0,10) (see Table 7). The statistical data provides evidence that the students who were excluded from the final survey (i.e. failures and drop outs) caused the strength of the negative correlation to diminish. It is evident that this was due to the fact that a number of the failures and/or drop outs had high anxiety scores with low performance scores.

5.7.2.4 **Observations**

Before an in-depth analysis of the results is presented it is perhaps appropriate to make a number of observations in respect of the statistical data presented. The following observations refer to the tables and graphs presented in Sections 5.7.2.1 and 5.7.2.2.

1. Mathematics anxiety at Standard 5 and Standard 10 level shows a negative correlation with mathematics performance. However, this is not significant due to the fact that the sample of 55 excludes mostly high anxiety sufferers tested in Standard 5. (Table 7 and 8)

2. There is a positive correlation between anxiety at Standard 5 level and anxiety at Standard 10 level. This may also be diminished due to the fact that the greater correlation would be with the high anxiety sufferers who were mostly excluded. (Table 6)

3. The correlation between mathematics performance at Standard 5 level and the rest of the secondary school years diminishes as one progresses, i.e. the Standard 5 year becomes less of a predictor of success in Standard 10. (Table 10)

4. Whilst there is a positive correlation between English performance at Standard 5 level and the rest of the secondary school years, it also diminishes as a predictor but not as markedly as mathematics. (Table 11)

5. The Standard 10 mathematics mark correlates more highly with Standard 9 and less as one looks back to Standard 5. (Table 10)

6. There is a diminishing correlation between I.Q. and mathematics performance as students progress through secondary school. (Table 9)

7. The correlation between mathematics performance and English performance diminishes as the students progress through school. (Table 12)

Note: Even if it were possible to test all those who had discontinued mathematics it would not have been a valid or a strong influence because they had not done mathematics since Standard 8 and had therefore not experienced anxiety in classroom situations. Testing may be more beneficial at Standard 7 level.

5.7.3 **Analysis and discussion of results**

For clarity and an inspection of the results of this case study it is necessary to present an analysis of each facet of the study. For this reason the following facets of the study will now be discussed and analysed as follows:

Standard 5 level research (MAR scale/Control Test)

College of Education research (MAR scale/Confidence to teach)

Standard 10 level research (MAR scale/Questionnaire)

Whilst the results of this research may not be used for generalization, they do provide an opportunity for comparison with other findings in the literature.

5.7.3.1 **Standard 5 level research**

The results of the Standard 5 mathematics anxiety rating scale and the control test at Standard 5 level were used to investigate a possible negative correlation between mathematics anxiety and mathematics performance.

The original sample of N = 100 revealed a strong negative Pearson product moment correlation of -0,39 between mathematics anxiety and mathematics performance.

The final sample of N = 55 excluded students who had discontinued mathematics, left school or failed. Using the exact same data from the tests at Standard 5 level, the Pearson product moment correlation was found to be -0,22.

These results clearly indicate a diminished negative correlation between mathematics anxiety and mathematics performance of the group that were able to continue with mathematics.

The results of the control test did not differ significantly from the normal test results of the class. However, three important findings have relevance to this study. They are:

(1) Certain questions may be posed to reveal pupil understanding of a topic.

(2) Teachers and students are unaware of the possible insight needed for this understanding.

(3) The link between classroom mathematics and the everyday use of practical applications needs to be developed.

A constructivist approach provides the opportunities for pupils to use investigational methods which are facilitated by the teacher. This problem-centred approach will enable teachers to interact to a greater extent with the class. The development of understanding is a critical variable in the teaching methodology emphasised in this thesis (see 1.1.3) and will be discussed further in Section 6.4.3.

It was also revealed by the mathematics anxiety rating scale that the areas of concern are mainly the classroom activities which are generated by the teacher. These areas of concern are particularly related to the testing aspects of mathematics which were revealed as high anxiety situations in the Standard 5 survey (see 5.6.3.5) and the Standard 10 survey (see 5.6.4.6). Clearly teaching methods in the class as proposed by constructivists (see 4.3) will provide greater opportunities for pupils to express their anxieties and to clarify their own understanding of the various topics. The critical variables of teaching methodology will be discussed further in Section 6.4.3.

5.7.3.2 <u>**College of Education survey**</u>

The mathematics anxiety levels of first year student teachers were alarmingly high. The average of 70,66 on a continuum of 150 to 30 is disturbingly much higher than Suinn *et al* (1972) discovered in his sample of college students. On a continuum scale the two findings may be compared as follows:

College of Education			Suinn et al (1972)		
150	70	30	490	187	98
◄ 80 ►	◄ 40 ►		◄ 303 ►	◄ 89 ►	

The College of Education anxiety levels average at the 0,33 level on the continuum whilst the Suinn *et al* (1972) sample averages at the 0,29 level on a continuum.

The reason that this is disturbing is the fact that the College of Education survey involved only students who had chosen mathematics teaching as a career. Furthermore, many of those students were confident of their teaching ability.

The high anxiety items in the mathematics anxiety rating scale were similar to those of the Standard 5 pupils in that they involved testing and active participation in the mathematics class. Once again the need for a more problem-centred approach which involves social interaction in the lecture will provide a better training for these teachers and enable them to carry this approach into their own classrooms.

5.7.3.3 <u>Standard 10 level intelligence and performance</u>

The mathematics anxiety ratings of the students who continued with mathematics to Standard 10 (N = 55) did not provide significantly reliable analysis. The negative correlations with mathematics performance were weak and significance levels were not acceptable (see Graphs 7 and 8). It is evident that the students who failed or discontinued with mathematics influenced the high negative correlation with mathematics performance at the Standard 5 level.

A number of other significant facts are revealed by the statistical data collected over the longitudinal case study and at the Standard 10 level. Whilst one may not generalise from this case study, an analysis of the data does provide evidence to support certain arguments.

The data provided in Table 9 reveals the expected positive correlation between I.Q. and mathematics and English performance. However, this positive correlation diminishes from Standard 5 to Standard 10 from the 0,6 level to the 0,3 level for both mathematics and English. Mathematics is a gradual decrease in correlation whilst the English correlation drops sharply after Standard 9.

The data indicates that the strength of I.Q. as a predictor of success weakens as the student approaches Standard 10. For mathematics the lower correlation is evident from Standard 7 level whereas the lower correlation between English performance and I.Q. is only revealed in the final Standard 10 examination. It therefore appears that for this particular group, their English marks from Standard 5 to Standard 9 correlated positively with their I.Q. but were not realistic of the final outcome at Standard 10 level. In contrast the mathematics results for Standard 5 to Standard 10 show a gradual decrease in correlation and already at Standard 8 and Standard 9 the positive correlation with the I.Q. is similar to that in Standard 10.

The significance of these results is that throughout the secondary school, pupils must feel comfortable with their English performance because it correlates with their I.Q. On the other hand, the testing of mathematics from Standard 7 reveals a lower correlation with I.Q. and hence could be a source of anxiety. This finding tends to confirm the findings of Brush (1981:38) described in Section 2.2.1.3. Brush found that anxiety ratings for English were higher than mathematics until Grade 9 (equivalent to Standard 7) when English anxiety dropped and mathematics anxiety increased.

5.7.3.4 **Standard 10 level mathematics and English performance**

As expected, the data in Table 10 reveals a high positive correlation between mathematics performance at various levels and Table 11 reveals a high positive correlation between English performance at various levels. However, the correlation in both cases does diminish as the student approaches Standard 10. This decrease is, however, less than that shown by I.Q. and would suggest that the mathematics and English performance at each level is a more reliable predictor of future success in that particular subject.

Graph 2 shows the decreasing correlation between English performance when Standard 5 performance is compared with performance at each of the other levels. This is supported by the data represented in Graph 3 where English performance at Standard 10 level is compared with English performance at other levels. Graphs 2 and 3 are representations of Table 11.

A similar pattern is revealed when the data on mathematics performance is analysed. The mathematics performance at Standard 5 level correlated positively with that at other levels but this positive correlation diminished as the student approached Standard 10. This is illustrated by Graph 4 comparing Standard 5 mathematics performance with mathematics performance at each level.

The control test written at Standard 5 level also correlated positively with mathematics performance at each level and the positive correlation diminished from Standard 5 level to Standard 10 level (see Graph 6).

Graph 5 supports this data and shows the positive correlation between mathematics performance at Standard 10 level compared with that at each other level and the data reveals a greater positive correlation as the students approach Standard 10.

It should be noted that this positive correlation between mathematics performance and English performance at each level is stronger than the positive correlation with I.Q. For example, the correlation between Standard 5 and Standard 10 performance for mathematics is 0,46 whilst I.Q. and Standard 10 performance is 0,33. The correlation between English performance at Standard 5 and Standard 10 is 0,61 whilst I.Q. and Standard 10 performance showed only a 0,34 correlation.

This data displayed in Tables 9, 10 and 11 indicates that whilst I.Q. remains a fairly reliable source for predicting performance, the actual mathematics and English performance at each level is a stronger predictor. This is especially true for students' English performance.

A high correlation between Standard 9 and Standard 10 results should be expected and this is revealed in mathematics (0,67) and English (0,62). However, the ability to identify future success in a subject requires an indicator earlier in the school career of a child. For the purpose of this study the start of the secondary school years (i.e. Standard 6) would be the most appropriate for comparison.

Correlation I.Q. and Standard 10 mathematics result	0,33
Correlation Standard 6 mathematics result and Standard 10 mathematics result	0,31
Correlation Standard 6 English result and Standard 10 English result	0,60

From these results it is evident that parents and teachers may assume that the results of the pupils at the entry level to secondary school English will remain very constant to Standard 10. However, the mathematics result is not a good predictor of future success in this subject.

The data also reveals further interesting analysis when mathematics and English performance is compared over the five year period. The performance level in both subjects drops but the decrease in mathematics performance is sharper than that in English performance (see Graph 1). However, the positive correlation between mathematics and English performance is fairly strong throughout the five year period.

Table 12 shows a positive correlation between mathematics and English performance at Standard 5 to be 0,76, this decreases gradually to 0,70 at Standard 6, 0,59 at Standard 7, 0,43 at Standard 8, 0,64 at Standard 9 and 0,58 at Standard 10. These correlations are significantly high and suggest a positive relationship between mathematics and English proficiency.

5.7.4 **General perspective**

As mentioned throughout this chapter it is not possible to make generalisations from the statistical data of a relatively small and selected group. However, there is enough evidence from the research to support other research findings which have been discussed in Chapters Two, Three and Four, pertaining to mathematics anxiety, curriculum reform and a constructivist approach to teaching.

In the final analysis the purpose of this research should provide benefit for the secondary school student. Too often research data is analysed and hypotheses are either rejected or accepted but there is very little follow through which provides recommendations for the research to be actively integrated into a purposeful project which will be of benefit to the student.

The aspects of this research (which were listed in the opening Section 5.1 of this chapter) that support the literature study are as follows:

1. Mathematics anxiety does exist at secondary school level and is measurable (see 5.6.2.4, 5.6.3.5 and 5.6.4.6).

2. Mathematics anxiety does have a negative effect on mathematics performance (see 5.6.3.5 and 5.7.3.1).

3. Mathematics anxiety is evident at the beginning and the end of secondary school (see 5.6.3.5 and 5.6.4.6).

4. Mathematics performance deteriorates through secondary school and results at the end of secondary school do not correlate favourably with the results at the start of secondary school (see 5.7.2.1, Table 10).

5. Certain learner characteristics are evident and will influence perceptions and attitudes which are a source of mathematics anxiety (see 5.6.4.4, Table 3).

6. Certain critical elements of teaching methodology can be identified as beneficial to alleviating mathematics anxiety (see 5.6.4.4, Table 3).

These aspects of the research provide the incentive to focus on the following:

1. Curriculum should be developed to provide a pupil-centred approach which takes cognisance of factors in the situation analysis stage that relate to a constructivist approach.
2. The main emphasis in curriculum development should be on teacher training and instructional methodology in the classroom. An anxious mathematics teacher is not desirable if the need is for the pupils' anxiety to be alleviated.

Chapter Six will provide the final synthesis of the research findings in literature and the conclusions reached by the findings of the empirical research. This synthesis will emphasise the main theme of the thesis but at the same time conclusions will be drawn to substantiate an approach to providing the necessary outcomes.

The outcome is based on each individual attaining intellectual autonomy and self-confidence in being able to work with mathematics. The consideration of this thesis is how the curriculum can provide the correct direction for achieving this aim and in particular how teaching methodology will provide for this student outcome.

For this reason the final synthesis in Chapter Six will refer back to the components of the curriculum (see 3.3.3) and the critical variables of mathematics teaching methodology (see 3.4.1) to analyse and synthesise the literature findings and the findings of this empirical research.

CHAPTER SIX

CONCLUSIONS AND SYNTHESIS

6.1 Introduction and review of aims of the study

As mentioned in Section 1.4.3 and in the introduction to Chapter Four, constructivism is a combination of a theory of knowledge or philosophical viewpoint and an approach to methodology of teaching. It provides a comprehensive view of curriculum development because of this dualistic role. In other words, constructivism does not simply provide a plan for teaching mathematics but also a reason why this plan should be adopted.

This is a very simplistic view of constructivism but it does emphasise the move away from a traditional approach to curriculum development (see 3.2.1) which ignored any human psychological processes and viewed building a store of knowledge as the ultimate goal. The philosophy of constructivism as a theory of how an individual acquires knowledge is the motivation for the constructivist approach to teaching and learning. Thus, a comprehensive view of curriculum development is all important if all aspects of constructivism are to be didactically beneficial to the child.

This thesis is, in essence, a curriculum study in which the role of mathematics anxiety as a variable is discussed from a constructivist perspective. Constructivism, as a comprehensive view of curriculum development becomes the synthesis of the central aspects of this study, namely, curriculum design, mathematics anxiety and teaching mathematics in the secondary school. The philosophical view of constructivism as well as the pragmatic ideas of a constructivist approach are both important aspects of this study.

Radical constructivism and an over-emphasis on discovery techniques and problem solving exercises could be negative aspects of a constructivist approach for the mathematically anxious child. However, the strengths of constructivism can be utilised to the advantage of all students and many of the elements of this approach are beneficial to those students whose anxiety negatively affects their performance and their attitudes (see 4.7).

The strength of constructivism also lies in the emphasis on the teaching-learning process and on social interaction. It is more pupil-centred and less orientated towards content and an emphasis on examinations and testing as a means of assessing performance. They are all factors which tend to alleviate mathematics anxiety and enhance mathematics performance (see 4.7).

Whilst the problem-centred approach may not serve the same purpose it is essential that students of today are provided with a more realistic view of mathematics. A move away from the learning of rules and rigid preparation for what will be examined is a move in the right direction. Students need to be empowered to think freely and approach problems with a more enquiring and investigational mind. They need the ability to reflect their solutions and to communicate and collaborate with others on a mathematical level (see 4.3.3).

The empirical research described in Chapter Five provides statistical support to the literature study and again emphasises mathematics anxiety as an important variable which should not be neglected in curriculum development and in particular the teaching methodology aspect of curriculum development (see 3.3.3.4).

Sovchick (1989:119) stresses the importance of recognising that the problem of mathematics anxiety exists before any strategy for a solution to the problem is proposed. In addition, he emphasises the importance of the teacher's role and teachers being aware of potential mathematics anxiety in their students as well as themselves.

It is these two important aspects of mathematics anxiety stated by Sovchick that have given direction to this thesis and should once again receive attention.

1. Recognise that mathematics anxiety exists.
2. Recognise that the teacher's role is critical in the alleviation of mathematics anxiety.

The aims of this study are concentrated on these two important statements. Firstly, mathematics anxiety is established as a widespread and continuing problem and secondly, remediation is proposed as a teaching strategy which incorporates critical variables of a constructivist approach which are compatible with the concerns of mathematics anxiety.

6.2 Implications for mathematics anxiety

It is unfortunate that a longitudinal study such as that undertaken in this thesis cannot investigate each individual's progress. This would be an enormous task and would have required an interview with each of the 100 participants throughout the six year period.

The students of particular interest are those who either failed mathematics or did not take mathematics as a subject to Standard 10. The significant drop-out rate and the poor mathematics performance of these students suggests that mathematics avoidance, as described by Tobias (1976:56), is a result of mathematics anxiety.

The longitudinal study was unable to provide conclusive evidence to support Stipek's claim that mathematics anxiety develops more over the critical secondary school years (Stipek 1988:111). Whilst there is some indication that for many of the students in the study mathematics anxiety increases, this trend would almost certainly have been more evident had the non-mathematics students had to continue their mathematics to Standard 10.

The fact that researchers such as Dougherty (1981:1) claim that it is safe to assume that mathematics anxiety is widespread and causes less mathematics to be learnt, may be supported by the findings of this research. In the three mathematics anxiety rating measurements undertaken at Standard 5 level, College of Education and at Standard 10 level, all three showed high levels of anxiety. The Standard 5 research and the College of Education Research revealed negative correlations with mathematics performance compatible with those reported by Richardson & Suinn (1972:551) and Richardson & Woolfolk (Sarason, 1980:24) which were discussed in Section 2.2.1.4 of this thesis. Once again it must be pointed out that the Standard 10 survey was less revealing because of the group of failures and non-mathematics students who were excluded at this stage.

These findings are particularly disturbing because of Hembree's analysis of research findings that led him to conclude that in all cases evidence was presented to prove that high-anxious students performed more poorly than low-anxious students (Hembree 1990:42). Mathematics anxiety also leads to mathematics avoidance which in turn leads to a reduced ability to perform mathematically and hence an increase in anxiety (see 2.1.3.3). Thus the empirical evidence of the existence of a high level of mathematics anxiety must prompt action before a spiralling effect of lower performance and mathematics avoidance becomes prevalent.

The parallel study of student teachers at a College of Education appears to strengthen arguments which call for efforts to improve teacher training and the selection of potential teachers. The "vicious circle effect" (see 2.1.3.4) is perpetuated by under-qualified teachers who may be suffering from higher levels of mathematics anxiety than their students. The syndrome of teachers passing on anxieties to students is a distinct possibility given the high level of anxiety of the prospective teachers involved in the research.

Of more concern is the unrealistic beliefs of the prospective teachers of their own levels of competency. This often leads to "defensive teaching" whereby the teacher prepares well, has a rigid lesson plan, does not accept alternative answers and allows his or her students to believe that mathematics solutions are arrived at with the utmost of ease. In this way the teacher's authority in the subject is never questioned and the students perceive the teacher as extremely intelligent.

The fears of Stodolsky (1985:13) that mathematics teaching lacks variety and is presented as a set pattern of work, are well founded when one analyses the responses of the student teachers in the College of Education survey. The high anxiety ratings are alarm bells for future generations who are likely to face teaching methodologies described by Greenwood (1984:663) as - explain - practice - memorize.

Researchers such as Johnson (1981:2), Widmer & Chadez (1982:276) and Kogelman & Warren (1978:18) all emphasise the role of the teacher in the development of healthy attitudes towards mathematics (see 2.2.2.4). The fact that the prospective teachers in the College of Education survey displayed high levels of anxiety, adds substance to the concerns of Kemp (1990:2) and Williams (1988:95) who both found that teachers transfer their negative feelings, attitudes and anxieties to their students (see 2.2.2.1).

The Standard 10 questionnaire provided a clearer view of some of the issues related to the development of mathematics anxiety. The intention of this research was to emphasise the socio-cultural and emotive factors influencing mathematics anxiety. Hembree's (1990:44) comprehensive analysis of 150 previous studies of mathematics anxiety provided sufficient influence to the direction of this research. Of particular interest is Hembree's findings that there is no conclusive evidence that special work to enhance students' competence was able to reduce their anxiety levels (Hembree 1990:45). The implication is that remediation should focus on the socio-cultural and emotive elements of teaching mathematics. These implications are also supported by Lewis (1987:62) and Jacobs (1978:125) who stress the social factors causing anxiety (see 2.2.2.1) and the number of researchers quoted in Section 2.2.2.2, who emphasise the emotive sources of mathematics anxiety.

A high number of students in Standard 10 responded positively to items in the questionnaire that pertained to increased competence in mathematics. Items 21 and 32 yielded a mean score of 2,84 and 2,95 respectively, However, the average anxiety levels of the students increased from 42,5% at Standard 5 level to 48,9% at Standard 10 level and 37 of the 55 respondents showed an increase in their anxiety levels.

Questions 11, 15 and 16 revealed high responses supporting the belief that mathematics is perceived as a rigid subject requiring a learning of procedures and formula. In contrast there was a fairly low response (Question 12) to the belief that mathematics requires creativity and discovery by the student.

The literature abounds with statements supporting the view that mathematics anxiety is perpetuated by teachers and parents (see 2.2.2.1). However, this does not appear to be the view of the students. Item 3 and 5 of the questionnaire both yielded low average scores of 1,80 and 1,67 which were amongst the 5 lowest scoring items of the 40 items presented.

The Questionnaire did provide support for Kogelman & Warren's (1978:30) concern for mathematics myths. In particular the idea that one either has a mathematics mind or not is strongly supported by the responses to item 29 which yielded a high average of 3,00. The belief that it is always important to get the answer exactly right (average 3,10) and the belief that mathematics does not require intuition (average 3,16) are also fairly well entrenched myths.

To summarise and to provide a clarity of thought it is important to link the observations from the empirical study on mathematics anxiety with the research findings on mathematics anxiety that were reported in Chapter Two. The following aspects arise from the discussion on the findings of the empirical research. In each case a link to relevant other research is provided.

The following aspects related to mathematics anxiety should be noted and comprise a summary of the main concerns of this thesis.

1. Mathematics anxiety does exist and has a negative influence on mathematics performance at the end of the primary school years and the beginning of the secondary school years. This phenomena is supported by all the mathematics anxiety researchers analysed by Hembree (1990:42).

2. Mathematics anxiety continues through secondary school and anxiety levels are likely to increase. Stipek (1988:111) insists that these are the critical

years and research by Brush (1981:38), Carpenter *et al* (1981:25) and Hembree (1990:38) confirm this belief.

3. There is an indication that teachers of pupils in the final stages of primary school or early years of secondary school lack the competence or confidence to provide the necessary support. This aspect was a concern of Stipek (1988:113), Widmer & Chadez (1962:276) and Johnson (1981:2) as reported in Section 2.2.2.4.

4. Teachers are more than likely to perpetuate rigid teaching techniques and perpetuate mathematics anxiety (this is linked to point 3).

5. A high level of mathematics anxiety at the beginning of secondary school suggests a higher chance of failure and a possible discontinuance of mathematics. Tobias (1976:56) and Dougherty (1981:1) expressed concern about the fact that students (in particular girls) discontinued their mathematics studies.

6. Students believe strongly that mathematics ability is inherent and cannot be taught. This myth must initiate an early anxiety for those students who believe that they were not given the "gift" to cope with mathematics. It will also frustrate teachers as students become more entrenched in the belief that there is nothing that will improve their mathematics mind. These concerns substantiate the findings of Kogelman & Warren (1978:30) who found myths a major cause of the manifestation of anxiety.

7. There is no strong evidence that parents perpetuate the feelings of anxiety. In fact, the survey revealed evidence that students believe that their parents do not contribute to their anxiety. This contradicts the beliefs expressed in the literature (Williams, 1988:95); (Jacobs, 1978:125) that parents perpetuate mathematics anxiety. However it may be that students responses are the measure of a more supporting and understanding parent. It is likely

that mathematics anxiety is passed on subconsciously and by innuendo as suggested by Kemp (1990:2), (see 2.2.2.1).

8. Students believe that mathematics consists of a set of correct procedures which should be learnt and practised. They believe that a correct answer is the all important aspect of mathematics and that creativity and intuition have little meaning when doing mathematics. These beliefs encourage a rigid approach to mathematics and a reluctance to explore various techniques and solutions. Any variation from routine will cause anxiety. This is supported by Stodolsky (1985:13) and Greenwood (1985:13) as discussed in Section 2.2.2.4.

9. There is strong evidence that students value the need for social interaction when doing mathematics. This interaction, both with fellow students and with the teacher, assist the individual to overcome anxieties and gain confidence to explore various solutions to posed problems. Tobias (1987:5) and Stodolsky (1985:13) support this notion (see 2.2.2.4).

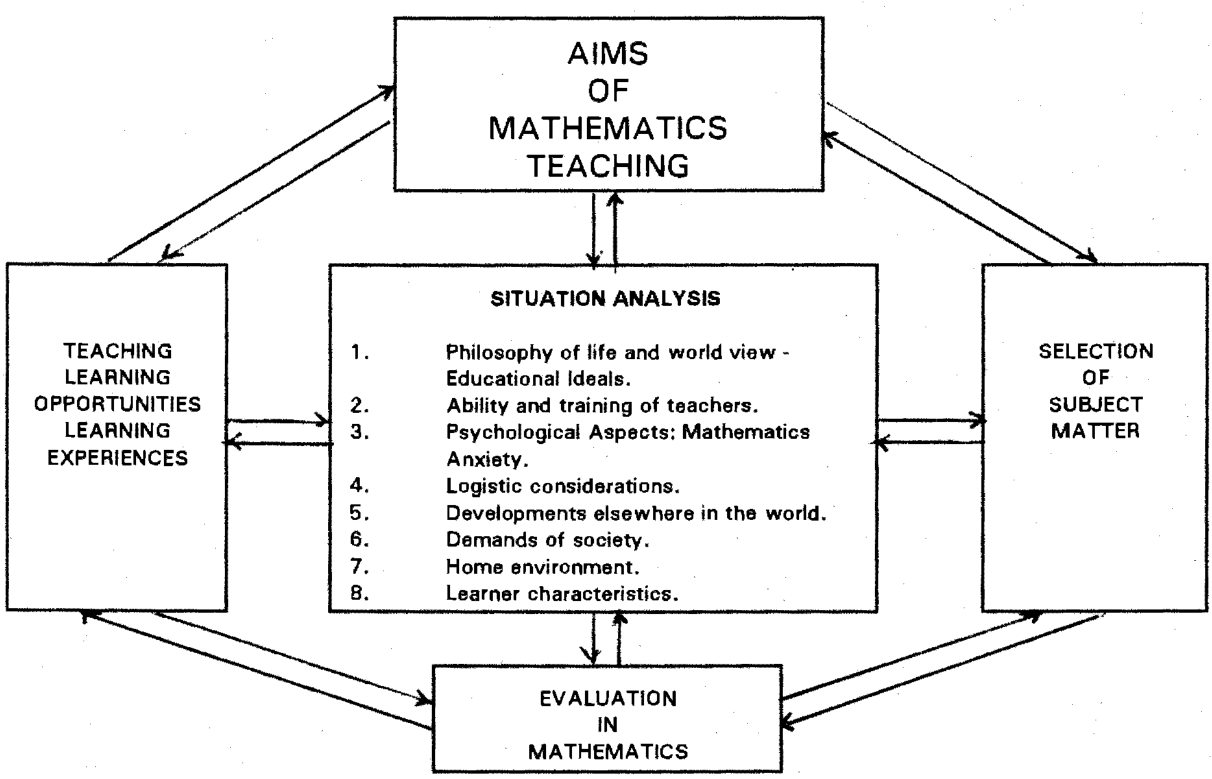
This summary of the findings on mathematics anxiety is provided to emphasise the central theme of this study. Mathematics is, indeed, a variable which exists and a number of the causes expressed by other researchers, as outlined above, have been verified in this empirical research. The second element of this study requires that mathematics as a variable has implications for curriculum development and in particular the teaching methodology component of curriculum development. For this reason it is important, at this stage, to synthesise the ideas of curriculum reform which were expressed in Chapter Three and Chapter Four.

6.3 <u>Implications for mathematics teaching and curriculum design</u>

In Chapter Three an overview of curriculum design was provided with the intent to emphasise the number of approaches which are now more learner-orientated (see 3.3.2). The emphasis on process, cognitive psychology, humanism, the affective domain and the socio-cultural domain demonstrates an overwhelming move to more pupil-centred approaches to curriculum reform.

In addition, curriculum was viewed as a much more complex phenomena than what was earlier termed, "the traditional view" (see 3.2.1). For this reason a more comprehensive view was expected from modern curriculum reformists and input is required from a broad base providing "bottom-up" information and research before formulating a curriculum (see 3.2.2).

As discussed in Chapter Three, curriculum has a far more comprehensive meaning than syllabus. This comprehensive structure may best be illustrated by means of a diagram.



Adapted from Harley (1983:1)

Whilst all aspects of the comprehensive approach are important for curriculum planning it is to be expected that the input and emphasis will vary from person to person in such a broad-based approach. The purpose of this research is to gain support for the central ideas expressed throughout this thesis with the intention that they may be given prominence whenever curriculum reform takes place in mathematics.

In essence, a case has been made for curriculum developers to pay attention to mathematics anxiety as an important variable in curriculum design. Secondly, a case has been made for constructivism to influence curriculum developers by providing a philosophy of how people acquire knowledge and a methodology for teaching. The constructivist approach is both a cognitive position and a methodological perspective (Noddings, 1990:18).

The previous diagram illustrates the dynamics of the curriculum development process. It involves an interaction of five very important components which all have their own content but are also interdependent of each other.

This comprehensive approach to curriculum development provides adequate avenues for input on the central ideas of mathematics anxiety and constructivism.

The component entitled situation analysis enables curriculum developers to focus on the essential aspects of the curriculum for a particular situation. In this case the situation is a mathematics curriculum for a South African society. Situation analysis provides a synthesis of the important aspects of this thesis which in turn should be the important aspects of curriculum design.

The elements of situation analysis which provide direction for research and development were discussed in Section 3.3.3.1 under the following headings:

Philosophy of life and world view - educational ideals
Ability and training of teachers
Psychological aspects
Logistic consideration
Developments elsewhere in the world
Demands of society
Home environment
Learner characteristics

These elements are all intricately interwoven and permeate the research in this thesis. However, to provide some structure it is appropriate at this stage to describe the necessary parameters for a situation analysis of mathematics curriculum development in South Africa.

6.3.1 **Philosophy of life and world view - educational ideals**

This aspect has to permeate all the thinking in mathematics curriculum design. Development cannot take place without a clear vision of ideals. Unfortunately, in South Africa at this time of change, it appears that little consideration will be given to this aspect and the concentration will be on content.

The philosophy of mathematics education should not stray from the latest trends in the world. Not because these trends are modern and idealistic, but because they provide a sound basis for mathematics education and because they satisfy the requirements of the vast majority of the people.

This philosophical basis is called constructivism and it has gained the support of mathematics educators throughout the world. It also has wide support in South Africa which is evident from the papers presented at the conference held in Broederstroom (N.E.C.C. Mathematics Commission, 1993). A major theme throughout this conference was a need to move towards a constructivist philosophy and away from fundamental pedagogics. In Olivier's pre-conference paper (N.E.C.C. Mathematics Commission, 1993) he expresses concern for the present teaching paradigm which tends to neglect the rich repertoire of informal knowledge that children of all races bring into the classroom. He goes on to state that it is an indictment on our teachers that opportunities to communicate and learn from others are not always made available in the classroom. Olivier states in his paper that the social and cultural environment of the child is not the primary cause of failure at school.

Steffe & Kieran (1994:728) support this view and believe that constructivism has positive implications in varying cultures and environments. The culture and environment of the individual causes thoughts and hence actions. Thus, the thoughts and actions in the classroom are mirrors of the individual's cultural and social environment. By listening to and observing students, teachers have a potential source both for content and organisation of various mathematical curricula.

> *Because of the priority given to childrens' activities as an occasioning source for constructivist models of mathematical activity, the very actions of constructivist teachers in listening to, in questioning, and in modelling childrens' mathematical activity, provide provocative examples for the practice of mathematics teaching* (Steffe & Kieran, 1994:728).

Making use of a constructivist approach in the classroom is based on a belief that each individual contributes to the building of knowledge and a belief that confidence and mathematical power to tackle problems is a necessary requirement for all students regardless of their cultural or environmental background.

Constructivism is seen as revolutionary because it regards mathematics as fallible and changing and largely a product of human inventions. Mathematics is construed as a way of thinking rather than a body of fixed knowledge or content. This revolutionary or radical viewpoint of constructivism may provide impetus for the adaptation of this philosophy but it is not the major strength of constructivism. The argument for constructivism as a mathematical philosophy is far more convincing when it is viewed as social constructivism.

Social constructivism accepts that certain rules and algorithms exist and that basic arithmetic skills are important pre-requisites for mathematics just like writing skills are pre-requisites for developing language skills. However, inter-personal social processes are required for the construction of mathematical knowledge. Communication between teacher and pupil, pupil and pupil, parent and pupil are all aspects of inter-personal social processes which develop mathematics knowledge and empower the pupil to learn mathematics by understanding the meaning of mathematical processes. As described in Section 4.7, it is these aspects of constructivism that have a strong appeal to those educators concerned about mathematics anxiety.

The Standard 10 survey in this thesis supports the need for social constructivism in that two aspects of social interaction were identified as important to students. The need to communicate with the teacher and with their class mates was clearly evident in the responses to items 37 and 38 of the questionnaire (see Table 3). This communication forms an essential support system which provides a student with the confidence to relate their mathematical experiences as well as their solutions to problems.

Another important aspect of the constructivist philosophy for the South African situation is the problem-centred approach. Whilst communication in a social context provides a confidence to express mathematical ideals it is the approach to problems that empowers people to be able to handle problems in later life. The motivation for curriculum reform is based on the needs of the individual. In a world that is changing rapidly both technologically and socially, the ability to think independently and have a problem solving attitude are necessary requirements (see 4.3.2 and 4.3.6).

It was evident from the Standard 5 Primary School test that not only are pupils ill equipped for a variation of a problem, the teacher is also unaware of the rote learning methods that are being adopted by the class. The lack of understanding of the placement of decimals was clearly indicated in the different responses to 0,3 x 0,3 and 0,4 x 0,4. This is a simple example of how teaching methods can fail to produce the required insight into a topic.

The socio-constructivist approach requires a more problem-centred learning environment which provides more opportunity for students to explore the different solutions to problems and to negotiate the answers with fellow students and the teacher. The learning environment should stimulate individual thinking and the social construction of meaning.

Problem-centred learning is an attempt to change the students perceptions away from mathematics as a collection of rules imposed on them by the teacher. Mathematics is presented as a human social activity and students are viewed by the teacher as active mathematical thinkers who are capable of constructing their own knowledge of mathematics through problem solving in a socially interactive environment.

6.3.2 **Ability and training of teachers**

It was clear from the College of Education survey and the responses from the Standard 5 teachers that the teaching of mathematics in schools is a major problem in South Africa. There is evidence in the literature reviewed in this thesis that this problem is universal. Educational ideals are worth nothing if they cannot be put into practice by the teachers.

The responses to the questions in the Standard 10 survey pertaining to discussion in the classroom, supported claims by researchers in the literature study. Cobb *et al* (1991:6), Schoenfeld (1987:205) and Brousseau (1984:112) all expressed concern about the lack of opportunity for discussion and negotiation in a problem solving environment (see 4.3.6). Steffe (1990:67) expressed the concern as a "universal cry" by mathematics educators that teachers do not provide an active environment but rather present mathematics as routine procedures which are either right or wrong. The student teachers involved in the survey at the College of Education displayed a disturbing lack of confidence in their mathematics performance and yet believed they were well equipped to teach mathematics to levels above their own capabilities.

The constructivist philosophy requires teachers who are confident enough in their own ability that they invite inquiry into methods rather than a passive acceptance of a routine. Without the enquiring mind of a teacher the static beliefs in formal procedure will be perpetuated and mathematical thinking will be rejected in favour of pre-determined methods.

It is vital for the future of South Africa that the country invests in a balanced and relevant education by upgrading the ability and the quality of the teachers. There is a need for radical change in teacher education because the strength of any curriculum approach depends on the ability of the teachers. It is vital that South Africa stands amongst other leading educational countries who believe that it is important for the future to produce independent thinkers able to make a meaningful contribution to the country as a whole.

The future of mathematics education is not a concern for content but rather how the content is delivered to the pupils. Teachers must be convinced not to resort to methods they observed during their own schooling or to necessarily adopt the teaching patterns of their older colleagues. For this reason, a new didactic approach should be part of the aims of a mathematics curriculum which emphasises how teachers should teach and how teaching and learning takes place.

The constructivist approach requires a reform in teacher education. Theory and research on cognition and instruction has now reached a point where it can be used profitably to develop principles for instruction to guide curriculum development and teaching practice. However, methods of instruction in the constructivist mode need to be reviewed as ongoing and as an ever-changing process. The aspects that teachers and student teachers must develop include encouraging communication among pupils and between him/herself and the pupils. There must be a flexibility of approach which can encompass the pupils' cognitive functioning in problem solving and provide opportunities for creative engagement by pupils. The teacher must learn to participate in lessons and creatively structure instruction to pose problems and investigate various pupil solutions. The environment in the classroom must be one of open discussion and negotiation in which the teacher's expertise can facilitate the mathematical thinking of his or her students.

6.3.3 Psychological aspects

The obvious psychological aspects relating to mathematics education is that of mathematics anxiety. The literature on this developmental problem is prolific and many programmes have been suggested and implemented to provide support for those suffering from mathematics anxiety.

The empirical research in this thesis, whilst relatively limited, did reveal high anxiety levels amongst Standard 5, Standard 10 and College of Education students. In Section 2.1.3, the idea that mathematics anxiety is widespread was supported by a number of researchers. More significantly Hembree (1990:42) integrated the findings of thirteen studies which all showed that mathematics anxiety has a negative effect on mathematics performance and Tobias (1976:56) found a number of mathematically anxious pupils avoiding future mathematics programmes.

In the case study presented in Chapter Five it appears that many of the pupils suffering from high anxiety levels do not complete their secondary mathematics schooling and this is prevalent in girls. These results are inconclusive due to the fact that the sample group was selected and small and contact was not made with the group of students who did not complete their mathematics to Standard 10 level, However, the findings of Hembree (1990:42) as described in Section 2.1.3.2, are particularly significant and highlight the concern for mathematics anxiety.

The perspective of this thesis is that the constructivist approach best serves the interests of educationists concerned with mathematics anxiety. This theme was established in Section 4.7 and emphasised at the end of Chapter Four.

There are several aspects of the constructivist approach that provide ideal opportunities to relieve tension in the classroom. In Chapter Four the sources of constructivism are described in conjunction with the sources of mathematics anxiety (see 4.7). The strength of the constructivist approach is in its emphasis on the learning environment. Constructivism calls for a learning environment which provides moral support for each individual by his or her teacher and class mates. There is an openness in discussion and a freedom to explore solutions and techniques without a fear of being ridiculed. The classroom is seen as a culture in which students have the opportunity to become involved in discovery and invention. Through social interaction the students become involved in explanation, negotiation and the sharing of ideas. They are also provided with the opportunity for self-evaluation.

In the constructivist philosophy the teacher becomes less of an authoritative figure and more of a facilitator. Students have the opportunity to express their anxieties and the teacher should be an empathic listener. Confidence is also gained by working through problems with fellow students and being aware that everyone experiences some anxiety when doing a mathematics problem.

The central goal of the constructivist approach is to provide students with confidence in their own mathematical ability and in their own cognitive processes. The idea that mathematical knowledge is not received from their teachers but rather from their own explorations and thinking, provides the mathematically anxious child with the will to develop mathematical autonomy.

6.3.4 Logistic considerations

In the South African context the logistic considerations are vast but the constructivist approach provides a key to the direction of future planning.

Whilst resources such as schools, classrooms and books must receive ongoing attention, it is our human resources that are most important. Teacher training must receive immediate attention and in-service courses need to provide present day mathematics teachers with the knowledge of the constructivist approach.

This means that the task of curriculum developers must be to give precedence to developing a didactic approach which describes why the constructivist philosophy is deemed suitable , what should be taught and learned and how this teaching and learning should take place. The ideas in Chapter Four and in particular Section 4.5 provide the background for such a didactic model.

Logistically this approach can be implemented immediately and is less costly.

Teachers can be upgraded and their classes will experience the immediate benefits of a more enlightened approach. Pupils will, in turn, experience a classroom technique which provides them with a far better opportunity to progress in mathematics. This, in turn, will see a new generation of mathematically empowered young people being available as teachers in a new society.

6.3.5 <u>Developments elsewhere in the world</u>

In Chapter Three the universal developments in curriculum reform were discussed. This was followed by a detailed discussion on constructivism in Chapter Four. A synthesis of the ideas from these two chapters would formulate the arguments for curriculum planners when considering developments elsewhere in the world. The main elements of the arguments of this thesis are as follows.

The humanistic view of constructivism has made this philosophy very appealing, especially in the democratised western world. The moderate viewpoint of social constructivism is a belief that knowledge is actively constructed by the individual and not passively received from the environment. Constructivism has evolved from research in cognitive psychology and from a move by curriculum designers from a traditional approach to a more comprehensive approach which has incorporated the beliefs and ideas of the pupil-centred and humanistic theorists. The investigational techniques and the problem-centred approach embodied in constructivism are world wide themes in mathematics education. This trend is evident in the Cockcroft Report (Cockcroft, 1982) in England and the Commission on Standards for School Mathematics (1987) in the U.S.A. The process of learning mathematics and not the content matter is becoming the new "battle cry" throughout the world. Constructivism is seen as the best approach for the provision of thinking and empowered students who will be able to cope with an ever-changing world.

6.3.6 <u>Demands of society</u>

The present day demands of society require educators to take a hard look at schooling and teaching techniques in particular.

Present day employers are not looking for passive absorbers of knowledge but rather for people who are able to cope with problem situations that are forever changing. Communication and social interaction are essential skills in a modern society and they are two of the cornerstones of the constructivist approach.

The constructivist approach requires curriculum developers to focus on issues of societal needs (see 4.3.4). The classroom must become a microcosm of society in that it provides the opportunities relevant to the working place. Students must learn to co-operate with others and work on problems together and be able to apply their knowledge to various situations. The new mathematics student must become confident in his or her mathematics ability and learn to communicate and reason mathematically.

Thus, the socio-constructivist ideals serve to reduce mathematics anxiety by addressing the problems of lack of understanding, lack of task-orientation and lack of confidence. The emphasis on classroom strategies and developing a task directed attitude are also concerns of mathematics anxiety researchers (see 2.3).

A synthesis of the ideas of constructivism and mathematics anxiety research was expounded in Section 4.7 where it was emphasised that both schools of thought propose classroom strategies and a pupil-centred approach to produce students who will ultimately possess mathematical autonomy and confidence.

Becoming a member of a community requires the capacity to interact with members of the community. This interaction in mathematics requires a mathematical literacy and a history of opportunities to participate in discourse in the classroom. The teaching of mathematics must provide this activity-based situation in the classroom.

6.3.7 **Home environment**

In the South African context the home environment can be very varied. In some instances lack of facilities at home make it very difficult for pupils who live in the rural areas or even in the poor urban areas. Whilst this is not a concern of this thesis it is important that curriculum planners take cognisance of the poor conditions suffered by pupils. In addition, teachers must be aware of the problems facing each of their students and a lack of facilities at home requires that the teacher provides the opportunities necessary for mathematics development in the classroom (see 4.3).

Of more concern in this study is the attitude of parents and the negative approach that parents may have towards mathematics. In particular the term "New Maths" has negative connotations which were formed in the 1970s and have been related to any new attempt to introduce mathematics in a non-traditional format of learning algorithms and skills. The typical concerns and misconceptions of parents were expressed by a parent whose letter to the Natal Daily News on 21 April 1994 was published in the Natal Education Department's publication Neon (1994:27). These are the criticisms of introducing constructivism as viewed by a parent.

1. It is only four years old and is thus only experimental.
2. It is an unproven experiment as other countries are now "going back to basics".
3. Mathematics is a vertical science which means that each generation does not have to rediscover rules.
4. The present system works so why change?
5. The problem lies in the quality of the mathematics teachers. Double their salary and increase their training and the mathematics problems would disappear.
6. The new system will produce mathematical illiterates who cannot do mental arithmetic and depend on a calculator.
7. School children need some rote learning, what good is lateral thinking if the child cannot learn and memorize facts?
8. In classes of over 50 pupils, how will it be possible to monitor the progress of each child?

The concerns expressed in query 5 and 8 refer to ability and training of teachers and logistic considerations and have been discussed in Sections 6.3.2 and 6.3.4

respectively and the incorrectness of query number 2 is evident by the discussion in Section 6.3.5. However, the other five concerns are common cries of worry heard by parents and it is important that schools address these queries by providing detailed information in brochure form and at parent information evenings. In answering the queries to this letter the Natal Education Department (1994:28) made the following comments:

1. The new approach is not four years old but dates back to the 1970s when worldwide research showed that young people are quite capable of constructing their own methods based on their own intuitive knowledge.

2. The Natal Education Department implemented this approach in five schools in 1990, a further two in 1991 and a further ten in 1992. The results of the research into the success of this implementation convinced the Natal Education Department of its worth.

3. Retaining the present system does not allow for the dramatic changes in technological advances in the world. Thus a major thrust of the change in approach to mathematics has to do with changing the mathematics - as - computation curriculum to a mathematics curriculum that genuinely embraces conceptual understanding, reasoning and problem solving as the fundamental goals of teaching mathematics.

4. It is a false assumption that a problem-centred approach does not require pupils to know their basic facts and algorithms. It simply provides a variety of ways for pupils to learn these facts and algorithms and requires a wider and better knowledge which is more successful than knowledge gained by rote learning.

This type of information is essential for parents who have a fear of any new approach and who will inevitably transfer this fear or negative attitude to their children.

In addition to providing information on a constructivist approach, the need for parents to provide a home environment conducive to alleviating anxiety is also of central concern in this study. This may also become part of the school's or education department's projects in providing a link between parent and teacher. This has been done very successfully in Canada where a brochure entitled Mathematics : The Invisible Filter was produced and distributed by the Mathematics

Department of the Toronto Board of Education (1983).

This publication includes several brochures which combine to make "Mathematics : The Invisible Filter kit" and provide parents with details and information regarding mathematics avoidance, mathematics anxiety and career choices. In a very "reader friendly" manner and with the aid of diagrams and drawings the mathematics anxiety concerns are expounded under a heading entitled "Survive and Succeed in Math". This brochure provides parents with the following:

1. What is mathematics anxiety?
2. What are the symptoms of mathematics anxiety?
3. The vicious circle effect.

These are important aspects for parents and were discussed early in this thesis in Sections 2.1.2, 2.2.3 and 2.1.3.4 respectively. By providing parents with an insight into the problem of mathematics anxiety the efforts of the teacher will be made easier as parents influence the home environment and the attitude of their children in a positive direction.

6.3.8 Learner characteristics

The diverse nature of South Africa's population may suggest that learner characteristics differ considerably throughout the country. However, the literature reveals that an analysis of the situation will indicate certain universal characteristics of needs, abilities, perceptions and experiences.

The questionnaire in Appendix 5 was specifically constructed to investigate the aspects of the literature pertaining to learner characteristics. This questionnaire was administered to the target group in their Standard 10 year and the data collected was documented in Section 5.6.4.4. This data provides a basis for this analysis of the needs, abilities, perceptions and experiences of the learner. For this reason, continued reference to items of this questionnaire and the collected data will be made throughout this section.

a) **Needs of the learner**

The following needs were established from the questionnaire and the mathematics anxiety rating scales (see Appendix 1,4,5 and 6).

1. A need to alleviate mathematics anxiety. High levels of anxiety are indicated by items 1,6 and 8 of the questionnaire and the negative correlation was measured in all three of the mathematics anxiety rating scales.

2. A need for discussion in the classroom. Items 37 and 38 indicate a strong desire for more discussion with peers and the teacher.

3. A need to make mathematics more interesting and more fun so pupils progress through school. Items 23 and 24 indicate a lack of these elements in the teaching of mathematics.

b) **Ability of the learner**

Whilst accepting that learners differ in ability level, the most critical aspect affecting learner abilities is the belief that mathematics ability is a gift that only some people possess. This belief may lead to the establishment of a syndrome which attributes any lack of ability to some uncontrollable factor or destiny. The following aspects pertaining to ability should be analysed.

1. The belief that mathematics ability is a gift (item 10) and that some people have a mathematics mind and others do not (item 29).

2. The belief that they will never be confident in coping with mathematics (item 22).

3. Mathematics ability is hindered by time constraints which lead to anxiety (item 32).

4. Poor mathematics performance is often blamed on careless errors (item 3) is of particular concern because these may occur because of time pressure.

5. The ability to understand mathematics procedure is regarded as important (items 15,18,19 and 20) and the belief that ability was enhanced by practising procedures should be developed. However, caution is required as it appears that perseverance and intense working until a problem is solved (items 18 and 25) may mitigate against a more creative and intuitive approach to procedural knowledge and a lack of development of conceptual knowledge.

6. Mathematics ability is linked to the memorization of rules and formulae (items 9 and 16).

c) **Perceptions of the learner**

There are a number of connotations, myths and perceptions which influence the learner's performance and attitude (see 2.2.2.2). These were investigated through the medium of the questionnaire and have already formed part of the needs and ability of learners. The most prominent perceptions that will influence mathematics performance are listed here in rank order according to the level of response from high percentage to low percentage.

1. Mathematics is done by working intensively until the problems is solved (item 25, mean 3,16).
2. Mathematics requires logic, not intuition (item 31, mean 3,16).
3. In mathematics something is either right or wrong (item 11, mean 3,15).
4. It is always important to get the answer exactly right (item 34, mean 3,11).
5. Some people have a mathematics mind and others do not (item 29, mean 3,00).
6. Mathematics ability is a gift that only some people have (item 10, mean 2,82).
7. Mathematics is not a creative subject (item 30, mean 2,73).
8. The best way to do mathematics is to memorize all the formulae (item 16, mean 2,69).

Clearly the perception of mathematics as a rigid and authoritarian subject will influence learning and the development of a problem solving attitude. The perception that mathematics requires little creativity or intuition will also mitigate towards a poor problem solving attitude. The perception that mathematics is a subject for the select few has wide ranging implications as pupils lose confidence in their ability and blame internal factors which cannot be changed.

d) **Experiences of the learner**

A number of experiences of learners are implied by the factors discussed as needs, abilities and perceptions. For instance, it was evident from the questionnaire that experiences from discussions with teachers and peers enhance the learning situation. It is also implied that mathematics is an anxiety provoking subject (items 6,7,8) which consists of rigid rules and procedures. However, the view that teachers and parents have a strong influence on learner experiences is not supported by the data collected for items 4 and 5 of the questionnaire. The five highest anxiety-provoking situations revealed important aspects of mathematical experiences that require attention.

1. The test situation. Waiting for a test and doing badly in a test.
2. Being faced with a new problem.
3. Using rote learning methods to cope with mathematics.

These are confirmed by items 1,6,7 and 8,9,16 in the questionnaire.

e) **Aspirations of the learner**

The aspirations of the learner should be analysed in conjunction with needs, ability, perceptions and experiences. Clearly the aspirations of learners should be to do mathematics with confidence and to develop a sense of mathematics power (see 1.4.4). These aspirations can only be met when teachers take cognisance of the other aspects of learner characteristics.

Thus a summary of important learner-centred aspects for situation analysis should include the following:

1. Addressing the problem of mathematics anxiety.
2. Providing for the needs of the learner by stimulating discussion and interest in mathematics.
3. Correcting misguided perceptions.
4. Avoiding and correcting bad experiences.

6.4 Implications for the teaching of mathematics in the secondary school

The other components of curriculum development all emphasise the task of the teacher in the classroom. Each facet of the teaching process has particular implications if one follows the philosophy of constructivism. In particular the constructivist approach depends heavily on the education and co-operation of the mathematics teachers.

The traditional structure of classroom mathematics instruction needs to make way for the new constructivist approach which makes provision for students to play an active and generative role in learning mathematics. Schwartz *et al* (1985:1) compares the teaching of mathematics with that of English and claims that mathematics teaching has traditionally fallen short when it comes to providing a climate for creativity and original thought. Mathematics is taught in such a way that teachers never expect students will ever have the occasion to invent new mathematics. English, on the other hand, is taught specifically to ensure that students do learn to create an original piece of prose or poetry.

In essence, this short description of how teachers approach mathematics instruction and English instruction highlights the inadequacies of the traditional approach to mathematics teaching. The rigid teaching of rules and algorithms no longer suffices in todays society. Students must be provided with mathematical opportunities which will empower them to become involved in mathematical discussion and enable them to make conjectures and evaluate mathematical arguments.

6.4.1 <u>Aims of mathematics teaching</u>

In Section 3.3.1.2 the central aim of mathematics teaching was stated as providing each individual with intellectual autonomy and a self-confidence in doing mathematics. Furthermore, four objectives were identified as critical aspects of mathematics teaching. These four objectives establish a basis for a didactic approach which develops constructivist theory which serves to enhance the mathematics performance of pupils suffering from mathematics anxiety. A brief synopsis of these four objectives is as follows:

1. To provide students with the ability to communicate mathematically.
   Central to the idea of constructivism is the emphasis on the importance of mathematics as an essential element of communication in a modern society. Learning the signs, symbols and terms of mathematics and being able to

discuss mathematical ideas will provide students with invaluable communication skills.

2. To guide the student into becoming a mathematical thinker.
The student must be seen as a mathematical thinker who tries to construct meaning of what he is doing on the basis of personal experience. As this experience broadens so will the opportunities arise for the student to build on previous knowledge. Students learn to monitor and reflect upon their own thinking and performance.

3. To enable the student to become confident in his or her own ability.
School mathematics must provide opportunities for students to grow in confidence and to realise that doing mathematics is a human activity. Having numerous and varied experiences in the classroom will provide the atmosphere for students to grow in confidence and trust their own mathematical ideas and methods.

4. To provide students with the opportunities to become problem solvers.
Constructivism is a problem-centred approach to mathematics in that it requires students to develop a problem solving attitude. This problem solving attitude is developed by allowing students a flexibility in exploring mathematical ideas, instilling a willingness to persevere with a mathematical task and creating an interest, curiosity and inventiveness regarding mathematics.

6.4.2 **Selection of subject matter**

The emphasis in this thesis has been the need for greater concentration on process rather than content by curriculum developers. However, content becomes important given the demands on teachers for a teaching strategy which requires a far greater effort. Curriculum designers are being short-sighted if they expect teachers to be creative in the classroom and yet cover a required body of content which essentially leaves little time for experimentation.

It is not the intention of this thesis to propose what content is essential to mathematics teaching. Suffice to say that a serious effort should be made by teachers and administrators to reduce the content of the secondary school mathematics curriculum in South Africa.

An overfull syllabus encourages rigid teaching styles and repetition and practice. It also encourages students to resort to rote learning of procedures and rules. This fact is reflected in the responses to items 15, 16, 18, 19 and 20 of the questionnaire given to Standard 10 pupils. In particular item 20 reflected the highest mean score of the entire questionnaire. This item confirmed that students believe that mathematics is essentially a learning and practising of procedures.

Most topics in the secondary school syllabus provide ample opportunities for exploration and investigation and the construction of knowledge. However, time constraints will inevitably restrict the teacher from providing these opportunities. In addition, unique problems need to be introduced to provide students with opportunities to reflect their own thinking and to work in collaboration with fellow students in seeking possible solutions (see 4.6.1.2).

6.4.3 Teaching, learning opportunities and learning experiences

The success of constructivism as a didactic approach depends greatly on the teaching of mathematics and the achievement of the stated aims in Section 6.4.1. An overfull syllabus and under-trained teachers do little to instill confidence in the future success of constructivism in the classroom. However, some positive results have been reported at primary schools in KwaZulu Natal and there is therefore reason to believe that this could continue on to secondary school.

Secondary school mathematics must provide continuous opportunities for inquiry-based activities. The student must be encouraged to question, to challenge and learn about real mathematical behaviour. In this way the pupils are motivated to strive for intellectual autonomy and self-confidence when doing mathematics.

Critical variables of mathematics teaching were identified (see 3.4.1) for the purpose of alleviating mathematics anxiety. These critical variables were again emphasised as important aspects of a constructivist view of mathematical development (see 4.3).

Thus a synthesis of these variables forms the crucial structure for instructional design as an element of curriculum development. Teaching, learning opportunities and learning experiences as elements of the curriculum need to be focused on these six variables. These variables have been fully discussed in terms of mathematics anxiety in Section 3.4.1 and in terms of a constructivist approach in Section 4.3. However, a number of important aspects were confirmed in the empirical research (see 5.6.4.4) and a synthesis of these important variables provides a direction for teaching, learning opportunities and learning experiences.

a) **Classroom environment**

The empirical research in this thesis does reveal that mathematics anxiety continues throughout secondary school and is indeed widespread (see 5.7.2). Items 4 and 5 of the Standard 10 questionnaire (see 5.6.4.4) indicate that pupils do not believe strongly that mathematics anxiety stems from the teacher or the parent. However, a number of learner perceptions are clearly established through classroom experiences (see 6.3.8) and these experiences cause anxiety. The classroom environment needs to be developed with attention to the following aspects:

1. Avoid causing anxiety by creating time pressures.

2. Avoid situations where a pupil is singled out to come to the front of the class.

3. Avoid derogatory and condescending remarks in the classroom.

4. Be aware of anxiety in test and examination situations and provide support and positive reinforcement.

5. De-emphasise the rigidity of right and wrongness in mathematics and encourage intuition and creativity.

6. Provide a supportive environment in which questions and discussion are eagerly pursued.

b) **Understanding**

The responses to items 9,16 and 31 of the Standard 10 questionnaire reveal that a high percentage of pupils depend on rote learning and believe that there is little room for intuition or creativity as indicated by the responses to items 12 and 31. A high percentage of responses to item 33 confirm that pupils find mathematics difficult to understand. This emphasises the need for teachers to develop the learning skills of their pupils to promote true understanding. For this purpose the following aspects of understanding are essential (see 4.3.2).

1. Representation. The need to understand symbols and systems used to express mathematical ideas.

2. Knowledge structure. The need to realise that each individual can construct their own knowledge.

3. Conceptual and procedural knowledge. The ability to understand procedures and to integrate various new concepts with prior knowledge.

4. Construction of knowledge. The ability to reorganise one's cognitive structures to assimilate new knowledge.

5. Situation cognition. The ability to adapt knowledge to various situations depending on physical environment and social interaction.

c) **Communication**

Responses to items 4 and 5 of the Standard 10 questionnaire indicate that parents and teachers are not perceived to be a cause of mathematics anxiety. The low average response to these two items is encouraging because it means that pupils obviously feel reasonably comfortable communicating with the teacher and with their parents.

Item 38 of the questionnaire confirms this belief as one of the highest average responses was recorded to indicate that pupils would be more confident doing mathematics if they were able to discuss problems with their teachers. By encouraging this communication, the teacher has the opportunity to dispel mathematics myths and perceptions whilst promoting the need for mathematics and the usefulness of the subject. At the same time the pupil becomes more confident to use mathematics terminology and symbols in his or her communications.

d) **Social interaction**

Once again the low responses to items 4 and 5 of the questionnaire are promising indicators. In Section 3.4.1.5 it was emphasised that parent attitudes and teacher attitudes influence the mathematical development of the child. In addition, the high average response to item 37 indicates that pupils would feel more confident doing mathematics if they are discussing problems with their class mates. However, some important aspects pertaining to social interaction (see 3.4.1.5 and 4.3.4) are confirmed by the responses to items in the questionnaire and should be given attention in the classroom.

1. Avoid sapiential authority. Making solutions appear easy leads to a confirmation in the belief that mathematics is only for a select few and that some people simply do not have a mathematics mind. This type of teaching also leads to a belief that all mathematics is done quickly in the head (item 26) and that there is a magic key to doing mathematics (item 27).

2. Be a facilitator of knowledge and not a transmitter of rules, facts and formulae. Responses to items 16 to 20 of the Standard 10 questionnaire indicate a high degree of belief in the fact that one is responsible for one's own progress. Whilst each item generated a high response on how mathematics is best done, it was very significant that items 18 and 20 revealed a very high response to a sense of responsibility for each person to persevere and practice. Hence, pupils are obviously prepared to do the work themselves but would require some discussion and some facilitation by the teacher.

3. Build on the trust that pupils have in the teacher by establishing mutual trust between teacher and pupil. Emphasise the interactive usefulness of constructivism by allowing pupils to come to mutual agreements with their peer group and with the teacher.

4. Allow pupils the opportunity to listen to each other's solutions, to try to make sense of these solutions and achieving a consensus about a solution.

e) **Intellectual autonomy and self-confidence**

This aspect of development overlaps with all the other critical aspects as it could be described as the focus and goal of mathematics teaching (see 3.3.1.2). A number of important factors pertaining to the development of self-confidence were described in Section 3.4.1.3 and these factors were included in the Standard 10 questionnaire. In Section 4.3.5 the development of intellectual autonomy was linked to the development of self-confidence as pupils participate in mathematics experimentation, exploration, creativity and intuitive thinking.

A summary of the important factors expressed in Sections 3.4.1.3 and 4.3.5 and confirmed in the Standard 10 questionnaire (see 5.6.4.4) are as follows:

1. Time restraints are contrary to the idea of developing mathematical autonomy. Each individual works at a different pace and competence and confidence is destroyed by unrealistic time constraints (see item 32 of the questionnaire in 5.6.4.4).

2. Listening and relating to pupils is an essential ingredient of teaching to develop autonomy and confidence. Item 38 of the questionnaire indicated a need for discussion between teacher and pupil.

3. Making mathematics less rigid and authoritative is important for combating rigid approaches to the doing of mathematics. A belief in rote learning and the memorization of rules and procedures will lead to a dependence on the teacher and hence a loss of self-confidence to do mathematics. This reliance on rules is confirmed by items 9 and 16 in the Standard 10 questionnaire and most noticeably by being the second highest anxiety situation described in the mathematics anxiety rating scale (see 5.6.4.6).

4. Practising unusual problems is most important for the development of intellectual autonomy and self-confidence. This is not intended to provide solutions for all problems but rather to develop an independence when faced with a new problem. The data collected from the mathematics anxiety rating scale indicates that facing a new problem is a highly anxiety provoking situation (see 5.6.4.6).

5. Providing positive feedback at all times and especially after tests will allay some of the fears of doing mathematics and of writing tests. The fact that mathematics certainly provides less positive feedback opportunities than a subject like English was established in Section 2.2.2.3 and is confirmed by the fact that test situations in mathematics are perceived to be a high anxiety provoking situations (see 5.6.4.6). Clearly a more supportive and empathic approach to test situations is important to establish a trust between teacher and pupil and to restore confidence in the child's ability.

f) **Problem solving**

In Section 4.3.6 it was clarified that developing problem solving skills was synonymous with developing a problem solving attitude. It is this all important attitude towards solving problems which provides pupils eventually with an autonomy to do mathematics and a confidence in their own ability.

The data from the Standard 10 questionnaire is encouraging when considering the factors relating to developing problem solving skills. Pupils do not strongly believe that problems can be done in one particular way (item 13). They believe in perseverance (item 18) and they believe that practice is an important aspect of mastering mathematics procedures. All these responses are encouraging when considering the development of healthy attitudes towards mathematics. However, these aspects are tempered by what pupils perceive to be the correct approach to problems. "Mathematics is done by working intensively until the problem is solved" may indicate a high level of perseverance but it also reveals a lack of understanding of the situation and creativity required when dealing with genuine mathematics problems (item 31).

Once again it should be stressed that the dependence on rules and memorization is a high anxiety risk method for pupils to adopt. This is confirmed in the mathematics anxiety rating scale data (see 5.6.4.6) and by items 9 and 16 in the Standard 10 questionnaire.

In Section 4.3.6 it was emphasised that teachers may view problem solving as a high anxiety provoking situation and there is no doubt considerable substance to this perception. Being confronted with a new problem is an anxiety provoking situation in any context. However, it is the development of a problem solving attitude that will alleviate this anxiety as pupils become aware and more responsible for their own mathematical skills and development and hence strive towards intellectual autonomy and self-confidence.

In addition to these six critical aspects of mathematics teaching, three classroom activities were identified in Section 4.6.1 as most important in the development of a constructivist approach to teaching and learning mathematics. These included providing a sense of number, dealing with novel problems and allowing pupils to reflect their methods. A summary of these concerns is also necessary at this stage.

1. Providing a sense of number

   This begins in primary school but must remain a cornerstone of the constructivist approach in the secondary school. The weakness in algebra, trigonometry and even geometry is more often than not a lack of a sense of number. The Standard 5 control test revealed some obvious gaps in the insight and understanding of number concepts both by the pupils and the teachers, e.g. 0,3 x 0,3 and 0,4 x 0,4 were considered to be testing multiplication and place-value whereas the results showed that only 0,3 x 0,3 truly tested place-value. Opportunities must be created in the classroom to provide exercise in number and a relevance to everyday needs. A confidence in quick mental arithmetic will provide a greater confidence in mathematics in general.

2. Dealing with novel problems

The teacher needs to introduce as many interesting and diverse problems as possible. Children should be willing to solve puzzles and in general be introduced to the fun of solving these puzzles (see 4.6.1.2). The use of books such as the Guinness Book of Records and Mathematics Olympiad type questions are useful in demonstrating a varied approach to mathematics and developing a sense of wanting to discover things mathematically.

The emphasis on problem solving must not be on strategies but rather on attitude. This is often a factor which proponents of the constructivist theory may be confused about. By teaching problem solving techniques and strategies a teacher may lapse into a rigid approach which promotes a type of rote learning of problems.

It is a problem solving attitude which is all important. The classroom becomes a problem centred arena where teachers create opportunities for investigation, discussion and discovery. The problems may be open-ended, hence allowing for free expression and not a predictable or given answer. Students are encouraged to investigate their own solution paths and to not become dependent on the teacher to provide the answers.

The problem solving attitude should be valued because it is the most important aspect of the school learning experience. The ability to face a problem with confidence will be the empowerment that future generations need to function competently in society.

3. Allowing pupils to reflect their methods

Reflective thinking should permeate all teaching ideals because it requires an ability to communicate and a concern for understanding. Students only become empowered in mathematics when they are able to use mathematical terms, symbols and ideas to communicate with others. They will only be able to communicate with others when they are confident about their own solutions and ideas. By providing opportunities for students to reflect their solutions to problems, a mathematics teacher allows the student to seek understanding and modification of his or her mathematical interpretation. It also provides the teacher with the opportunity to listen to solutions and be able to provide guidance.

The idea of reflection will also provide opportunities to dispel many of the mathematics myths which were confirmed in the Standard 10 questionnaire. For example, reflection will illustrate that there is no magic key to mathematics and that mathematics is not done quickly in mathematicians heads. It will also enable students to see that mathematics can be creative and not purely working intensively until the problem is solved.

The issues discussed in Section 2.2.2.2 and Section 4.4, emphasise the importance that reflection can have on mathematics anxiety. Caring deeply for children and learning to listen will give teachers the opportunity to dispel harmful myths whilst at the same time providing an atmosphere where the mathematically anxious child may be more willing to take risks and contribute to the solution of a problem.

6.4.4 <u>Evaluation in mathematics</u>

There is no doubt from the various instruments of empirical research used in this thesis that the testing and examining of mathematics is a highly emotive issue. In addition, the idea that topics will be tested comprehensively will have a negative effect on constructivist teaching.

The critical aspects of the constructivist theory which have just been discussed all emphasise interaction and participation by the class. Negotiation of solutions and an open-ended problem solving attitude is essential. Evaluation by writing tests and/or examinations is contrary to this theory and will always lead to teachers compromising their ideals and students concentrating their efforts on what is important for the examination rather than what is important for the development of their minds.

It is not the task of this thesis to present various options for testing but at the same time it is necessary to make the observation that present testing methods need to be changed because the tested curriculum has such a strong influence on what students are taught and how they are taught.

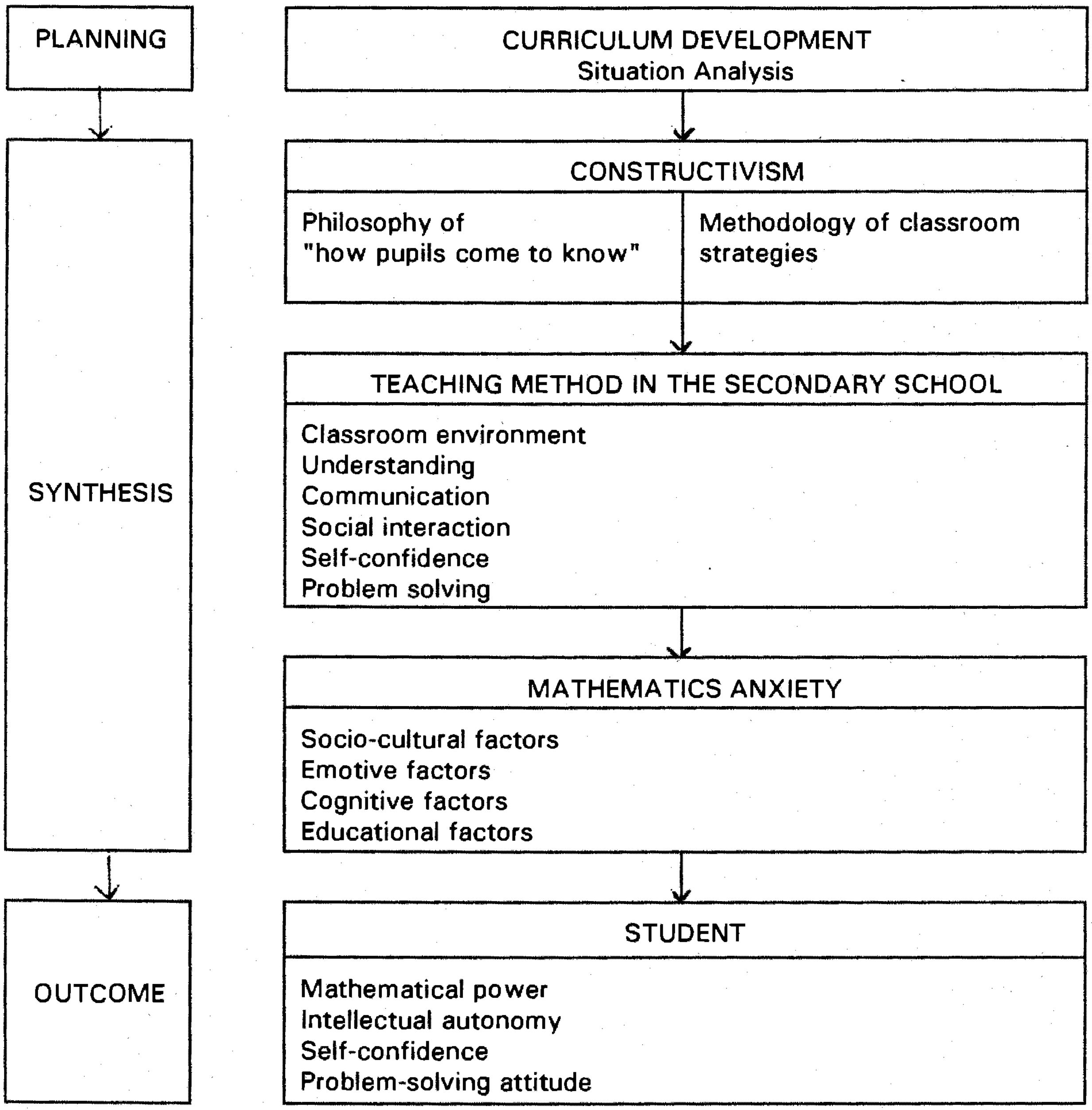
The most effective evaluation techniques will be ones which reflect the constructivist philosophy by embracing an assessment of all that the student is involved in mathematically. It should focus on a broad range of tasks and assess development in problem situations that require the application of a number of mathematical ideas. The everyday teaching program should have the assessment fully integrated into it and students should be assessed by monitoring how they think about mathematics. Standardised testing may still form part of the assessment but should be de-emphasised in favour of ongoing assessment which encourages freedom of thought and a will to participate in problem solving.

6.5 Synthesis

Constructivism is in itself a synthesis of the central themes of this research. It is a didactic approach which encompasses a philosophy for curriculum design as well as a strategic plan for instructional methodology. As a philosophy it provides an impetus and direction for curriculum reform and as an instructional methodology it provides an impetus and direction for classroom strategies. In both cases the benefit is for the development of the individual as mathematically functional and free from mathematics anxiety.

The comprehensive nature of curriculum development requires one to address a wide range of criteria. However, the central aspects of this thesis is a concern for the mathematically anxious child in future curriculum reform. The intention of this study is to show that the elements of constructivism provide the philosophy and teaching approach which serve the needs of the mathematically anxious child.

Thus the synthesis of the central elements of this thesis may best be illustrated in the diagram below.



Curriculum development requires a comprehensive analysis of the situation. This analysis involves elements of philosophy, social influences, psychological considerations and teacher evaluation.

These elements provide the grounds for arguments which favour constructivism as a didactic approach. Constructivism embodies a philosophy and a methodology which are specifically aimed at providing teachers with ideas for classroom strategies as well as providing a reason why these strategies are adopted (see 4.2 and 4.3).

The teaching strategies, in turn, provide the elements of concern when sources of mathematics anxiety are considered. These include socio-cultural, emotive, cognitive and educational factors (see 4.7). In the final analysis the synthesis of constructivism, mathematics anxiety and teaching strategies must serve to equip the student for the future by instilling in each individual a sense of mathematical power, intellectual autonomy, self-confidence and a problem solving attitude.

6.6 Conclusion

The conclusions reached in this thesis are based on a synthesis of the comprehensive literature study on mathematics anxiety, curriculum design and constructivism. In addition, the empirical research provided evidence to support certain theoretical aspects which were emphasised in the literature.

This synthesis is the ultimate aim of this thesis as stated in Section 1.5.5. It was proposed that a synthesis of the research in mathematics anxiety, constructivism and curriculum development will provide the didactic approach which will alleviate mathematics anxiety through a comprehensive concentration on critical aspects of the teaching and learning of mathematics.

Providing an overview of mathematics anxiety and establishing it as an important variable in the mathematics development of the child initiated the need to provide an overview of mathematics teaching and curriculum design in mathematics. This led to an overview of the constructivist approach as a proposed didactic theory which would provide a comprehensive remediation for mathematics anxiety. The empirical research described in Chapter 5 was used to confirm and support the literature views on mathematics anxiety, mathematics teaching and constructivism.

Constructivism forms the synthesis of the various aspects of this research because it focuses on all the elements which will provide students with an empowerment to work confidently with mathematics. Constructivism provides the didactic approach on which to base curriculum design. Constructivism becomes the basis of situation analysis and permeates the various aspects of situation analysis in the curriculum. In particular, constructivism provides the answers to many of the problems facing the mathematically anxious child. Socio-cultural, emotive, cognitive and educational sources of anxiety are all addressed by a constructivist approach (see 4.7).

Constructivism emphasises the role of the teacher and the need for reform in teaching practice. The research in this thesis pertaining to trainee teachers indicates that there is a need for improving the mathematical competence and confidence of our teachers and at the same time making them aware of the value of a constructivist approach to teaching mathematics.

The research at secondary school level indicated an unsatisfactory correlation between early success in mathematics and the final outcome in Standard 10 (see Graph 4; 5.7.2.2). This highlights the importance of the secondary schooling and the inadequacies of the mathematics form of study. In comparison, English maintained a far better correlation throughout the secondary school years (see Graph 2; 5.7.2.2) and it could be argued that the success in English is due to the freedom of thought and less rigid approach that English teachers have been able to present to their students.

In summary, constructivism calls for a reform in present teacher training and a need for in-service courses to re-educate the old-style teachers or teachers who adopt a traditional approach of presenting mathematics as static and consisting of routine procedures (Steffe, 1990:167) which Greenwood (1984:663) describes as an explain-practice-memorize teaching paradigm. Mathematics anxiety research also reveals a need for the re-education of teachers (see 2.2.2.4).

Clearly the elements of constructivism and mathematics anxiety research are interwoven in the expressions of reform in curriculum design and in particular the teaching philosophy and strategies of curriculum planning.

Hence, teachers need to become the focus of future reform because the success of a new didactic approach needs the support and belief of the teachers. The teaching philosophy and strategies of constructivism form a cognitive position and a methodological position which should enthuse teachers and motivate them to accept change.

In addition, the community at large needs to be informed and to also accept the positive elements of the constructivist theory. That these present day students are being prepared for a different world situation than their parents is the key to why change is necessary. Furthermore, the socio-cultural factors influencing both mathematics anxiety (see 2.2.2.1) and the constructivist approach (see 4.7.1) need to be addressed with the assistance of the parent and teacher to ensure that the problem of mathematics anxiety is not perpetuated (see 2.1.3.4).

More importantly, classroom activity must move towards a far less rigid, authoritarian approach to a more flexible and caring centre which promotes social-interaction, reflection, a sense of number and a problem solving environment.

Teachers must have more time to devote to these activities and not be over-concerned with a very full syllabus. There should be a de-emphasis of old examination techniques and a greater emphasis on project work and the teacher listening to the input from each individual child on a day to day basis.

Finally, constructivism provides the ingredients to serve the needs of each individual in a changing society. The key word associated with the constructivist approach is "empowerment". Providing the individual with "mathematics power" is central to the aims of constructivism. Mathematics power embodies the needs of the individual because it means an ability to communicate mathematically, to interact confidently in mathematical situations and to enter problem situations with a confidence in ones ability to make a worthwhile contribution.

In the South African context this mathematical empowerment of students is the overall attraction of constructivism. As Beane (1990:182) points out:

> *The constructivist capacity of people matches the themes of democracy, dignity and diversity more closely than behaviouristic theories that favour conditioning toward compliance with externally imposed behaviour rules without reason or consent and in defiance of dignity and choice.*

Sears & Marshall (1990:15) view the process of providing students with empowerment as a critical aspect of the development of the child and as the central reason for curriculum studies:

> *The intent of empowering students through curriculum studies is to enable them to recognise, create and channel their own power. This is achieved in large part because the learning environment expects students to be active, responsible participants.*

In the final analysis the mathematics power provided by constructivism is not only the required outcome for all students of mathematics but it is also a necessary outcome for all functioning members of society. Communication, confidence and a problem solving attitude are all essential elements for future success in an ever-changing world.

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APPENDIX FILE

1. Mathematics Anxiety Rating Scale for Standard 5 students.
2. Mathematics Control Test for Standard 5 students.
3. Easy/Difficult Rating Scale for Standard 5 Control Test.
4. Mathematics Anxiety Rating Scale for Standard 10 students.
5. Questionnaire for Standard 10 students.
6. Mathematics Anxiety Rating Scale for College of Education students.
7. Details of students excluded at Standard 10 level.
8. Comparison of items on local MARS tests and original MARS test.
9. College of Education results.
10. Standard 5 results.

APPENDIX 1

MATHEMATICS ANXIETY RATING SCALE (STANDARD 5)

BOY	GIRL

NAME: ______________________________ **TOTAL SCORE:** ________

The items in the questionnaire refer to things and experiences that may cause fear or apprehension. For each item place a tick [✓] in the box under the column that describes how much you are frightened by the situation described.

	NOT AT ALL	A LITTLE	A FAIR AMOUNT	MUCH	VERY MUCH
1. Checking your change after buying several items at a shop.					
2. Having someone watch you when you do mathematics.					
3. Calculating the sales tax on a purchase.					
4. Watching a teacher doing mathematics on the board.					
5. Sitting in your mathematics class.					
6. Watching someone work with a calculator.					
7. Listening to someone explain how a computer works.					
8. Studying for a maths test.					
9. Writing a maths test.					
10. Reading a maths book.					
11. Raising your hand in a maths class to ask a question.					
12. Playing cards where numbers are involved.					
13. Thinking about a maths test one week before.					
14. Thinking about a maths test one day before.					
15. Thinking about a maths test one hour before.					
16. Waiting to get a maths test returned in which you expected to do well.					

	NOT AT ALL	A LITTLE	A FAIR AMOUNT	MUCH	VERY MUCH
17. Waiting to get a maths test returned in which you expected to do badly.					
18. Opening a maths book and seeing a page full of problems.					
19. Being given a sudden quiz in a maths class.					
20. Asking your maths teacher to help you with a problem you do not understand.					
21. Being asked how you arrived at a particular answer to a problem.					
22. Having your teacher watch you as you work through a problem.					

APPENDIX 2

PRIMARY MATHEMATICS CONTROL TEST (STANDARD 5)

NAME: ________________________________ BOY: _____ GIRL: _____

DO THE FOLLOWING PROBLEMS: **ANSWER**

1. Write $\frac{37}{100}$ as a decimal. ____________

2. What number must be placed in the box to make the following true?

   $\frac{\boxed{x}}{8} = \frac{3}{12}$ ____________

3. 41 x 40 ____________

4. How many squares of length 3cm could I fit into a square with sides of length 6cm? ____________

5. 0,7 ÷ 0,7 ____________

6. The ratio of boys to girls in a class is 2:5. If there are 15 girls, how many boys are there? ____________

7. Change $\frac{3}{8}$ to a decimal ____________

8. 0,4 x 0,4 ____________

9. 513 ÷ 27 ____________

10. What number must go in the box to make the following number true?

    $12 \div 6 - \square = 15$ ____________

11. 0,3 x 0,3 ____________

12. What number must x be to make the following true?

    $x + 8 + x = 12$ ____________

13. What number must go in the box to make the following true?

    $\frac{2}{\boxed{x}} = \frac{1}{3}$ ____________

14. In John's class there are 30 children. 5 of them wear glasses. What fraction of the class wear glasses? ____________

15. $2\frac{1}{16} - \frac{3}{8}$ ______

16. If a girl can cycle at a speed of 8km per hour, what time will she take to travel 2km? ______

17. Add the following numbers:

    16,36   1,9   243,075 ______

18. Which of the following fractions is the smallest?

    $\frac{7}{16}$ $\frac{5}{32}$ $\frac{63}{64}$ $\frac{3}{8}$ ______

19. If the area of a square is $25m^2$, what is the length of each side? ______

20. In a Supermarket I bought a tin of paint costing R3,69 and a brush costing R1,15. The Cashier rang up an incorrect price, making a total bill of R5,21. How much was I overcharged? ______

21. Add the following numbers:

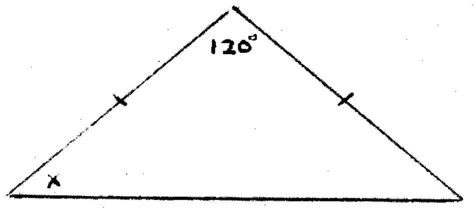
    298,78   72,36   13,89 ______

22. You are going to the U.S.A. on holiday and you take R100,00 to spend. If the exchange rate is 0,52 dollars what will you receive for your R100,00? ______

23. $816 \div 8$ ______

24. $\frac{1}{6} + \frac{2}{3}$ ______

25. How many degrees is x? ______



APPENDIX 3

EASY/DIFFICULT RATING SCALE FOR PRIMARY MATHEMATICS CONTROL TESTS (STANDARD 5)

Now that you have answered the 25 questions above, read the questions again carefully and rate each question on the scale below.

Place a tick [✓] in the block below the heading that best describes your opinion of each question.

	VERY EASY	EASY	NOT SURE	DIFFICULT	VERY DIFFICULT
1.					
2.					
3.					
4.					
5.					
6.					
7.					
8.					
9.					
10.					
11.					
12.					
13.					
14.					
15.					
16.					
17.					
18.					
19.					
20.					
21.					
22.					
23.					
24.					
25.					

APPENDIX 4

MATHEMATICS ANXIETY RATING SCALE (STANDARD 10)

BOY: _____ GIRL: _____

NAME: ______________________ TOTAL SCORE: __________

The items in the questionnaire refer to things and experiences that may cause fear or apprehensions. For each item place a tick [✓] in the box under the column that indicates <u>how much your anxiety is aroused</u> by the situation described.

		NOT AT ALL	A LITTLE	A FAIR AMOUNT	MUCH	VERY MUCH
1.	Checking your change after buying several items at a shop.					
2.	Having someone watch you when you do mathematics.					
3.	Calculating the sales tax on a purchase.					
4.	Watching a teacher doing mathematics on the board.					
5.	Sitting in your mathematics class.					
6.	Watching someone work with a calculator.					
7.	Listening to someone explain how a computer works.					
8.	Studying for a mathematics test.					
9.	Writing a mathematics test.					
10.	Reading a mathematics book.					
11.	Raising your hand in a mathematics class to ask a question.					
12.	Playing cards where numbers are involved.					
13.	Thinking about a mathematics test one week before.					
14.	Thinking about a mathematics test one day before.					
15.	Thinking about a mathematics test one hour before.					
16.	Waiting to get a mathematics test returned in which you expected to do well.					

		NOT AT ALL	A LITTLE	A FAIR AMOUNT	MUCH	VERY MUCH
17.	Waiting to get a mathematics test returned in which you expected to do badly.					
18.	Opening a mathematics book and seeing a page full of problems.					
19.	Being given a sudden quiz in a mathematics class.					
20.	Asking your mathematics teacher to help you with a problem you do not understand.					
21.	Being asked how you arrived at a particular answer to a problem.					
22.	Having your teacher watch you as you work through a problem.					
23.	Working with a calculator.					
24.	Thinking about the time you have to complete a mathematics test.					
25.	Being asked to estimate an answer without using your calculator.					
26.	Doing a word problem in algebra.					
27.	Deciding how to start to answer a mathematics question.					
28.	Having to do a geometry problem that looks difficult.					
29.	Not having the formula needed to solve a mathematics problem.					
30.	Being faced with a mathematics problem unlike one that you have done before.					

APPENDIX 5

QUESTIONNAIRE (STANDARD 10)

Please complete the following questionnaire. Your answers will help to analyse problems in the teaching and learning of mathematics so please indicate your true thoughts. This is not a test and there are no right or wrong answers. Your answers will be treated with the utmost confidentiality. Please complete the personal details and then tick the appropriate blocks in the sections that follow.

NAME: ________________________________

SCHOOL: ________________________________

AGE: _______ YEARS _______ MONTHS

SEX: MALE ___ FEMALE ___

MATHS GRADE AT PRESENT: HIGHER ____

STANDARD ____

FUNCTIONAL ____

LOWER ____

		VERY TRUE	SORT OF TRUE	NOT VERY TRUE	NOT AT ALL TRUE
1.	I get anxious when confronted with a mathematics problem.				
2.	I understand mathematics but my anxiety has a negative effect on my performance in class.				
3.	I understand maths but I make too many careless errors.				
4.	My teachers cause me to be anxious about mathematics.				
5.	My parents cause me to be anxious about mathematics.				
6.	I get anxious when confronted with a mathematics test.				
7.	I get anxious when writing a mathematics examination.				
8.	I feel more anxious when writing a mathematics examination.				
9.	The mathematics I learn at school is mostly facts and methods that have to be monitored.				
10.	Mathematics ability is a gift that only some people have.				

		VERY TRUE	SORT OF TRUE	NOT VERY TRUE	NOT AT ALL TRUE
11.	In mathematics, something is either right or wrong.				
12.	In mathematics you can be creative and discover things by yourself.				
13.	Mathematics problems can be done correctly in only one way.				
14.	Real mathematics problems can be solved by common sense instead of the rules you learn at school.				
15.	To solve mathematics problems you have to be taught the right procedures or you cannot do anything.				
16.	The best way to do mathematics is to memorise all the formulae.				
17.	The best way to do mathematics is to use your own initiative.				
18.	The best way to do mathematics is to persevere till you hit on the right procedure.				
19.	The best way to do mathematics is to ask someone to help you find the right procedure.				
20.	The best way to do mathematics is to practice different procedures.				
21.	Over the years I have become more confident in my ability to do mathematics.				
22.	I have coped with school mathematics but will never be confident in my ability to do mathematics.				
23.	Mathematics has become less fun than it was at first.				
24.	Mathematics has never been a subject I enjoyed.				
25.	Mathematics is done by working intensively until the problem is solved.				
26.	Mathematicians do problems quickly in their heads.				
27.	There is a magic key to doing mathematics.				
28.	My mathematics ability is related to the mathematics ability of my parents.				
29.	Some people have a mathematics mind and others do not.				
30.	Mathematics is not a creative subject.				

		VERY TRUE	SORT OF TRUE	NOT VERY TRUE	NOT AT ALL TRUE
31.	Mathematics requires logic, not intuition.				
32.	I can do mathematics but I need more time.				
33.	The questions in mathematics are always difficult to understand.				
34.	It is always important in mathematics to get the answer exactly right.				
35.	I am never sure how to start a mathematics problem.				
36.	Calculators have made mathematics more difficult.				
37.	I am more confident about mathematics when I am able to discuss problems with my class mates.				
38.	I am more confident about mathematics when I am able to discuss problems with my teacher.				
39.	Boys are better than girls at mathematics.				
40.	I do not feel comfortable asking a question in a mathematics class.				

APPENDIX 6

MATHEMATICS ANXIETY RATING SCALE
COLLEGE OF EDUCATION

Please answer the following questions on this page.
There is no need to write your name on this questionnaire.
All answers will be treated confidentially.

1. Are you Male or Female? ________

2. Are you taking the general maths course or the specialised maths course? ________

3. What was the highest level of mathematics you achieved at school? [e.g. Std.8, Std.10 (Standard Grade) Std.10 (Higher Grade)] ________

4. If you wrote mathematics for matric, what symbol did you get? ________

5. Up to what level of mathematics will you feel confident teaching? ________

The remaining questionnaire consists of several things or experiences that may cause fear or apprehension.

This is <u>not a test</u>. Please answer the questions truthfully and try to assess your feelings as accurately as possible by imagining the situations described. Work quickly but be sure to consider each item individually.

The items in the questionnaire refer to things and experiences that may cause fear or apprehension. For each item place a tick [√] in the box under the column that describes how anxious you feel about the situation described.

		NOT AT ALL	A LITTLE	A FAIR AMOUNT	MUCH	VERY MUCH
1.	Watching a lecturer do mathematics on the board.					
2.	Having someone watch you when you do mathematics.					
3.	Opening a mathematics book and seeing a page full of problems.					
4.	Studying for a mathematics test.					
5.	Thinking about an upcoming mathematics test one day before.					
6.	Entering a mathematics class.					
7.	Studying for a mathematics examination.					
8.	Working with a calculator.					
9.	Having your lecturer watch you as you work through a problem.					
10.	Writing a mathematics test.					
11.	Waiting to get the results of a mathematics test in which you expected to do well.					
12.	Waiting to get the results of a mathematics test in which you expected to do badly.					
13.	Being asked how you arrived at a particular answer to a problem.					
14.	Being given a sudden quiz in a mathematics class.					
15.	Asking your mathematics lecturer to help you with a problem you do not understand.					
16.	Listening to a lecture in a mathematics class.					
17.	Not having the formula needed to solve a particular problem.					
18.	Listening to someone explain how a computer works.					
19.	Reading a formula in chemistry.					

		NOT AT ALL	A LITTLE	A FAIR AMOUNT	MUCH	VERY MUCH
20.	Having a friend try to teach you a mathematics problem and finding you do not understand.					
21.	Discussing a mathematics problem with someone in your class who does well at mathematics.					
22.	Receiving your final mathematics results in the post.					
23.	Reading and interpreting graphs or charts.					
24.	Trying to estimate the answer to a square root problem before looking for it in your tables book.					
25.	Being called upon unexpectedly to recite in a mathematics class.					
26.	Doing a word problem in Algebra.					
27.	Watching a lecturer work an Algebraic equation on the blackboard.					
28.	Having to use a 3 figure table book.					
29.	Being given a homework assignment of many difficult mathematics problems which is due in the next class meeting.					
30	Calculating the sales tax on a purchase.					

APPENDIX 7

DETAILS OF STUDENTS EXCLUDED AT STANDARD 10 LEVEL

BOYS	REASONS FOR EXCLUSION
Watson	Left country
Cain	No trace
Christianson	No trace
Friel	Failed
Leclezio	Failed
Schroen, M.	Left country
Van Dam	No trace
Wheeler	Failed
Clarke	No trace
Gomez	Left school
Preston	Left school
Schroen, G.	Left country
Sheath	Left school
Smith	Left country
McKenzie	No trace
McKie	Left school
Morby-Smith	Left school
Parker	Failed
Rodrigues	Left school
Wisby	Left school
GIRLS	**REASONS FOR EXCLUSION**
Blackwood	No maths
Dockendorf	Left Durban
Horner	No maths
Rosenthal	No maths
Russell	No maths
Culver	Left Durban
Jones	No maths
Nevett	No maths
Nizzi	No maths
Pace	No maths
Warddropper	Left Durban
Beard	No maths
Kriel	No maths
Lowens	No maths
Pienaar	No maths
Salvador	No maths
Weakley	No maths
Weatherby	No maths
Blenkensdorp	No maths
Clarkin	Left school
de Goede	No maths
Saul	No maths
Scott	No maths
Webb	No maths
Diana	Left school

33 (Left school / Failed / Discontinued mathematics)

APPENDIX 8

COMPARISON OF ITEMS ON LOCAL MARS TESTS AND ORIGINAL MARS TEST

STANDARD 5 ITEMS	MARS	STANDARD 10 ITEMS	MARS	C of E ITEMS	MARS
1	1	1	1	1	25
2	2/3	2	2/3	2	2/3
3	6	3	6	3	82
4	25	4	25	4	34
5	28	5	28	5	74
6	30	6	30	6	28
7	40/49	7	44/49	7	34
8	34	8	34	8	93
9	53	9	53	9	2/3
10	82	10	82	10	53
11	45	11	45	11	78
12	60	12	60	12	79
13	73	13	73	13	96
14	74	14	74	14	91
15	75	15	75	15	95
16	78	16	78	16	89
17	79	17	79	17	84
18	82	18	82	18	44/49
19	91	19	91	19	51
20	95	20	95	20	61
21	96	21	96	21	77
22	2/3	22	2/3	22	85
		23	93	23	46
		24	New	24	40
		25	56	25	83
		26	25	26	57
		27	New	27	25
		28	40	28	93
		29	84	29	72
		30	New	30	6

APPENDIX 9

COLLEGE OF EDUCATION SURVEY (Page 1)

ANALYSIS OF ANXIETY SCORES

PERSONAL ATTAINMENT LEVEL	STD.7	STD.10 STD. GRADE D OR BELOW	STD.10 STD. GRADE C OR ABOVE	STD.10 HIGH GRADE D OR BELOW	STD.10 HIGH GRADE C OR ABOVE	TOTAL
No. of Students	16	44	46	30	12	148
Mean	91,75	73,25	69,06	65,53	49,25	70,6
Std. Deviation	17,88	14,54	15,31	13,9	10,05	17,64

COLLEGE OF EDUCATION (Page 2)

TOTAL ANXIETY RATINGS FOR EACH ITEM ON THE MATHEMATICS ANXIETY RATING SCALE

NO. OF ITEM	TOTAL ANXIETY RATING	POSITION	NO. OF ITEM	TOTAL ANXIETY RATING	POSITION
1	678	27	16	684	29
2	511	13	17	431	4
3	530	15	18	540	17
4	484	11	19	557	19
5	448	6	20	482	10
6	692	30	21	669	26
7	427	3	22	385	2
8	681	28	23	547	18
9	492	12	24	559	20
10	448	6	25	440	5
11	465	9	26	534	16
12	323	1	27	648	25
13	579	21	28	638	24
14	527	14	29	452	8
15	623	23	30	620	22

SCORING: NOT AT ALL TO VERY MUCH (1 TO 5)

NO. OF STUDENTS: 148

POSSIBLE LOWEST SCORE: 148

(Reflecting Highest Anxiety Provoking Item)

POSSIBLE HIGHEST SCORE: 740

(Reflecting Least Anxiety Provoking Item)

COLLEGE OF EDUCATION SURVEY (Page 3)

TABLE SHOWING ANXIETY LEVELS OF FIRST YEAR STUDENTS AND LEVEL THAT STUDENT IS CONFIDENT TO TEACH

(Anxiety Scores Range: 150 = High to 30 = Low Anxiety)

Personal Attainment Level	Standard 7		Standard 10 - Std. Grade D or below		Standard 10 - Std. Grade C or above		Standard 10 - Higher Grade D or below		Standard 10 - Higher Grade C or above	
	Anxiety Rating	Level Confident to Teach	Anxiety Rating	Level Confident to Teach	Anxiety Rating	Level Confident to Teach	Anxiety Rating	Level Confident to Teach	Anxiety Rating	Level Confident to Teach
	129	3	101	7	106	5	96	7	67	5
	118	1	100	5	96	5	93	1	60	7
	113	4	100	3	95	5	88	9	58	1
	110	3	100	5	92	7	84	6	56	5
	100	0	98	3	92	8	82	1	55	10
	95	7	90	2	90	8	81	10	48	9
	93	5	89	8	88	2	79	3	46	7
	89	1	87	4	85	8	76	5	45	10
	83	3	86	3	84	7	68	4	44	10
	83	5	85	1	81	8	68	7	40	7
	81	2	84	4	81	6	67	7	40	6
	80	7	84	4	78	10	67	5	32	8
	80	6	81	5	78	8	65	5		
	77	5	79	7	77	8	65	5		
	70	5	79	7	77	0	64	7		
	67	0	79	5	75	1	63	5		
			78	4	75	7	62	8		
			78	1	72	7	61	4		
			77	3	71	8	60	7		
			76	5	71	7	59	4		
			72	5	69	1	58	8		
			72	5	68	8	58	7		
			71	6	68	8	56	7		
			70	7	66	8	54	6		
			69	4	65	5	52	7		
			69	5	65	10	52	5		
			68	7	64	8	52	9		
			67	7	64	5	50	8		
			66	8	63	5	43	10		
			64	5	62	5	43	10		
			64	8	62	0				
			62	4	61	5				
			62	7	61	7				
			62	7	60	8				
			60	5	59	7				
			59	7	59	7				
			59	5	58	5				
			58	7	56	1				
			57	5	56	8				
			56	5	56	4				
			55	8	54	5				
			53	6	44	8				
			51	5	44	5				
			46	5	44	8				
					43	8				
					42	5				

APPENDIX 10

STANDARD 5 SURVEY (Page 1)

PRIMARY MATHEMATICS TEST

ANALYSIS OF RESULTS AND DEGREE OF DIFFICULTY RATINGS NUMBER OF PUPILS (109)

Question No.	% Correct Answers	% Considered Question very easy or easy but gave the wrong answer	Four Teachers Ratings	
			Very Easy	Easy
1	69	13	2	2
2	45	14	0	4
3	81	15	2	2
4	23	42	0	2
5	59	29	0	2
6	41	12	0	1
7	19	21	0	2
8	41	40	1	2
9	69	15	1	3
10	92	5	2	2
11	3	77	1	2
12	81	9	1	2
13	79	4	1	2
14	72	8	1	2
15	23	17	0	1
16	45	15	0	1
17	59	33	1	3
18	38	33	0	1
19	14	27	0	2
20	70	20	0	4
21	77	14	2	2
22	14	21	0	1
23	66	25	2	2
24	35	41	1	2

STANDARD 5 SURVEY (Page 2)

ANALYSIS OF MATHEMATICS ANXIETY RATING

Item No.	Total Anxiety Rating	Position
1	161	19
2	249	10
3	230	12
4	140	22
5	151	20
6	165	18
7	175	17
8	300	5
9	341	3
10	179	16
11	197	15
12	146	21
13	225	13
14	320	4
15	383	2
16	296	7
17	410	1
18	297	6
19	266	9
20	219	14
21	244	11
22	271	8

No. of Pupils:	100	
Possible High Anxiety Score (Reflecting High Anxiety)	500	(5 x 100)
Possible Low Anxiety Score (Reflecting Low Anxiety)	100	(1 x 100)

STANDARD 5 SURVEY (Page 3)

PEARSON PRODUCT MOMENT CORRELATIONS BETWEEN MATHEMATICS PERFORMANCE (AS MEASURED ON THE PRIMARY TEST) AND MATHEMATICS ANXIETY (AS MEASURED ON THE RATING SCALE)

X = Score on Primary Test

Y = Score on Mathematics Anxiety Rating Scale

	GIRLS	BOYS	TOTAL
Number of Pupils	52	48	100
Mean X	49	54	51
Mean Y	50	48	49
Standard Deviation X	15,3	16,3	15,9
Standard Deviation Y	11,1	12,1	11,6
Pearson Product Moment Correlations	-0,32	-0,49	-0,39

Significant at the 0,05% Level.

STANDARD 5 SURVEY (Page 4)

RESULTS OF 1ST TERM EXAMINATIONS/ PRIMARY TEST/MATHEMATICS ANXIETY RATING/ DEGREE OF DIFFICULTY RATING

'A' CLASS

GIRLS				BOYS			
1st Term Exam	Primary Test %	Anxiety Rating	Easy Rating	1st Term Exam	Primary Test %	Anxiety Rating	Easy Rating
70	63	63	99	80	71	43	118
80	75	38	83	84	83	52	90
71	54	61	80	78	67	39	112
67	38	49	103	88	79	57	93
76	46	42	93	82	88	36	118
88	58	49	100	86	88	33	106
73	71	58	91	72	63	50	84
76	71	36	104	70	71	42	98
82	54	24	106	87	67	31	99
78	50	57	103	69	67	31	101
73	71	45	101	72	75	54	88
86	63	76	73				
86	75	54	105				
79	75	40	102				
86	67	42	97				
78	54	52	86				
92	50	57	88				
81	54	55	111				
80	79	53	102				
70	42	48	98				
95	54	57	102				
83	54	50	115				
73	54	59	93				
70	50	37	92				

'B' CLASS

GIRLS				BOYS			
1st Term Exam	Primary Test %	Anxiety Rating	Easy Rating	1st Term Exam	Primary Test %	Anxiety Rating	Easy Rating
72	54	44	84	75	42	57	105
84	46	48	89	68	50	38	101
61	46	46	86	73	54	48	103
58	46	46	79	62	54	38	94
73	54	53	96	73	46	36	94
76	42	43	83	57	54	41	95
79	58	52	96	71	38	63	90
62	46	44	90	77	79	43	106
76	42	61	90	68	54	31	109
65	54	51	96	73	46	38	105
67	83	30	109	64	38	37	89
				55	54	57	84
				53	50	42	93
				82	67	33	112
				68	54	28	107
				56	54	36	109
				56	46	44	94
				69	67	43	93
				73	63	62	99

'C' CLASS

GIRLS				BOYS			
1st Term Exam	Primary Test %	Anxiety Rating	Easy Rating	1st Term Exam	Primary Test %	Anxiety Rating	Easy Rating
44	29	60	91	43	54	67	79
69	46	60	79	62	54	53	76
66	29	47	83	34	13	78	101
47	42	72	95	48	50	42	105
54	42	62	70	74	33	57	100
56	29	78	99	54	54	40	92
51	29	70	88	64	46	46	90
51	29	62	61	77	46	57	87
53	13	56	89	64	38	55	96
52	29	64	93	56	58	58	96

STANDARD FIVE SURVEY (Page 7)

'D' CLASS

GIRLS				BOYS			
1st Term Exam	Primary Test %	Anxiety Rating	Easy Rating	1st Term Exam	Primary Test %	Anxiety Rating	Easy Rating
37	38	63	71	44	33	56	99
50	17	46	84	64	38	53	99
44	38	41	104	35	25	71	64
45	46	53	85	30	50	45	90
42	42	42	99	26	21	56	87
63	50	42	95	44	54	52	96
38	46	46	89	63	50	76	99
				51	46	56	88

Total Number of Pupils:	100	(9 Spoilt Papers)
Possible High Anxiety Score	110	
Possible Low Anxiety Score	22	
Possible High Degree of Easy Rating	120	
Possible Low Degree of Easy Rating	24	
First Term Exam Average	66%	
Primary Test Average	51%	