AN EVALUATION OF THE EFFICACY OF THE AIMS AND OBJECTIVES OF THE SENIOR CERTIFICATE MATHEMATICS CURRICULUM

by

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DECLARATION

I declare that

An evaluation of the efficacy of the aims and objectives of the senior certificate mathematics curriculum is my own work and that all sources I have used or quoted have been indicated and acknowledged by means of complete references.

H. RAMBEHARI

To

my wife - Gyanthee,

my children - Chandhira, Shereeka and Jesheeka,

my son-in-law - Rajesh

and

my grandson - Darshan

SUMMARY

AN EVALUATION OF THE EFFICACY OF THE AIMS AND OBJECTIVES OF THE SENIOR CERTIFICATE MATHEMATICS CURRICULUM

In this study, senior certificate (standard 10) pupils' attainment of the cognitive and affective aims and objectives of the senior certificate mathematics curriculum was investigated. With regard to the attainment of the cognitive objectives and aims, senior certificate pupils' performance in their mathematics examination, in terms of three broad categories of cognitive abilities (lower level, middle level and higher level mathematical abilities) was analysed and examined. As no norms (criteria) for mathematical attainment in respect of these three categories of cognitive abilities could be identified, these norms had to be firstly developed by the researcher. However, suitable standardised scales were identified and administered to determine senior certificate pupils' attainment of the affective aims and objectives (attitude towards and interest in mathematics). Besides the quantitative analysis, qualitative assessments of senior certificate pupils' attainment of the cognitive and affective aims and objectives were also made using information obtained, by way of a questionnaire, from teachers of senior certificate mathematics classes.

The main findings that emerged from this investigation were:

* The senior certificate pupils are attaining the desired proficiency levels in the cognitive objectives and aims of the senior certificate mathematics curriculum. However, these pupils are not adequately attaining the affective aims and objectives of the mathematics curriculum.

*	Qualitative information elicited from senior certificate teachers of mathematics tends to support the above findings which were obtained from the quantitative analysis.
*	There is a need for curriculum development in certain areas of the senior certificate mathematics curriculum, particularly in Euclidean geometry, for standard grade pupils.

In terms of the general findings, certain recommendations were also formulated.

In several ways, the present research is a pioneering effort in evaluating the efficacy of the cognitive and affective aims and objectives of the senior certificate mathematics curriculum. It is hoped that this study will serve as a catalyst for future research.

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Key Terms:

Mathematics aims and objectives; Affective aims; Cognitive objectives; Efficacy of objectives; Mathematical attainment norms; Mathematics achievement; Mathematics curriculum; Curriculum evaluation; Secondary school mathematics; Secondary school students.

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GENERAL INTRODUCTION AND ORIENTATION

1.1 INTRODUCTION

> ***"Mathematics is an ancient discipline vested with modern authority. Mathematics empowers people with the capacity for control in their lives; it offers science a firm foundation for effective theories; and it promises society a vigorous economy. In all cultures, in all generations, children study mathematics to gain access to a better life" (Steen 1989:19).***

Throughout the world, mathematics occupies a prominent place in the official school curriculum (Campbell & Fey 1988:53; Travers & Westbury 1989:1; Kamens & Benavot 1991:166). According to Travers and Westbury (1989:1), the importance of mathematics in the school curriculum reflects the vital role it plays in contemporary society. At the most basic level, a knowledge of mathematical concepts and techniques is indispensable in commerce, engineering and sciences. From the individual's point of view, the mastery of school mathematics provides both a basic preparation for adult life and a broad entrée into a vast array of career choices. From a social perspective, mathematical competence is an essential component in the preparation of a numerate citizenry and it is needed to ensure the continued production of highly-skilled personnel required by industry, technology and science.

By focusing on the importance of mathematics, the abovementioned authors have, in fact, hinted on some of the broad goals and aims of mathematics education. Historical surveys of mathematics as a school subject, reported by writers such as Griffiths and Howson (1974: 1-24) and Kramer (1978:4) indicate that the aims of mathematics teaching and learning have been undergoing continuous change. The disciplinary, utilitarian, mathematical, and social aims have each been dominant at various times. However, Kramer (1978:4) observes that, in recent times, the mathematical and social aspects of the subject are emphasised. According to Hunting (1987:32), the traditional goals and aims of mathematics education before the reform period of the 60's were mainly to develop computational proficiency. At the time of the "New Maths" revolution, goals of conceptual understanding were accepted and added, and these emphasised "learning-by-discovery" and "learning-by-doing". During the late 70's and early 80's, problem-solving applications became a major thrust. Conceptual understanding was necessary for successful problem-solving, and so too were computational skills. Leung (1987:37), who made a study of pertinent literature, observes that the abovementioned aims constitute an important part of the set of aims of mathematics education currently adopted by many countries for the senior secondary school level.

In view of the importance of mathematics in society and in the schools, the efficacy of the aims and objectives of mathematics teaching and learning requires sustained scrutiny as well as empirical evaluation from time to time. In this study, the efficacy of the aims and objectives of the Senior Certificate Mathematics Curriculum will be investigated.

1.2 <u>FACTORS LEADING TO THIS STUDY</u>

1.2.1 Curriculum development and syllabus design in the RSA

Yule (S.a.: 9) avers that in South Africa, the State, through the various education departments, plays a dominant role in curriculum development and evaluation. A similar assertion has also been made by authors like Jarvis (1989:33) and Laridon (1990:22) who state that, in South Africa, syllabuses have been developed by the "top-down" process in that it is decreed by Committees at the top of the educational hierarchy. However, White (1985:3) asserts that those attempting to change any aspect of the curriculum must work (as it is done in overseas countries) in consultation with all who are expected to participate in the change - in most cases including principals of schools, teachers, teacher-trainees and parents (perhaps even the pupils, too). He adds that involvement must be from the beginning, when the aims and objectives are being formulated and continue through the implementation stage to evaluating the effectiveness of the intended changes. According to Krüger (1985:11), although some formative evaluation is done before finalising a new curriculum, the system (in South Africa) cannot function properly if there is no follow-up in the schools.

In terms of existing legislation (1993), the Committee of Heads of Education Departments (of the White provincial education departments) is responsible for the approval of primary and secondary school syllabuses, and the Network Committee for Curriculation administers and co-ordinates the syllabus revision programme on behalf of the Committee of Heads of Education. The Joint Matriculation Board (JMB), subsequently replaced by the South African Certification Council (SAFCERT), is responsible for determining university admission requirements and as a result is also involved in the syllabus revision programme (JMB 1989). Although "non-white" education departments are invited to various curriculum committee meetings, they are awarded observer status only - and hence their inputs are minimal.

Since the Department of Education and Culture, Administration: House of Delegates (hereafter referred to as HOD) has not made direct inputs in the development of the broad mathematics curriculum and the core syllabuses that are currently being used at the senior certificate level, an evaluation of the aims and objectives becomes necessary in order to ascertain their efficacy amongst Indian pupils.

1.2.2 Implementation of revised syllabuses

Recent curriculum reform projects, in overseas countries as well as in South Africa, have emphasised the learning of mathematics with "insight and understanding" and a breakaway from rote learning and only simple recall. According to Cresswell (1985:i), the teaching and learning of mathematics (and physical science) in South Africa has been a source of dissatisfaction for a long time. He adds that in view of this, the Human Sciences Research Council Investigation into Education (1983) gave special attention to the teaching of these subjects and made a number of recommendations - many of which were related to curriculum development. Subsequently, new (revised) syllabuses for mathematics (higher grade - HG, and standard grade - SG) were introduced into the junior and senior secondary phases in South African schools, including those falling under the control of the HOD, as from 1985. These syllabuses were examined at the senior certificate level for the first time at the end of 1987.

Although the said syllabuses have been in operation for several years, the performance of pupils in mathematics, particularly at the standard grade level, in the Senior Certificate Examination of the HOD, is viewed by the HOD mathematics educators as being generally unsatisfactory (see paragraph 1.2.3 and Tables 1.1 and 1.2). This was the main reason that prompted this investigation.

1.2.3 Mathematics attainment in the Senior Certificate Examination of the HOD

On the average, approximately 75% of all the standard ten pupils who sit for the Senior Certificate Examination of the HOD offer mathematics. Of the pupils who offer mathematics, approximately one-third of them offer the subject on the higher grade and about sixty percent take the subject on the standard grade. The remainder of the pupils ($\pm$ 6%) offer mathematics on the lower grade. A fairly stringent selection procedure is applied by the HOD in order to determine the grades on which a pupil may offer a subject, including mathematics. The comparatively smaller percentage of pupils who offer mathematics HG in the Senior Certificate Examination of the HOD could therefore be considered a select group, comprising the more able pupils. However, despite the above, it can be observed from the five-year average in Table 1.1, that on the average, some 25% of the HG candidates fail to obtain a pass mark of 40% in mathematics while about 45% of the HG candidates score less than 50% in mathematics in the Senior Certificate Examination.

TABLE 1.1 **MATHEMATICS HG RESULTS (SYMBOL-WISE) IN THE SENIOR CERTIFICATE EXAMINATION OF THE HOD (1989 TO 1993)**

%	0 to 19	20 to 29	30 to 33	33⅓ to 39	40 to 49	50 to 59	60 to 69	70 to 79	80 to 100	NO. OF CANDIDATES	MEAN %
SYMBOLS	H	G	FF	F	E	D	C	B	A		
1989	2,48	8,06	3,93	11,99	20,51	20,61	14,70	10,47	7,27	3 028	51,50
1990	2,11	6,63	3,67	8,25	19,34	20,03	16,84	12,68	10,45	3 320	55,00
1991	3,57	11,03	5,10	13,57	20,13	17,80	13,91	8,99	5,90	3 472	49,25
1992	3,55	10,37	5,25	12,96	20,96	17,39	12,38	8,53	8,61	3 635	50,25
1993	3,77	9,74	5,67	11,19	16,84	17,24	13,70	11,03	10,81	3 736	52,25
5 - YEAR AVERAGE (1988 - 1992)	2,51	7,97	4,14	10,97	19,92	19,61	15,46	10,96	8,48		52,50

TABLE 1.2 MATHEMATICS SG RESULTS (SYMBOL-WISE) IN THE SENIOR CERTIFICATE EXAMINATION OF THE HOD (1989 TO 1993)

%	0 to 19	20 to 29	30 to 33	33⅓ to 39	40 to 49	50 to 59	60 to 69	70 to 79	80 to 100	NO. OF CANDIDATES	MEAN %
SYMBOLS	H	G	FF	F	E	D	C	B	A		
1989	10,96	13,16	5,45	12,23	17,03	15,66	13,10	8,57	3,70	6 106	44,80
1990	10,96	16,16	6,80	12,69	19,03	15,45	9,85	5,93	3,13	6 194	42,67
1991	26,37	20,33	6,16	10,91	15,19	10,07	6,49	3,22	1,27	6 398	34,00
1992	16,85	16,48	5,95	10,67	16,44	13,60	10,83	6,46	4,52	6 521	41,67
1993	29,95	19,07	5,52	10,27	11,97	9,25	7,04	4,62	2,31	7 232	34,00
5 - YEAR AVERAGE (1988 - 1992)	14,77	15,23	5,85	11,62	17,16	14,28	10,84	6,66	3,60		41,50

With regard to the SG pupils, the position appears to be comparatively worse. A scrutiny of Table 1.2 reveals that the mean performance of these pupils, in terms of the five-year average, converges around 42% which is considerably lower than the mean performance of HG pupils (52,50%). Furthermore, if one compares the mean performance of SG pupils of the HOD with that of their counterparts in the Cape Education Department, which according to Norton (1991a:44) is 55,9%, then the mean performance of SG pupils in mathematics in the HOD could be considered as being unsatisfactory, especially in view of the fact that a fair percentage of pupils who qualify to take mathematics on the HG opt, for various reasons, to offer the subject on the SG. In this regard, the observation made by Chetty (1992:4) is pertinent. According to him, more pupils in Indian secondary schools are opting for standard grade mathematics instead of higher grade - because it is easier. Notwithstanding the abovementioned, a scrutiny of the five-year average in Table 1.2 reveals that some 36% of the SG pupils fail to obtain a pass mark (of 33⅓ %) in mathematics while about 65% of them score less than 50%. Furthermore, on the average, only about 3,6 % of the SG candidates obtain "A" symbols.

In view of the apparent poor performance of S.C. candidates, it is possible that some of the aims and objectives of the senior certificate mathematics curriculum are not being adequately attained by the candidates.

1.2.4 <u>Mathematics performance in the junior secondary phase in HOD schools.</u>

The performance of pupils in mathematics in the junior secondary phase in schools of the HOD is monitored by educators at schools as well as by Superintendents of Education for mathematics. However, from time to time, the HOD administers common tests in order to monitor standards at schools.

As part of its monitoring programme, in 1991, the HOD administered criterion-referenced tests in three subjects including mathematics, to standards 5 and 7 pupils in all its schools. An analysis of the test results revealed that the mean performance of standard 5 pupils in the mathematics criterion-referenced tests was 41,1%, with 46,69% of the pupils failing to obtain a pass mark of 40%. On the other hand, the standard 7 pupils obtained a mean of only 29,1%, with 65,82% failing to obtain a pass mark of 40%. The HOD's comments in respect of the above were as follows: Performance of pupils in mathematics in both standards 5 and 7 is a matter of much concern. The factors responsible for the poor performance need to be fully investigated and steps taken to remedy the situation (HOD 1992a:2). The above findings reveal that the performance of the junior secondary pupils in mathematics is unsatisfactory. This could also indicate that the aims of the junior secondary mathematics curriculum, which are more or less the same as the aims of the senior secondary curriculum, are also not being adequately attained at that level.

1.2.5 Some relevant research findings

Several studies, conducted locally as well as overseas, have also highlighted certain shortcomings in the mathematics performance/attainment of secondary pupils and matriculants. A summary of the more relevant research and research findings is presented below.

1.2.5.1 After making a study of various projects on mathematics in the USA, Campbell and Fey (1988:53) note that the more recent analyses of school mathematics have shown that students are not acquiring the skills and understanding they will need to participate in the cultural, economic, political and scientific environments of the future. They add that data from the National Assessment of Education Progress (NAEP) and from college-entrance testing programmes reveal a discouraging pattern of mathematics achievement, particularly in important problem-solving and higher-order thinking skills.

1.2.5.2 Likewise, Brown, Carpenter, Kouba, Lindquist, Silver and Swafford (1988:346-7) have observed that while secondary school students in the USA have generally secured basic computational skills, these students are often not able to apply this knowledge in problem-solving situations, nor do they appear to understand many of the structures underlying these mathematical concepts and skills.

1.2.5.3 Lutfiyya (1989:689) reports on his study carried out in 1984/85 on the acquisition of mathematical skills by Jordanian students in the elementary (grades 1-6), preparatory (grades 7-9) and secondary (grades 10-12) educational stages. While the results of the analysis obtained showed that the educational stages had significantly positive effects on the development of basic mathematical skills of students proceeding from one educational stage to another, he observed that the students' level of performance remained significantly ($p<0,001$) below the educationally accepted level of performance in all of the educational stages.

1.2.5.4 According to Norton (1991a:156), the amount of research undertaken in mathematics education at the senior secondary level in South Africa appears to be rather limited. However, he cites the study of Kriel (1978) who found that first-year tertiary students in general lacked the ability to think critically and reason logically. Also, they exhibited shortcomings in calculation skills in fractions and in approximating answers as well as a lack of manipulative skills in the use of mathematical formulae and equations (Norton 1991a:158).

1.2.5.5 An analysis of errors in high school mathematics made by first-year university students was carried out by Glencross and Fridjhon (1989:36). Besides identifying certain "standard" wrong answers tendered by many different pupils, these authors also observed the consistent use of a wrong method or process by individual pupils. They also found that despite an apparent ability to solve many problems correctly, numerous students did not understand <u>why</u> they were doing <u>what</u> they were doing when understanding was probed. This has led the abovementioned authors to conclude that the creative aspect of mathematics has largely been ignored in our (South African) school curricula and we see the results in matriculants applying rules with little or no understanding.

1.3 <u>STATEMENT OF THE PROBLEM</u>

On reviewing literature on mathematics achievement, it is evident that senior secondary pupils' abilities in problem solving and in the successful application of the higher-order thinking skills are areas of concern in several countries, including the RSA.

It is apparent that the mathematics achievement of standard 10 pupils in Indian schools is unsatisfactory (see paragraph 1.2.3 and Tables 1.1 and 1.2) and, as such, is of much concern to mathematics educators in the HOD. The following are some of the recurrent comments made by the mathematics examiners (HOD 1991: 13-24; HOD 1992b: 21-32): candidates failed to understand concepts, to deduce, to solve, to understand and analyse the problem; and candidates lacked understanding and insight. Seen against the background of the above, the following question arises: **Are the aims and objectives of the senior certificate mathematics curriculum being adequately attained by standard 10 pupils?** It is in seeking solutions to this question that this research is motivated.

1.4 ANALYSIS AND IMPLICATIONS OF THE PROBLEM

Mathematics plays a vital and important role in contemporary society. In reflecting on the importance of mathematics, Cockcroft (1982:1) refers to the more general perceptions of the value of mathematics: at the one end of the scale, he identifies mathematics as a tool needed in everyday life while at the other end of the scale, he sees it as an essential basis of scientific development, modern technology and commerce and industry. Currently, great and rapid technological changes are taking place in society and the demand for students with an adequate knowledge of mathematical skills is growing. Such a demand has implications on the skills and content that are targeted in the teaching and learning of mathematics. Furthermore, performance levels in mathematics have implications on employment opportunities as well as on entry to certain faculties at tertiary institutions. In summary, the NCTM (1989:123) notes that, historically, the purposes of secondary school mathematics have been to provide students with opportunities to acquire mathematical knowledge, skills and modes of thought needed for daily life and effective citizenship, to prepare students for occupations that do not require formal study after graduation and to prepare students for post secondary education.

1.4.1 Mathematical demands in a technological society

According to Glencross and Fridjhon (1990:307), with the world entering what has been described as the second industrial, or technological, revolution, great changes are taking place in society. Consequently, Steen (1989:19) states that this new reality in which we all live, this high-tech prelude to the next century, augurs important changes for the context in which mathematics is taught around the world. Altizer-Tuning (1984:3) asserts that the rapid pace of technological changes accentuates and widens the gap between what is being taught and the "cutting edge of knowledge". In this regard, Silver, Lindquist, Carpenter, Brown, Kouba and Swafford (1988:727) state that many skills that pupils have learnt are in danger of becoming obsolete as technological advancements alter the mathematics that adults need to function productively in society. Taking note of the changes in modern society, the researcher avers that schools too must make changes to accommodate the new social goals of this society.

It is clear then that society seeks leadership from educationists and educational institutions in preparing the youth to enable them to function optimally in a rapidly changing technological society. Hence, it becomes incumbent on educators to also teach those skills that are important and required by the technological environment. In this regard, Corbitt (1985:243) states that the major influence of technology in mathematics education is the potential to shift the focus of instruction from an emphasis on manipulative skills to an emphasis on developing concepts, relationships, structures and problem-solving skills.

It is pertinent to note that one of the five goals adopted by the Standards (NCTM 1989) (discussed in detail in paragraphs 2.2.2.2 and 2.5.2.1) is that American students should "learn to value mathematics". In terms of this goal, students are urged to recognise the varied roles played by mathematics in society so that they will have the incentive to continue studying mathematics as long as they are in school (Steen 1989:19; Johnson 1990:530). In South Africa, however, the position is not very encouraging. According to the Department of National Education, given the scientific and technological needs of the country (South Africa), it is disturbing to note that mathematics (and physical science) are taken by relatively few pupils in the senior secondary phase (DNE 1991:36). It adds that this is especially so among children from the developing population. For example, in 1988, the percentage of Black pupils in standard 10 taking the abovementioned subjects were: physical science - 15,1 % and mathematics - 32,3 %.

1.4.2 <u>The importance of mathematics in the environment</u>

It is generally accepted that mathematics is part of our environment and way of life. This is also reflected by Naidoo (1985:1) who notes that mathematics is used daily in the home, in business, industry, governmental activities and even in sports and other leisure-time activities. This author adds that the use of mathematics in business and industry has become so important that mathematical training has become a required qualification in most fields of employment.

According to Human (1975:14-15), mathematical activities are a significant matter in the life of man, in handling of quantitative aspects of everyday situations, in respect of communication, and vocational pursuits. In this regard, Kramer (1978:5) states that skills such as thinking critically, appraising the correctness of mathematics solutions and applying acquired techniques in verbal problems, to situations in daily life should be developed in pupils. It is important that while pupils should be able to transfer knowledge and skills gained in the classroom to familiar situations in the environment, they must also learn to apply these skills to new situations (problems) as they arise in the future.

1.4.3 Demands in respect of mathematics at the tertiary level

There is a tendency, sustained by universities and other tertiary institutions, to regard achievement levels in mathematics as a yardstick in determining a child's overall ability (Barnard & Strauss 1989:228). While a student who passes the senior certificate examination with matriculation exemption is eligible for enrolment at a university, it is observed that many tertiary institutions in South Africa place greater emphasis on mathematics achievement for further study in certain directions, as they have laid down certain minimum pre-requisites in mathematical performance for first-year enrolments. For example, one of the minimum requirements for first-year enrolments to the science or commerce faculties at many universities is a pass in mathematics with at least 40% on the higher grade or a minimum of 50% on the standard grade at the senior certificate level.

1.5 PURPOSE OF THIS STUDY

1.5.1 In the light of the information presented in the previous paragraphs, it is clear that mathematical proficiency, especially in respect of problem solving and higher order thinking skills, is becoming an important requirement in our continually changing environment. Since the aims of the senior certificate mathematics curriculum cater for the development of a variety of mathematical skills, it is important that Indian matriculants attain the desired proficiency in all these skills at the end of their secondary schooling. Hence, the aim of this study is:

To evaluate the efficacy/attainment of the cognitive and affective aims and objectives of the current mathematics curriculum offered at the senior certificate level.

1.5.2 It is, however, observed that current literature tends to report on students' mathematical achievement in terms of broad categories of mathematical abilities, e.g., in terms of higher-order thinking skills, lower mental processes, etc. In view of this and taking into consideration that the cognitive aims and objectives of the mathematics curriculum can be more readily and easily categorised into three broad levels of mathematical abilities, a classification that is also favoured by HOD examiners, this study <u>more specifically</u> proposes:

1.5.2.1 to evaluate the overall mathematical attainment of senior certificate (S.C.) candidates in terms of three broad categories of cognitive outcomes, namely:-

a) lower level cognitive outcomes (knowledge, skills);
b) middle level cognitive outcomes (comprehension/understanding); and
c) higher level cognitive outcomes (application, analysis, synthesis and evaluation/creative);

1.5.2.2 using the data obtained from the above, to evaluate the S.C. pupils' attainment of the cognitive aims of the S.C. mathematics curriculum;

1.5.2.3 to ascertain, for curriculum development purposes, more indepth information on S.C. pupils' attainment in the 3 categories of cognitive outcomes in terms of selected variables, e.g., subjects, IQ, grade;

1.5.2.4 to evaluate the S.C. pupils' attainment of the affective aims (interest and attitude) of the S.C. mathematics curriculum; and

1.5.2.5 to ascertain qualitative data from S.C. mathematics teachers, in order to supplement the abovementioned (quantitative) evaluations.

1.5.3 In the light of the stated purposes of this study, the following research <u>hypotheses</u> can be formulated.

<u>HYPOTHESIS 1</u>	The lower level cognitive outcomes are being adequately attained by S.C. candidates.
<u>HYPOTHESIS 2</u>	The middle level cognitive outcomes are being adequately attained by S.C. candidates.
<u>HYPOTHESIS 3</u>	The higher level cognitive outcomes are being adequately attained by S.C. candidates.
<u>HYPOTHESIS 4</u>	The cognitive aims of the S.C. mathematics curriculum are being attained by S.C. candidates.
<u>HYPOTHESIS 5</u>	S.C. candidates are displaying an adequate positive attitude towards mathematics.
<u>HYPOTHESIS 6</u>	S.C. candidates are displaying an adequate positive interest in mathematics.
<u>HYPOTHESIS 7</u>	The affective aims of the S.C. mathematics curriculum are being attained by S.C. candidates.

NB: The above research hypotheses will be reformulated in terms of <u>null</u> hypotheses (see Chapter 5) in order to test them against the norms (criteria) that would be developed or identified.

1.5.4 Apart from providing information on the efficacy of the cognitive and affective aims and objectives of the S.C. mathematics curriculum, it is considered that this study will yield some valuable suggestions, for the present and the future, in respect of:

a) A systematic approach in analysing aims and objectives of the mathematics curriculum;

b) Criteria (norms) to evaluate mathematics attainment in terms of three broad categories of mathematical abilities, namely lower, middle and higher level cognitive outcomes;

c) Curriculum development in mathematics;

d) Assessment in mathematics; and

e) Future aspects and problems for research.

1.6 DEMARCATION OF FIELD OF STUDY, ASSUMPTIONS AND LIMITATIONS

1.6.1 Field of Study

The field of study in this research concerns the cognitive and affective aims and objectives of mathematics teaching and learning. The sample for the study is limited to standard 10 pupils offering mathematics higher grade and standard grade at Indian secondary schools under the control of the HOD.

1.6.2 Assumptions

The study is based on the following assumptions:

1.6.2.1 That teachers responded objectively to the items included in the teacher questionnaire which was completed by them;

1.6.2.2 That, as requested, the teachers rated the mathematical abilities of pupils in terms of their global performance (in class exercises, tests, examinations, homework exercises, assignments) and that their ratings were based on trends observed in pupil performance over the last few years.

1.6.2.3 That all sections of the relevant syllabuses had been covered (taught) by teachers who handle senior certificate mathematics classes.

1.6.2.4 That the mathematics question papers set by the HOD in its senior certificate examination are considered as valid and reliable instruments that can be used to measure pupils' achievement of the cognitive objectives;

1.6.2.5 That in view of the importance attached to the senior certificate examination, the pupils put in their best effort in this examination (test) and as such their performance is truly reflective of their mathematical ability at the end of the senior secondary phase (standard 10); and

1.6.2.6 That pupils responded without bias or prejudice to the items in the questionnaire (attitude and interest scales) that was completed by them.

1.6.3 Limitations

The limitations on which the present investigation is based are as follows:

1.6.3.1 Although the aims of the senior certificate mathematics curriculum fall into 3 distinct groups, viz, cognitive, affective and utilitarian, this study is restricted to the attainment of certain aims in only the cognitive and affective domains of educational outcome in mathematics. This decision was based on the fact that certain aims are long-term and therefore cannot be fully realised within the school context; also, the attainment of certain aims are not/cannot be catered for in the examination.

1.6.3.2 This study is also confined to the aims of the mathematics higher grade and standard grade syllabuses only, and does not include those of the lower grade syllabus which is also offered by some pupils at the senior certificate level. It is observed that the aims of the higher grade and standard grade syllabuses are almost the same.

1.6.4 Note on impending curriculum change(s)

It is generally accepted that curriculum and/or syllabus revision has to take place from time to time, in order to meet the demands of the changing environment. In the RSA, the Department of Education and Culture, Administration: House of Assembly (HOA) recently published new curricula (which include revised content and aims) for the senior secondary level , for implementation in standard 9 and standard 10 in 1994 and 1995 respectively. Whilst these curricula are intended for schools falling under the control of the HOA, they have been made available to the other education departments in the country. However, many of these departments, including the HOD, have not adopted them in their schools. Nevertheless, Marsh (1991:154) intimates that there is some likelihood of the new curricula being prescribed (albeit in modified form) in the future, for all schools nationwide.

It is observed that the aims of the new (revised) curricula can also be firstly grouped into the cognitive, affective and utilitarian categories, as has been done with the aims of the current mathematics curriculum under consideration in this study. Furthermore, the cognitive aims of the revised curricula can be further classified into three broad categories of mathematical abilities, namely, lower level, middle level and higher level mathematical abilities, which accord with the theoretical framework of this study (see Chapter 2). With regard to the revision of the content, it is worthy of note that content is a means by which the aims can be realised.

In the light of the above, and although further curricula changes are envisaged shortly, the writer is of the view that the introduction of any revised syllabuses at the senior certificate level will not limit the contribution made by this study, as the criteria (norms) as well as the methods and techniques developed and used in this investigation have the scope for future applicability, particularly with regard to the attainment of the cognitive and affective aims.

1.7 DEFINITION OF TERMS

1.7.1 Indian schools

Indian schools are those schools offering education to Indian pupils, in accordance with the Indians Education Act (Act 61 of 1965) as amended. In terms of this Act, a separate department was established by the State to control and administer education for Indians in the RSA. The nomenclature of the department had changed over the years as follows:

1965 - 1981 Department of Indian Affairs

1981 - 1984	Department of Internal Affairs, Division of Indian Education.
1984 - 31/03/1994	Department of Education and Culture, Administration: House of Delegates (HOD)
1/4/1994 -	Education and Culture Service (Ex - HOD)

It is to be noted that as from 1989, as a result of the HOD's non-racial policy in respect of enrolment, pupils from other race groups, mainly Black pupils, were admitted to Indian schools. While in 1992 there were some 23 000 Black pupils in Indian schools, only a very small number of Black pupils were enrolled in standard 10. Hence it can be accepted that the findings of this research pertain to Indian pupils only.

It needs to be pointed out that this study was commenced with when the tricameral system of political and education dispensation was operative in the RSA, whereby separate departments controlled education for the different population groups. Hence the reference to the HOD and Indian pupils. During the period in which this thesis was being written, the different education departments continued to operate "separately" (e.g., as ex-HOD, ex-HOA, etc.) for a while until the end of 1994. During 1995, the different ex-departments in a province merged under the respective provincial education department.

1.7.2 <u>Senior secondary phase</u>

The educational programme followed by pupils attending schools of the various education departments in South Africa, including those of the HOD, consists of TWELVE years which is divided into the following FOUR school phases of THREE years each, with the respective classes/standards indicated in brackets.

1. Junior primary phase (class i/Sub A; class ii/Sub B, standard 1)
2. Senior primary phase (standards 2,3 and 4)
3. Junior secondary phase (standards 5, 6 and 7)
4. Senior secondary phase (standards 8, 9 and 10)

Hence it is noted that the senior secondary phase, which is the fourth phase, consists of the last 3 years of schooling, namely, the 10th, 11th and 12th years which correspond to standards 8, 9 and 10 respectively.

1.7.3 Senior certificate examination

The senior certificate examination is a public (external) examination which is written by pupils at the end of the 12th year of schooling, i.e., at the end of the standard 10 year. The HOD is one of several education departments in the RSA that are accredited examining authorities that conduct the senior certificate examination. Standard 10 pupils from Indian schools write the Senior Certificate Examination of the HOD.

It is to be noted that a pupil may pass the senior certificate examination with matriculation exemption, provided the minimum statutory requirements for admission to bachelor degree studies at a university are complied with.

1.7.4 Curriculum

A survey of literature reveals that a variety of meanings is associated with the term curriculum. On the one end of the spectrum the term is used to describe the broad or total curriculum, e.g., the entire programme of study, and on the other end it is defined as a written plan of action. In the context of this study, the term curriculum is defined as a plan or written programme for teaching and learning which includes the aims and the corresponding selected and organised subject matter.

1.7.5 Mathematics curriculum

Several subjects, including mathematics, are offered by pupils in schools. The mathematics curriculum refers to the written programme for the teaching and learning of the subject mathematics for a phase. As the education programme is divided into four phases, different but developmental (vertically differentiated) mathematics curricula (programmes) have been developed for use in each of the phases. Each curriculum document contains a set of aims for the teaching and learning of mathematics in that phase as well as a delineation of the subject matter into standards. For example, the mathematics curriculum for the senior secondary phase consists of the aims for the learning of mathematics as well as the contents of the subject that ought to be taught in each of standards 8, 9 and 10.

1.7.6 Senior certificate mathematics curriculum

For purposes of the senior certificate examination, in mathematics pupils are tested on the contents of the subject taught in standards 9 and 10 only. (It is to be noted that since mathematics is developmental, concepts taught in the lower standards are indirectly tested). Hence, the senior certificate mathematics curriculum is defined as the written programme for the teaching and learning of mathematics which includes the aims for the senior secondary phase and the mathematical content taught in standards 9 and 10.

1.7.7 Aims and objectives

A study of literature shows that, generally, three categories of aims are distinguished at the vertical level, namely, macro-level aims, meso-level aims and micro-level aims which correspond to what educationists commonly refer to as goals, aims and objectives. For purposes of this study, the following definitions of these terms are used:

Goals (or general aims) are described as general statements expressing the overall ideal aspired for. They are seen as norm- or value-laden ideals which are directives for all teaching. Hence they tend to be vague.

Aims (or specific aims) are more explicit about the end in view than goals. They are described as the teaching outcomes which are achieved on completion of a school phase or course (in this study, at the end of the fourth school phase in mathematics).

It is to be noted that the aims of the senior certificate mathematics curriculum, being evaluated in this study, fall into both the above categories, namely, general aims and specific aims.

Objectives embrace the immediate, specified and measurable outcomes of classroom teaching.

1.7.8 Cognitive aims and objectives

Cognitive aims and objectives specify behaviours that indicate the functioning of any changes in the various mental processes (Bell 1978:168). They include the objectives which deal with recall or recognition of knowledge as well as the development of intellectual abilities and skills.

1.7.9 Affective aims and objectives

Affective aims and objectives specify behaviours that indicate changes in attitude (Bell 1978:168). They include objectives which describe changes in interest, attitudes and values, and the development of appreciation and adequate adjustment (awareness, feeling).

1.7.10 Utilitarian aims

Utilitarian aims refer to those aims that have a utility value, i.e., usefulness. They include aims such as the use of mathematics in daily life as well as in other subjects.

1.7.11 Efficacy

Efficacy is defined as the ability to produce the results intended, i.e., the extent to which the desired effect/outcome/result is attained/realised/actualised.

1.7.12 Evaluation

Evaluation is described as the process by means of which information is assembled and used to make judgements. It is directed at the teaching and learning of events themselves as well as of their outcomes in terms of particular aims. While two types of evaluation, namely, formative and summative evaluation, are identified, in the context of this study, the term evaluation is used to refer to summative evaluation which comes at the end of a course or year of study. Summative evaluation focuses on the determination of pupils' achievement in terms of the aims of the course/subject.

1.8 RESEARCH METHODS USED IN THIS STUDY

1.8.1 Empirical methods

1.8.1.1 In the absence of any available norms, certain criteria (numerical indexes representing norms for three groups of cognitive outcomes) were first developed. Using these norms, against which pupils' achievements were compared, the attainment of the cognitive objectives and hence the cognitive aims, was determined.

1.8.1.2 The cognitive aims of the senior certificate mathematics curriculum were analysed and elaborated with a view to identifying the more important objectives inherent in them. These were used, amongst others, to develop a teacher questionnaire in order to obtain qualitative information on pupils' mathematical achievements.

1.8.2 Literature survey

In addition to the above, research materials have been obtained from the following sources.

1.8.2.1 Published materials

Books, particularly on mathematics education, and relevant articles in journals and periodicals have been used to gather information on the teaching and learning of mathematics as well as on attainment in the subject at the senior secondary level. Information was gleaned from both local and overseas publications.

1.8.2.2 Unpublished theses and dissertations

Information has also been obtained from unpublished theses and dissertations (of local researchers) related to performance in mathematics in particular, and to mathematics education in general. Information from such sources proved to be invaluable as it provided updated information on the state of mathematics at the local level.

1.8.2.3 Circulars, circular minutes, guides and syllabuses

These are unpublished documents of the HOD. While some of the documents contain policy decisions, others are information documents submitted to educators with a view to improving the service at schools. The relevant documents in respect of mathematics were consulted and pertinent information contained therein was used in this research.

1.8.2.4 Tests (senior certificate mathematics question papers and answer scripts)

In order to obtain information on the achievement of pupils, the mathematics question papers of the HOD as well as the answer scripts of the candidates who wrote the senior certificate examination in mathematics were used. Special permission had to be obtained from the HOD for the use of the latter. The information obtained from the answer scripts, amongst others, formed the basis of the quantitative analysis.

1.8.2.5 Questionnaire completed by teachers

A questionnaire eliciting information on pupils' mathematics performance was completed by teachers of standard 10 classes. The information obtained therefrom was used to supplement the quantitative analysis of pupils' achievements in the test.

1.8.2.6 Questionnaire completed by senior certificate candidates

A questionnaire, comprising a standardised attitude scale and an interest scale pertaining to mathematics, was administered to senior certificate candidates. The questionnaire was analysed and the information obtained from these scales was used to ascertain pupils' attainment of these affective aims.

1.9 THE COURSE OF THE RESEARCH

1.9.1 In this chapter (Chapter 1), the writer has provided a general orientation to the study. In the introduction, the importance of mathematics is discussed and the manner in which the aims of mathematics teaching and learning has changed over the years is presented. This is followed by a discussion of the factors leading to this study, the statement of the problem, and an analysis and the implications of the problem. After the aim and purposes of the study together with the research hypotheses are presented, the limitations and assumptions of the study are discussed. Finally, a definition of terms as well as the research methods used in the study are presented.

1.9.2 Chapter 2 deals with educational aims and objectives and their evaluation in a subject didactical perspective, with particular reference to mathematics. This chapter concerns, in the main, a survey of pertinent literature related to the above. At the beginning of this chapter, a subject-didactical perspective and theoretical framework against which this study is juxtaposed, is presented. Thereafter, aims and objectives in general are discussed, followed by a discussion on aims and objectives of mathematics teaching and learning in particular. Various taxonomies of educational objectives, including the more recent taxonomies of educational objectives for mathematics teaching and learning are presented. Finally, a discussion on the methods and techniques used in the evaluation of mathematics attainment in the cognitive and affective domains is presented.

1.9.3 The development of criteria and identification of suitable research instruments are discussed in Chapter 3. Four research studies on mathematics attainment carried out in South Africa are examined and analysed with a view to establishing norms (numerical indexes) against which the attainment of the cognitive objectives, and hence the cognitive aims, can be determined. Also in this chapter, the aims of the current senior certificate mathematics curriculum are elaborated and analysed with a view to identifying the more important objectives inherent in them, to be included in a teacher questionnaire and administered so as to provide a qualitative assessment of pupils' attainment of the cognitive aims and objectives of the mathematics curriculum. Finally, details are provided in respect of the attitude scale and the interest scale used in this study to ascertain the pupils' attainment of the affective aims.

1.9.4 In Chapter 4, a detailed discussion is provided in respect of the selection of the samples, the administration of the test and other research instruments (questionnaires) as well as the handling of data.

1.9.5 An analysis and interpretation of the data obtained is presented in Chapter 5. This includes a detailed discussion of pupils' achievement of the cognitive objectives and aims as well as the attainment of the affective aims. Comparisons are made between the quantitative and the qualitative data obtained. A discussion of the statistics used, and their application, is also presented. The findings are interpreted in terms of the stated purposes and hypotheses of the study.

1.9.6 In the final chapter (Chapter 6), the writer's findings and conclusions as well as the implications of the study are discussed. Recommendations on aspects for future research are also made.

CHAPTER 2

EDUCATIONAL AIMS AND OBJECTIVES AND THEIR EVALUATION IN A SUBJECT DIDACTICAL PERSPECTIVE, WITH PARTICULAR REFERENCE TO MATHEMATICS

2.1 INTRODUCTION

As mentioned in the previous chapter, the purpose of this study is to evaluate the attainment of the cognitive and the affective aims and objectives of mathematics teaching and learning (of the senior certificate mathematics curriculum). Duminy and Söhnge (1986:89) observe that, in the RSA, the teaching and learning components in the different school phases reside under the South African Education System which determines in broad outline the curriculum, the syllabuses and education goals and aims; but immediate aims and objectives as well as the presentation of the learning material are resolved largely by the educator/teacher. The latter points to the fact that aims and objectives are realised in the context of the classroom, where teaching and learning takes place. In view of the above didactical implications, it becomes necessary to first place this study in the context of a subject didactical perspective and theoretical framework. A brief exposition of this is considered to be adequate for purposes of the study. However, an explicit account is presented in respect of aims and objectives and their evaluation which, in this study, constitute the point of entry in the subject didactical framework.

2.2 SUBJECT DIDACTICAL PERSPECTIVE AND THEORETICAL FRAMEWORK USED IN THIS STUDY

In presenting the subject didactical perspective, attention is given firstly to the cognitive demands (or types of learning) involved in the learning of mathematics.

This is followed by a discussion on the philosophical and didactical shifts in mathematics education, particularly since the middle 1980's. Thereafter, information from the above two sections is synthesised and a theoretical framework for the study is arrived at. The respective aspects are presented below.

2.2.1 Cognitive demands involved in the learning of mathematics

The cognitive demands or types or learning involved in mathematics learning are those direct and indirect things which we want students to learn in mathematics. Bell (1978:108), who refers to the cognitive demands as "objects" of mathematics learning, mentions that direct objects of mathematics learning are facts, skills, concepts and principles while some of the many indirect objects are transfer of learning, inquiry ability, problem solving ability and self discipline. He points out that the above four direct objects of mathematics learning are the four categories into which mathematical content can be separated. It is noted that recent authors prefer to use the term "mental activities" (Orton 1987) or "types of learning" (Costello 1991) when referring to the cognitive abilities involved in the learning of mathematics. After making a scrutiny of various classifications, both these authors conclude that, in the main, the schemes distinguish four types of mathematics learning, namely, the learning of facts (simple recall), skills (algorithms) , conceptual structures and problem solving strategies (Orton 1987:24; Costello 1991:1). These four cognitive categories provide a suitable structure for further discussion, although, according to Orton (1987:24), in reality all four are inextricably linked in the learning process.

2.2.1.1 Mathematical facts: Mathematical facts are those arbitrary conventions in mathematics such as symbols and formulae in mathematics. In other words, facts are items of knowledge - pieces of information which are known and remembered. Facts are learned through various techniques of rote learning (e.g., memorisation, drill, rehearsal). Bell (1978:108) states that students are considered to have learned a fact when they can state the fact and make appropriate use of it in a number of different situations. It is important that students learn facts with meaning. Such a view is supported by Costello (1991: 2-3) who mentions that while a lot of knowledge is available to us by immediate recall, factual knowledge is of no great value without a context in which it can be understood and used. Likewise, Orton (1987:25) states that committing knowledge to memory is important in terms of efficient processing, but at the same time rote learning without meaning is relatively unhelpful. This has led him to assert that retention and recall are easier if what is learned is meaningful in terms of the network of knowledge already held in the mind of the learner.

2.2.1.2 Mathematical skills: Mathematical skills can be defined as those operations and procedures (set of rules) which students are expected to carry out with speed and accuracy. Such skills are generally associated with arithmetical calculations. Hence, it is noted that the word "algorithm" has come to be used to describe any routine, well defined mathematical procedure. However, Bell (1978:108) and Costello (1991:4) aver that besides their use in arithmetical calculations, mathematical skills may equally well be applied in procedures such as constructing right angles, finding the union or intersection of sets of objects, for carrying out an accurate measurement or statistical test. Skills are learned through demonstrations and various types of drill and practice. According to Bell (1978:108), students have mastered a skill when they can correctly demonstrate the skill by solving different types of problems requiring the skill or by applying the skill in various situations.

2.2.1.3 Conceptual structures: It is observed that while Bell (1978:108-9) discusses concepts and principles under separate headings, both Orton (1987:31) and Costello (1991:4) discuss these aspects under one heading - the learning of conceptual structures. The researcher is of the opinion that such differentiation in approach is of no real consequence as Bell (1978:107) defines principles as sequences of concepts together with relationships among these concepts.

A concept in mathematics is an abstract idea which enables people to classify objects or events. Examples of concepts are sets, equality, triangles and percentages while examples of principles (conceptual structures) are congruency and similarity. Bell (1978:108) mentions that concepts can be learned either through definitions or by direct observation. He asserts that younger children (who are in Piaget's stage of concrete operations) usually need to handle physical representations of a concept in order to learn it, whereas older formal operational people may be able to learn concepts through discussion and contemplation. This assertion, as viewed by the researcher, reinforces the commonly held view that children, particularly young children, learn best by proceeding from the concrete to the abstract.

Mathematics learning consists very largely of building understanding of new concepts onto previously understood concepts. In this regard, Costello (1991:4) states that an understanding of conceptual structures is necessary to give meaning and value to mathematical knowledge and skills. He adds that such structures involve interconnecting relationships and properties, which is similar to the definition provided by Bell (1978:109) in respect of principles. Likewise, the NCTM (1989:7) asserts that informational knowledge (mastering concepts and procedures) has no value, except that its value lies in the extent to which it is useful in the course of some purposeful activity. Bell (1978:109) states that principles (or conceptual structures), which are the most complex of mathematical objects, can be learned through the process of scientific enquiry, guided discovery lessons, group discussions, the use of problem solving strategies and demonstration. Furthermore, a student has learned a principle when he or she can identify the concepts included in the principle, put the concepts in their correct relation to one another and apply the principle to a particular situation.

2.2.1.4 Problem solving strategies: Problem solving is generally intended to imply a process by which the learner combines previously learned elements of knowledge, rules, techniques, skills and concepts to provide a solution to a novel situation (Bell 1978:119; Orton 1987:35). A perusal of literature indicates that it is now generally accepted that mathematics is both product and process: both an organised body of knowledge and a creative activity in which the learner participates. Hence it is observed that words like "creativity" and "discovery" are often associated with problem solving, which is a higher order and more complex type or learning. Orton (1987:35) points out that while discovery, investigation and problem solving, all of which require "thinking", may be separate terms, there are clear relationships between the processes involved in all three of the terms. According to the said author, by definition, problems are not routine, each one being to a greater or lesser degree a novelty to the learner. Successful solution of problems is dependent on the learner not only having the knowledge and skills required, but also being able to tap into them and establish a network or structure.

Characteristically, then, mathematical activity demands the ability to devise strategies for solving problems. Thus, currently, there is considerable interest in aiming to improve the problem solving skills of pupils in schools. Together with Polya (1945), who led the way in the consideration of how to establish a routine for problem solving, authors like Bunker (1969 : 157) and Johnson and Rising (1972 : 247) have proposed that the strategy for problem solving in mathematics includes four main activities, namely:

1. understanding the problem ;
2. devising a plan ;
3. carrying out the plan ; and
4. analysing and evaluating the solution (i.e., looking back).

At this juncture, the researcher would like to point out that problem solving as discussed above must not be confused with the problem solving approach which has been proposed by the NCTM (1989 & 1991) as a new approach in the teaching and learning of mathematics. The latter aspect is discussed in more detail in paragraph 2.2.2.2.

Returning to the cognitive demands involved in mathematics learning, Costello (1991 : 6) mentions that perhaps more significant than the classification of particular questions are the balance and presentation of the mathematics curriculum and the relative emphasis which is given to the different forms of mathematical learning. He adds that a curriculum may be designed or expressed in a way which leads to concentration of routine skills, or an understanding structure, or on devising strategies for problem solving. Also, an individual teacher may, consciously or otherwise, adopt a teaching approach which emphasises a particular aspect of mathematical learning at the expense of other forms of competence. It is possible too that different pupils may prefer and respond positively to different forms of activity and styles of learning. The abovementioned author concludes by stating that in the implementation of all the abovementioned, a conscious awareness of the different aspects of mathematical learning is necessary in order to provide a realistic, balanced and varied programme of activity in school mathematics.

The researcher also avers that there should be a balance in the emphasis given to the different forms of mathematical learning in schools. He notes that this did not happen in mathematics education in the past. However, major philosophical and didactical changes in mathematics education took place since the mid-eighties, as can be evidenced from the discussion presented below.

2.2.2 Philosophical and didactical shifts in mathematics education since the 1980's

Over the years, the content and emphasis of the mathematics curriculum have changed. This is to be expected, since school mathematics must reflect the changes in need and usage. In this regard, Dessart (1981 : 1-2) states that the selection of mathematical content for instruction in schools is subject to many demands and pressures, which fall into three categories: psychological, sociological and structural. He goes on to say that in an ideal situation, they play equal and complementary roles in the curricular process ; but in reality there is a tendency to stress one demand over another, depending on the pressures of society. Hence, before the reform period of the sixties, it is observed that the mathematics school curriculum was concerned mainly with developing computational skills and proficiency. At the time of the "New Maths" revolution, curriculum goals of conceptual understanding were accepted and added. Two processes were identified as vehicles for achieving these goals: "learning-by-discovery" and "learning-by-doing". During the late seventies and early eighties, problem-solving applications and investigations became a major thrust. Perhaps the greatest philosophical and didactical shifts in mathematics education have been evidenced since the mid-eighties when focus became centred on not only "what" had to be taught, but also on "how" mathematics is to be taught. Hence, the more recent and current paradigms in didactics and mathematics have led to a greater emphasis being placed on the teaching and learning of mathematics. Earnest (1989) (in Costello 1991 : 81) writes that the mathematics teaching community in Britain is virtually unanimous in its support for a more problem-solving orientation to mathematics teaching and learning, incorporating discussion, practical and investigative work. Likewise, in the USA, the NCTM (1989 & 1991) has advocated major changes in curricula, teaching methods and approaches, and assessment techniques in mathematics teaching and learning. A brief exposition of the developments in these two countries becomes necessary in order to elucidate the major didactical and philosophical changes that have taken place and which have influenced mathematics teaching and learning in certain other countries, including the RSA.

2.2.2.1 Philosophical and didactical changes in mathematics education in Britain

Anxiety about standards of achievement in mathematics in British schools precipitated changes in mathematics teaching and learning. Shiu (1990 : 18) mentions that if there is a problem, it does not lie in the content which is taught, but rather in the different degrees to which children have responded to the teaching. She adds that differences may reside either in the ways in which the mathematical content is presented or in the capacities of pupils to respond to those ways, and most probably in a mixture of the two. According to Costello (1991 :

83), the blueprint which Cockcroft provided for the 1980's, by calling for problem solving and investigational work, encouraged activities through which pupils might learn to create their own mathematical strategies.

In Britain, the discussion of ways of teaching and learning mathematics has been a central issue in mathematics education since the Cockcroft Report. Costello (1991 : 12) mentions that as part of its work, the Cockcroft Committee set up studies into the mathematical needs of employment and adult life, and also commissioned a review of existing research on the teaching and learning of mathematics. While the report picks up the distinctions amongst different aspects of mathematics learning, the implications of these distinctions for effective mathematics teaching and learning are discussed in its paragraph 2.4.3 which states that:

Mathematics (teaching and learning) at all levels should include opportunities for:-

* exposition by the teacher ;
* discussion between teacher and pupils and between pupils and themselves ;
* appropriate practical work ;
* consolidation and practice of fundamental skills and routines ;
* problem solving, including the application of mathematics to everyday situations ; and
* investigational work.

No prescriptions were made in the Cockcroft Report regarding teaching styles. While the report was very well received, Costello (1991 : 12) states that in reaction to the report, there were some who would have wished the report to indicate a definitive style for the teaching of mathematics, yet the report recognises that this is neither desirable nor possible. The latter is considered a positive step as the researcher avers that it should be left to teachers to make professional decisions about <u>how</u> to teach and choose methods of teaching according to the individual needs of pupils. However, the non-statutory Guidance Document (June 1989) provides many useful suggestions designed to support the teaching and learning of mathematics in the National Curriculum. According to Costello (1991 : 12-13), the National Curriculum consultation document (1988) recognised the influence of the Cockcroft Report in highlighting and disseminating good practice in British schools. The statutory requirements were published in a document entitled Mathematics in the National Curriculum, early in 1989. The National Curriculum legislation operates by prescribing "attainment targets (AT's)" - specific forms of competencies which pupils are expected to achieve. These are then translated into "programmes of study" - broadly defined classroom activities which allow pupils to acquire the levels of learning required. Costello (1991 : 14 - 15) reports that there are 14 attainment targets in all which are organised into 2 main groups, as follows:

Knowledge, skills, understanding and use of number, algebra and measures

AT 1 Using and applying mathematics	Use number , algebra and measures in practical tasks, in real-life problems, and to investigate within mathematics itself.
AT 2 Number	Understand number and number notation.
AT 3 Number	Understand number operations (addition, subtraction, multiplication and division) and make use of appropriate methods of calculation.

AT 4 Number	Estimate and approximate in number.
AT 5 Number/Algebra	Recognise and use patterns, relationships and sequences, and make generalisations.
AT 6 Algebra	Recognise and use functions, formulae, equations and inequalities.
AT 7 Algebra	Use graphical representation of algebraic functions.
AT 8 Measures	Estimate and measure quantities, and appreciate the approximate nature of measurement.

Knowledge, skills, understanding and use of shape and space and data handling

AT 9 Using and applying mathematics	Use shape and space and handle data in practical tasks, in real-life problems, and to investigate within mathematics itself.
AT 10 Shape and space	Recognise and use the properties of two-dimensional and three-dimensional shapes.
AT 11 Shape and space	Recognise location and use transformations in the study of space.
AT 12 Handling data	Collect, record and process data.
AT 13 Handling data	Represent and interpret data.
AT 14 Handling data	Understand, estimate and calculate probabilities.

According to Costello (1991:15), all the attainment targets are to be considered relevant throughout the compulsory (5-16) years of schooling, and their demands are specified at ten different levels. Thus, for example, AT10 begins with level 1, at which pupils should be able to:

- sort three-dimensional and two-dimensional shape; and
- build with three-dimensional solid shapes and draw-two dimensional shapes and describe them.

At level 10, pupils should be able to:

- use angle and tangent properties of circles;
- sketch the graphs of sine, cosine, and tangent functions for all angles;
- generate trigonometric functions using a calculator or computer and interpret them; and
- use sine and cosine rules to solve problems, including simple cases in three-dimensions.

With regard to assessment, Costello (1991:16) states that at or near the end of each key stage (four in all), arrangements are made to assess the achievement of each pupil in each attainment target. These arrangements involve a combination of externally determined standard assessment tasks and teachers' own assessments.

Shiu (1990: 16-17) mentions that the successful implementation of the National Curriculum in mathematics (attainment targets) would require, amongst others, constant monitoring and continuing provision for training and support for teachers during the progressive implementation of the National Curriculum. A perusal of relevant literature reveals that the implementation of the National Curriculum in Britain is well under way; that the necessary training and support for teachers are being continuously provided (for example, Bailey 1994:18; Meyer 1994: 8-9); and that the new Curriculum is being monitored and evaluated by both the State education authorities and the subject teaching associations, as an ongoing operation (for example, Neal, Bradshaw & Jones 1994:1).

2.2.2.2 Philosophical and didactical changes in mathematics education in the USA

Concern about American students' achievement in various projects in mathematics (see paragraphs 1.2.5.1 and 1.2.5.2 for examples) precipitated reform in school mathematics in the USA. In March 1989, the National Council of Teachers of Mathematics (NCTM) released its Curriculum and Evaluation Standards for school mathematics in the USA. This was followed by the publication of Professional Standards for Teaching Mathematics, in March 1991. The 1989 document describes what a high quality mathematics education for North American students, K-12, should comprise. Central to the Curriculum and Evaluation Standards is the development of mathematical power for all students. The NCTM (1991:1) asserts that to reach the goal of developing mathematical power for all students requires the creation of a curriculum and an environment, in which teaching and learning are to occur, that are very different from much of current practice. For example, the 9-12 standards call for a shift of emphasis from a curriculum dominated by memorisation of isolated facts and procedures and by proficiency with paper-and-pencil skills to one that emphasises conceptual understanding, multiple representations and connections, mathematical modelling and mathematical problem solving (NCTM 1989:123). The NCTM (1989:2) cites the following 3 reasons for the need for standards : to ensure quality; to indicate goals; and to promote change. It describes standard as a statement that can be used to judge the quality of a mathematical curriculum or methods of evaluation. Thus, standards are statements about what is valued. The 1989 document presents fifty-four standards divided among four categories K-4, 5-8, 9-12, and evaluation.

The NCTM (1989:9) states that "knowing" mathematics is "doing" mathematics and hence instruction should persistently emphasise "doing" rather than "knowing that". It cites the following general principles which should guide student activities:

* activities should grow out of <u>problem situations;</u> and
* learning occurs through active as well as passive involvement with mathematics.

With regard to the former, the NCTM (1989: 9-10) argues that knowledge should emerge from experience with problems. In this way, students may recognise the need to apply a particular concept or procedure and have a strong conceptual basis for reconstructing their knowledge at a later time. It adds that a genuine problem is a <u>situation</u> in which, for the individual or group concerned, one or more appropriate solutions have yet to be developed. Furthermore, problems should arise from situations that are not always well formed and students should have opportunities to formulate problems and questions that stem from their own interest (NCTM 1989:66). In summary, the NCTM (1989:23) emphasises that problem solving should be the central focus of the mathematics curriculum. As such, it is a primary goal for all mathematical instruction and an integral part of all mathematical activity. It adds that problem solving is not a distinct topic but a process that should permeate the entire programme and provide the context in which concepts and skills can be learned. Thus we see that the problem solving or the problem-centred approach, as proposed by the NCTM, is different from problem solving as defined and used in the "old" approach.

Resnick (in NCTM 1989:10) reports that research findings from psychology indicate that learning does not occur by passive absorption alone. Instead, in many situations individuals approach a new task with prior knowledge, assimilate new information and construct their own meanings. This constructive, active view of the learning process must be reflected in the way much of the mathematics is taught. Thus the NCTM (1989:10) asserts that instruction should vary and include opportunities for:

* appropriate project work ;
* group and individual assignments ;
* discussion between teacher and students and among students;
* practice on mathematical methods ; and
* exposition by the teacher.

Taking cognisance of the above, the Standards also suggests a change in instructional patterns and in the roles of both teachers and students, some of which are summarised below (NCTM 1989 : 129).

Increased attention needs to be given to :-

* the active involvement of students in constructing and applying mathematical ideas ;
* problem solving as a means as well as a goal of instruction;
* effective questioning techniques that promote student interaction ;

* the use of a variety of instructional formats (small groups, individual explorations, peer instruction, whole class discussion, project work) ;
* the use of calculators and computers as tools for learning and doing mathematics ;
* student communication of mathematical ideas orally and in writing ;
* the establishment and application of the interrelatedness of mathematical topics ;
* the systematic maintenance of student learning and embedding review in the context of new topics and problem situations ; and
* the assessment of learning as an integral part of instruction.

Likewise, the fourteen evaluation standards emphasise aspects of assessment and programme evaluation that depart from current practice. The following are some of the aspects that are to receive increased attention (NCTM 1989 : 191) :-

* Assessing what students <u>know</u> and <u>how</u> they think about mathematics.
* Having assessment be an integral part of teaching.
* Focusing on a broader range of mathematical tasks and taking a holistic view of mathematics.
* Developing problem situations that require the application of a number of mathematical ideas.
* Using multiple assessment techniques, including written, oral and demonstration formats.
* Using standardised achievement tests as <u>only one</u> of many indicators of programme outcomes.
* Using calculators, computers and manipulatives in assessment.

A perusal of pertinent literature, for example, Showalter (1994) and Rosnick (1994), reveals that the NCTM's proposals are widely acceptable and propagated and problem solving is in action from K-12 in the USA.

2.2.2.3 Changes in mathematics education in the RSA

International developments, particularly those in the USA, have influenced shifts in thinking on mathematics teaching and learning in the RSA. While no major national research had been carried out in this regard, as was done in Britain or the USA, the new thinking has been encapsulated in the introductory sections of a new syllabus for mathematics for standards 8, 9 and 10 which has been prepared by the Free State Department of Education for the House of Assembly (HOA) schools. Laridon (1990 : 19) mentions that the introductory section of this new syllabus suggesting broad principles and goals show an awareness and sensitivity to international tends in curriculum development, not evident in previous syllabus statements. Aspects of the new thinking, expressed and included in the form of principles in the syllabus, are as follows (HOA 1992 : 1-2) :

* The pupil should be regarded as an active mathematical thinker who tries to construct meaning out of what he is doing on the bases of personal experience, always building on the knowledge which he has already constructed (reference to the constructivist approach).
* In implementing the mathematics curriculum, due attention should be accorded not only to the provision of mathematical knowledge, skills and concepts, but also to the mathematical processes by means of which pupils are actively and productively involved in learning.

* The mathematics curriculum assumes universal access to calculators by all pupils at all levels.
* Problem solving should be the central focus of teaching and learning mathematics. Not only is the ability to solve problems a major reason for studying mathematics, but problem solving provides a context for learning and doing mathematics.
* Besides direct teaching (of individuals, groups and whole classes), teaching approaches should include activity-based learning, discussion, application and problem solving, and open-ended investigations.

The preamble states that assessment should reflect the broad classroom approaches to the teaching and learning of mathematics.

It is to be noted that the abovementioned new curriculum/syllabus is being utilised in HOA schools only, with progressive implementation from standard 8 in 1993. Hence the other education departments, including the HOD, are still following the existing syllabuses, which to a large extent utilise the "old" approach in the teaching and learning of mathematics.

From the discussions presented above, the researcher notes that the most recent curriculum reforms in mathematics education aim to bring about a realistic balance in emphasis given to the different forms of mathematics learning, particularly the higher-order thinking skills. Furthermore, he observes and concurs that due emphasis is also placed on teaching and learning strategies, with a variety of approaches to be used being recommended.

Attention will now be focused on the theoretical framework used in this study.

2.2.3.4 Synthesis and theoretical framework

In terms of the information presented in the above paragraphs, it is evident that mathematics teaching and learning should focus not only on the lower level cognitive abilities, but also on the higher order thinking tasks and problem solving. The researcher observes that in Britain and the USA, assertions are made for greater emphasis being placed on the latter tasks, as they had been neglected to a large extent in the past. While the NCTM in the USA also emphasises problem solving, it goes beyond British thinking in that it proposes the problem solving approach in the teaching and learning of mathematics. It avers that by using this approach, new knowledge, skills and algorithms, conceptual understanding and problem solving strategies could be derived, arising out of a need.

It is observed that, as presented in the Standards document in the USA, each standard starts with a statement of what the mathematics curriculum should include. This is followed by a description of the student activities associated with that mathematics and a discussion that includes instructional examples. A similar pattern of presentation is noted in respect of the attainment targets used in Britain. A close scrutiny of the verbs used to describe the student activities in each of the attainment targets (in Britain) and the curriculum standards (in the USA) indicates a subtle and indirect reference to objectives (different levels of intellectual tasks). This can be observed from the attainment targets presented in paragraph 2.2.2.1 as well as from the following extracts from the Standards (NCTM 1989: 137-184).

In grades 9-12, the mathematics curriculum should include ....so that all students can :-

* express mathematical ideas orally and in writing;
* follow logical arguments ;
* construct proofs for mathematical assertions ;
* represent situations that involve variable quantities with expressions, equations, inequalities and matrices;
* analyse the effect of parameter changes on the graph of functions; and
* translate between synthetic and co-ordinate representations.

With regard to assessing students' mathematical knowledge and understanding, the Standards document proposes the following five evaluation standards that relate directly to mathematical content.

Standard 1 : mathematical knowledge
Standard 2 : conceptual understanding
Standard 3 : procedural knowledge
Standard 4 : problem solving
Standard 5 : reasoning

The writer concurs with Kulm (1990 : 14) who states that the above standards are in one sense hierarchical in that they range from mathematical knowledge to problem solving and reasoning, i.e., from lower level abilities to higher level abilities. A closer examination of these standards reveals that they could be grouped into the three broad categories of the types of learning involved in mathematics discussed in paragraph 2.2.1, namely, mathematical knowledge (facts) and skills, conceptual structures (understanding) and problem solving strategies. These 3 broad categories also accord with the categories which could be classified as lower level mathematical abilities, middle level mathematical abilities and higher level mathematical abilities. Hence, for purposes of this study and in order to relate the didactical outcomes inherent in the new approaches with those present in the current approach(es), it is considered expedient to use the following theoretical framework in the categorisation of intellectual abilities, in evaluating their attainment :

1. Lower level mathematical or cognitive abilities : LL
2. Middle level mathematical or cognitive abilities : ML
3. Higher level mathematical or cognitive abilities : HL

The NCTM (1989 : 193) mentions that the assessment of students' mathematics learning should enable educators to draw conclusions about, amongst others, their progress in achieving the goals and objectives of the curriculum. The degree to which meaningful inferences can be drawn from such an assessment depends on the degree to which the assessment methods and tasks are aligned or are in agreement with the curriculum. Thus we see that the evaluation of student attainment takes place in the context of the curriculum and as such is related to the content, aims and objectives. Since this study is concerned with an evaluation of the attainment of aims and objectives (which constitute part of the curriculum), it is considered appropriate to first provide a brief exposition of curriculum, a discussion of which follows, before focusing on aims and objectives.

2.3 CURRICULUM AND THE CURRICULUM COMPONENTS

2.3.1 The concept of curriculum

As mentioned in paragraph 1.7.4, a study of literature reveals that a variety of definitions is tendered for the concept of the curriculum. After examining the various definitions and approaches to the concept, Zais (1976:14) identifies the following categories into which the various definitions can be classified:

- curriculum as a programme of study;
- curriculum as course contents;
- curriculum as a structured series of projected learning outcomes or aims; and
- curriculum as a written plan of action.

The NCTM (1989:1) defines a curriculum as an operational plan for instruction that details what mathematics students need to know, how students are to achieve the identified curricular goals, what teachers are to do to help students develop their mathematical knowledge, and the context in which learning and teaching occur. Also, according to the NCTM (1989:193), for teachers, curriculum can refer to the material covered in a chapter, unit, semester or year; for administrators, it can be the context of the entire mathematics programme; for test developers, it can refer to the content that is common to the instructional programmes of a number of school districts.

In defining the concept curriculum for purposes of this study (see paragraph 1.7.4), the researcher has made use of the exposition provided by Jansen (1984:90) who defines a curriculum as a plan or programme for teaching and learning which is conceived in the light of certain aims and which includes selected and organised subject matter. Notwithstanding the above, the researcher acknowledges the more comprehensive definitions of the concept as presented by other authors, for example, Niebuhr (1986:99) who states that a curriculum is a scientific written programme for teaching and learning in which are included the aims, the corresponding selected and organised subject matter together with didactic guidelines for the creation of learning opportunities and for evaluation. The researcher also notes the assertion made by the NCTM (1989:241) which states that there is a difference between a curriculum as specified in a plan or textbook and a curriculum as implemented in the classroom. A curriculum specifies goals, topics, sequences, instructional activities and assessment methods and instruments. An implemented curriculum is what actually happens in the classroom.

2.3.2 Components of the curriculum

From the above definitions, it can be seen that a curriculum can have several components. However, it is observed that the following four components at least may be distinguished in any curriculum: aims (and objectives), contents (subject matter), teaching strategies/learning experiences and evaluation. In this regard, Harley (1983:4) mentions that although each component has its own content, they are all interdependent and together they constitute an indissoluble whole which we regard as the curriculum. The following interpretation (see Figure 2.1) shows the interaction and continuous reciprocal relationships among the four basic components of any subject curriculum.

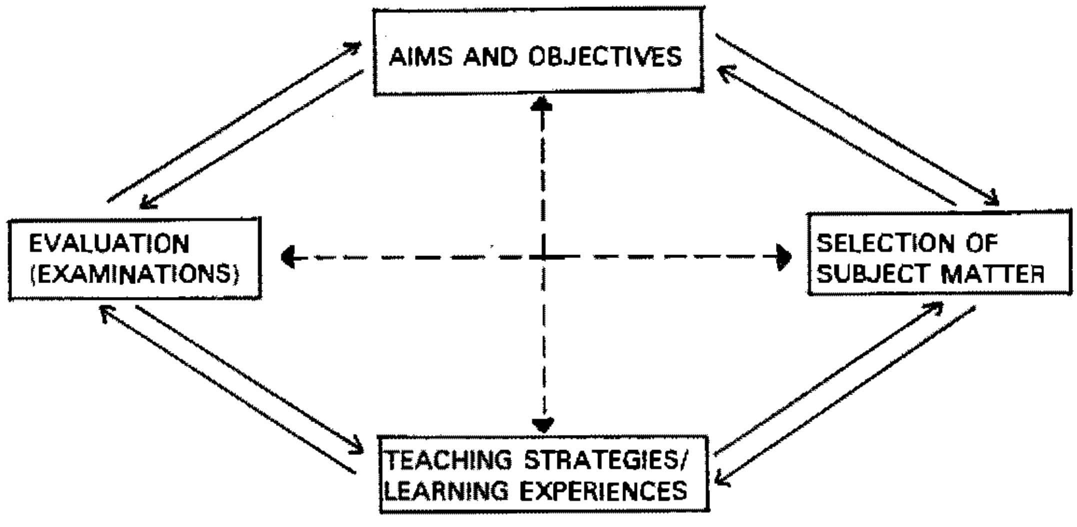


FIGURE 2.1: The four basic components of the curriculum. (Adapted from Harley 1983:7)

According to Furst (1958:13), aims and objectives serve as the bases for developing learning experiences and evaluation procedures which in turn provide feedback on the effectiveness of the curriculum. As this study focuses on aims and objectives and their evaluation, a detailed discussion on only these two components of the curriculum is presented below.

2.4 GOALS, AIMS AND OBJECTIVES

2.4.1 Sources of aims

The school is a purposive institution and education is an intentional activity. Acknowledging this, Stenhouse (1975:53) asserts that the educational purposes the school seeks to attain can be equated with goals and objectives. Hence, he maintains that the aims of education should be formulated as a result of consideration of learners themselves, contemporary life outside the school, the nature of the subject, the psychology of learning, and a philosophy or a set of values. A study of literature, for example, Moodley (1975:14), Zais (1976:301) and the Department of National Education (DNE) (1989:1-7) reveals that, in general, aims originate from philosophical, sociological and psychological considerations.

2.4.1.1 Philosophical sources

To a large extent, educational aims are derived from philosophical sources which are related to norms and values. Smit (1984: 126-127) mentions that education is dynamic, indicating a purposeful progress through a series of pedagogic situations. He adds that purposefulness implies that the aim of education is the realisation of values; and values are always embedded in the philosophy of life (life-world) of a particular group of people. Elaborating on the above, Gunter (1990:104) mentions that every educationist, every educator (parent, teacher, etc.) and every community views and interprets the phenomenon of education in the light of their particular world- and life-view as their particular perspective on the reality of man-in-the-world; and from this they derive the values and norms (moral, social, cultural, etc.) that must be imparted to the child by education, and at the same time, the material goal(s) at which education must be aimed and which it must realise. It is noted that while Zais (1976:303-304) holds a similar view, he cautions on the values that are to be fostered. According to him, society and the learner (which he refers to as empirical sources) tells us what is, but a philosophical enquiry is necessary to help us decide what ought to be. He therefore asserts that decisions regarding the desired life outcomes affected by the school are most effectively determined by bringing philosophical reflections to bear on existing conditions.

2.4.1.2 Society as a source of aims

Educational aims are also derived from certain needs or forces in society. According to the DNE (1989:1), the following forces in society determine the demands of education:

- Philosophical convictions/assumptions underlying the religion which is practised, the economic system, the social system and the political system;
- The level of technological development; and
- The demographic characteristics and the growth pattern of the population.

It adds that society is complex and should be analysed in all its facets in order to ensure the identification of justifiable aims which could be achieved by a "needs determination", or what Harley (1983:8) refers to as "situation analysis".

Worthy of note is the emphasis placed by Zais (1976:301) on the conducting of empirical studies, which he observes has become current in the last one hundred years, in the framing of curriculum aims. According to him, the curriculum should aim at educating in those areas required for functioning in life outside school (adult life). Keitel (1987:398) observes that social needs have typically been given priority, and they have mostly been identified with economic interests. As society (contemporary life) is an important source for curriculum purposes, the researcher asserts that curriculum developers must take cognisance of the changes in society in order to keep aims relevant.

2.4.1.3 The learner as a source of aims

Curriculum purposes are also derived from the needs of learners, which are intrinsic (innate) and extrinsic (learned or norm-related). The DNE (1989:6) draws attention to the following in respect of learners' needs:

- The internal (inborn) needs of the learner are influenced by society to a lesser degree. With regard to such needs, reference could be made to Maslow's taxonomy of needs which relate to the following needs: psychological, security, love and solidarity, self-respect, self-actualization and knowledge and understanding.
- The norm-related needs of the learner are determined by society. These include a certain form of behaviour, as well as tasks, that the learner must execute in order to find happiness, to be accepted by society and to be able to survive.

2.4.1.4 Subject matter source

According to Zais (1976:304), subject matter (or more accurately, subject matter specialists) is probably the most commonly used source of aims in public school curricula. He adds, however, that since it is based on specialised professional expertise, one of the serious objections to such a source is that the curriculum purposes it generates is specialised, narrow and technical. Hence, he avers that subject matter can serve only as a source of subordinate aims, namely, specific aims and objectives. Harley (1983:15) points out that views on the nature and structure of mathematics (a subject) also play an important role in the formulation of aims for the subject.

In conclusion, the researcher emphasises that subject aims, including those for mathematics, are selected with the intention of fostering general formative educational goals. This is supported by Human (1975:6) who mentions that the aims of mathematics teaching and learning must only be distinguished, and not be separated, from the aims (goals) of formative education in the school context. He adds that, for example, if one of the aims of school education is to assist children towards maturation and eventual entry to adult life, and if mathematics as such is meaningful in the life of adults of a particular community, then mathematics will be included as a subject in the school curriculum and that an aim of mathematics education would be the actualization of one or more aspects of maturation (the whole of insight, skills and disposition).

It now becomes necessary to define the terms goals, aims and objectives.

2.4.2 Definition of goals, aims and objectives

A scrutiny of literature reveals that, generally, three levels of aims are distinguished on the vertical level, namely, (in descending hierarchical order) macro-level, meso-level and micro-level aims. However, it is observed that inconsistencies exist in the use of and the descriptions attached to the terms goals, aims and objectives. According to Nowlan (1990:7), some authors also use the terms aims and objectives as if they had the same meaning. It would appear, as noted by Davies (1976:11), that part of the problem is that the British educators have been concerned to define aims, and Americans to write objectives.

The flexibility in the use of terminology to describe the different levels of aims is illustrated in Table 2.1.

TABLE 2.1 TERMINOLOGY ASSOCIATED WITH THE THREE LEVELS OF AIMS

Macro-level aim	Meso-level aim	Micro-level aim	Source/Author
Aims	Goals	Objectives	Zais 1976:307-11
Broad goals	-	Objectives	*MRP 1978:27
General aims	Specific aims	Objectives	Harley 1983:14-15
General/long term aims	Mid-term aims	Short-term aims/ objectives	Nowlan 1990:7-9
Ultimate goal	Remote goal	proximate aim	Gunter 1990: 101-102
General objectives/goals	Intermediate / course objectives	Specific objectives	Gay 1991:76-79
Goals	Aims	(Objectives)	HOA 1992:3

*MRP: Mathematics Research Project

It is observed that in the RSA, generally, the term "goals" is used when referring to the general formative educational aims (goals) and that the term "aims" is used when referring to the aims of the different educational phases and/or the aims of subject curricula.

Goals: Goals (or general aims) are global statements of long-term outcomes, expressing the overall ideal aspired for. According to Zais (1976:306), general aims are statements that describe expected life outcomes based on some value schemes either consciously or subconsciously borrowed from philosophy. While general aims reflect what is worthwhile in society, Harley (1983:14) states that in terms of education and teaching, general aims may also be described as the community's educational ideals. He adds that since such aims are vague, general and philosophical, it is difficult to establish whether or not they have been attained.

Aims: Aims (or specific aims) actually acquire significance from their relationship with general aims, but they are more explicit about the end in view. Specific aims indicate teaching outcomes which are achieved in the long term, e.g., on completion of a course, a school phase or an entire school career. Zais (1976:306) points out that specific aims involve the achievement of a relatively large number of objectives for their attainment. Likewise, Gay (1991:77) mentions that specific aims are usually measurable and typically represent more complex behaviours expected as a result of the achievement of a number of prerequisite objectives.

It is to be noted that the cognitive and the affective aims of the senior certificate mathematics curriculum being investigated in this study fall into this category of aims, viz, specific aims.

Objectives: At the micro-level, aims are specified and often described in operational terms and are referred to as objectives. According to Zais (1976:306), objectives are described as the most immediate specific outcomes of classroom instruction.

The researcher also recognises the difference in the terms goals, aims and objectives. He prefers to use the terms in the descending order presented above to denote aims that move from the most general to the most specific. The definitions of these terms adopted by the researcher for use in this study are reflected in paragraph 1.7.7.

Worthy of note is the assertion made by Gunter (1990:102) who states that every objective serves as a necessary step or means towards the achievement of a wider, specific aim, which in turn constitutes an essential rung on the ladder on the way to the final goal. He adds that, at school, each lesson in every subject has both its objective(s) and specific aims and by the achievement of these, brings the pupils a step nearer to the general aims of their education. The above exposition has been used to formulate one of the theoretical constructs used in this study (see Chapter 3).

Having provided a general background on aims and objectives, attention will now be focused on the aims and objectives of mathematics education at the secondary school level.

2.5 AIMS OF MATHEMATICS EDUCATION AT THE SECONDARY SCHOOL LEVEL

2.5.1 Current aims of mathematics education

2.5.1.1 Overseas countries

After reviewing relevant literature from certain countries, particularly Britain and the USA, Leung (1987:37) observes that, in general, the aims of mathematics education at the secondary school level can be grouped into the following five broad categories:

* To help students to develop the ability to communicate logically and concisely.
* To provide students with the skills for daily application.

* To use mathematics as tools for other subjects, especially science and technology.
* To help students to develop the ability to think logically.
* To help students to appreciate mathematics as part of their cultural heritage and for its intrinsic beauty.

However, Rambaran (1989:28-33), who also reviewed overseas and local literature, states that while many aims of mathematics education can be given, most of the aims (of the secondary phase) fall under the following ten categories.

- To enable the child to solve mathematical problems in daily life.
- To provide a suitable type of discipline to the mind of the learner so that it develops the child's powers of reasoning and thinking.
- To develop in the child a scientific and realistic attitude towards life.
- To prepare the child for economical, purposeful, productive, creative and constructive living.
- To develop the habits of concentration, self reliance and discovery.
- To create in the child a love for hard work.
- To prepare the child for the professions.
- To enable the pupils to understand everyday literature.
- To develop the learner's powers of expression.
- To develop in the child an acquaintance with his culture and an appreciation of the cultural arts.

A comparison of the above two lists reveals that although Rambaran (1989) provides a longer list of aims, both the lists include the utilitarian, cognitive and affective aims of mathematics teaching and learning.

2.5.1.2 South Africa

In South Africa, a major research into the aims of mathematics instruction as well as intervention in respect of the teaching of this subject was undertaken in the early 1970's by Human (then a researcher in the HSRC). In his report, Human (1975: 14-19) identifies the following categories of aims for mathematics instruction without making claims to their completeness as he asserts that further insights into the nature and significance of mathematics and the maturation of children may reveal further pedagogically accountable aims of mathematics instruction:

* Handling the quantitative and formal aspects of everyday situations.
* Development of communication potentialities.
* Mastery of contemporary reality by insight and comprehension.
* Vocational orientation.
* Development of mathematical potential at different levels.
* Development of logical and abstract thinking.
* Propaedeutic (preparation) for further study.

With regard to the above, Human (1975:14) cautions that the order in which the aims are presented does not at all indicate priority or related importance, but rather a possible order in which aims will be emphasised at school, i.e., from grade i to standard 10.

There is no doubt that the contributions of Human (1975) have played an important role in determining the aims of mathematics education, both at the primary and secondary school levels, in the RSA. Taking into account the above list of aims as well as other pedagogical and didactical considerations, curriculum developers and syllabus compilers in the RSA identified and formulated an initial set of aims for mathematics education for the senior secondary phase. These aims were revised/modified from time to time, over the years. Currently, the following aims have been adopted for mathematics teaching and learning at the senior certificate level, which have been in use since 1985 (HOD 1984:1).

- To develop a love for, an interest in and a positive attitude towards mathematics, by presenting the subject meaningfully.
- To enable pupils to gain mathematical knowledge and proficiency.
- To develop clarity of thought and the ability to make logical deductions.
- To develop accuracy and mathematical insight.
- To instil in pupils the habit to estimate answers where applicable and where possible to verify answers.
- To develop the ability of the pupils to use mathematical knowledge and methods in other subjects and in their daily life.
- To provide basic training for future study and careers.

Certain aims from the above list are under consideration in this study. A detailed analysis and elaboration of these aims, together with the relevant objectives, is presented in Chapter 3. At this point, suffice it is to mention that the above aims can be categorised into three groups as follows:

Affective : The first aim in the abovementioned list.
Cognitive : Four aims - the second to the fifth.
Utilitarian : The last two aims.

Recent reforms in mathematics education, both in overseas countries and in South Africa, have resulted in revised aims being proposed for the teaching and learning of mathematics. Whilst these "new" aims do not come under investigation in this study, the inclusion of a discussion on these new developments is not considered out of place.

2.5.2 Proposed new aims/goals for mathematics education

2.5.2.1 United States of America

As already stated in paragraph 2.2.2.2, major reform in school mathematics in the USA was undertaken recently by the NCTM and the recommendations thereon are contained in the NCTM's report, the Curriculum and Evaluation Standards for School Mathematics (referred to in short as the "Standards") which was released in 1989. It is noted that one of the new social goals of education, as proposed in the Standards, is "to educate students to become mathematically literate workers". In this regard, Romberg (1990:473) mentions that the authors of the Standards adopted "mathematical power" as the phrase most evocative of the quality of mathematical literacy sought for the entire population. Their perception of mathematical power regards individuals and societies as empowered by mathematics. In keeping with the above, the Standards identifies the following five broad goals (aims) for mathematics education for all students in the USA (Johnson 1990:530; Steen 1990:19):

* To value mathematics.
* To reason mathematically.
* To communicate mathematically.
* To solve mathematical problems.

* To develop confidence in the ability to do mathematics.

With regard to the above, it is also noted that a greater emphasis is placed on the fourth goal, namely, To solve mathematical problems. Romberg (1990 : 475) mentions that the Standards argues that to be mathematically powerful in a mathematical and technical culture, students should develop the power to explore, conjecture, reason logically, and integrate a variety of mathematical methods to solve problems. Authors like Steen (1990: 19-22) and Romberg (1990: 471-477) state that besides proposing new goals for mathematics education, the Standards advocates changes in curricula, teaching methods and approaches, and assessment techniques. Also, it recommends the use of calculators at all levels and in all contexts (see also paragraph 2.2.2.2.).

Current literature on mathematics education in the USA, for example, Rosnick (1994) and Showalter (1994), indicates that attention is being given by educationists and educators to realise the vision of the Standards.

2.5.2.2 South Africa

As mentioned in paragraphs 1.6.4 and 2.2.2.3, in the RSA, the HOA has recently published new curricula, which include revised content and aims, for use in schools falling under its control. The HOA's syllabus document (HOA 1992:3) reflects the following aims for the teaching and learning of mathematics at the senior secondary level.

The mathematics curriculum aims to develop in pupils:

- an understanding of number, of measurement and of spatial concepts and relationships;
- the ability to use a variety of mathematical processes, e.g., comparing, classifying, specialising, abstracting, generalising, inferring, analysing and proving, in learning and doing mathematics;
- the ability to apply their knowledge of mathematical concepts, methods and processes (by individual or collective effort) to solve relevant problems;
- the ability to think and reason logically, to express ideas in meaningful ways and to reflect critically on the quality and validity of their own thinking and written work;
- the ability to understand, interpret, read, speak and write mathematical language;
- an appreciation of the place of mathematics, and its widespread applications in other subjects and our world;
- the ability to use a range of mathematical methods and technological aids sensibly and appropriately.
- the mathematical skills necessary for future employment and further study; and
- a love for and a positive attitude towards mathematics.

Attention is drawn in the document that the goals relate to principles which are included in the introductory section of the syllabus. These principles have already been presented in paragraph 2.2.2.3.

A closer examination of the new emphases, which include new aims and approaches, for mathematics education proposed in the HOA's document and the

Standards (1989) reveals several parallels between local and international developments. Greater emphasis is placed in both these documents on the problem solving approach (to ensure the development and attainment of the higher mathematical abilities) as well as on the use of a variety of teaching methods.

Mention needs to be made that during the period in which the researcher was writing up this thesis, the Department of National Education (of the new government in South Africa) approved a new mathematics curriculum for the senior secondary phase, to be tested in the S.C. examination as from 1996. A scrutiny of the specific aims for mathematics education reflected in the syllabus document (Department of Education 1995) reveals that, in essence, the new specific aims are more or less the same as those contained in the "old" syllabus and in the HOA's 1992 syllabus document. However, some of the aims are elaborated or stated in other terms. Of significance is the additional aim which reads: to create an awareness of and an appreciation for the contributions of all peoples of the world to the development of mathematics. However, one wonders what is meant by the specific aim: to develop an <u>inquisitive</u> attitude towards mathematics, which is included in the document. For additional information on these aims, the interested reader is referred to the relevant document of the Department of Education (1995).

Since a mutual relationship exists between aims and objectives, it becomes necessary to take a closer look at objectives.

2.6 <u>EDUCATIONAL OBJECTIVES</u>

2.6.1 <u>Definition and characteristics of educational objectives</u>

Educationist agree that aims and objectives are essential and basic to any educational programme. While aims are important in mathematics instruction, the teacher in the classroom needs clear specifications (objectives) for his daily lessons. In this regard, Dean (1982:137) states that aims have been regarded as "general declarations of intent" which if heeded can give both direction and shape to a teaching programme. On the other hand, educational objectives specifically refer to the type of behaviour expected of pupils at the end of a course of instruction.

According to Bloom (1956:12), objectives may be defined as the intended <u>result</u> of teaching rather than the <u>process</u> of teaching itself. He adds that the most important characteristic of a useful objective is that it describes the type of performance that will be accepted as evidence that the learner has mastered the content.

In view of the importance of objectives in the teaching-learning situation, there is a need for effective objectives, which should satisfy certain requirements/criteria. According to Thorndike, Cunningham, Thorndike and Hagen (1991:201-204), objectives should have the following characteristics.

* Objectives should be stated in terms of student behaviour rather than learning activities or teacher purposes.
* Objectives should begin with an active verb that indicates what a student should exhibit at the conclusion of instruction.
* Objectives should be stated in terms of behaviour that is observable in the classroom or school.
* Objectives should be stated precisely.

* Objectives should be unitary.
* Objectives should be stated at an appropriate level of generality.
* Objectives should represent intended direct outcomes of instruction.
* Objectives should be realistic (and attainable).

After studying relevant publications on behavioural objectives, Orton (1987:41) concludes that writers seem to be generally agreed that objectives are important because they -

- provide the teacher with guidelines for developing instructional material and and teaching method;
- enable the teacher to design means of assessing whether what was intended has been accomplished; and
- give direction to the learners and assist them to make better efforts to attain their goal (objective).

Generally, it is found that there are endless outcomes (objectives) that may be considered; however, a systematic approach to objectives reduces the confusion. One such approach is to compile ordered classification systems or taxonomies, a discussion of which follows.

2.6.2 Classification of educational objectives

The most important contribution to thinking on behavioural objectives has been by Bloom and his co-workers (Bloom 1956), who have classified objectives into three principal domains: the cognitive, the affective and the psychomotor. The cognitive domain deals with such objectives as recall of knowledge and the exercise of intellectual skills. The affective domain is concerned with objectives which involve values, attitudes and interests. The psychomotor domain deals with objectives that involve motor skills and manual dexterity. Since this study is concerned with aims and objectives in the cognitive and affective domains, only these two aspects are discussed here.

2.6.2.1 Bloom's Taxonomy of educational objectives (the cognitive domain)

Six major categories of objectives are identified in Bloom's taxonomy for the cognitive domain: knowledge, comprehension, application, analysis, synthesis and evaluation. This arrangement of objectives is regarded by Bloom (1956) as forming a hierarchy from simple (knowledge) to complex (evaluation). The categories/levels are cumulative in the sense that each higher category absorbs those below it, for example, in order to apply knowledge successfully, a pupil needs to have the basic knowledge and also understand its significance.

Except for the category application, all the other major categories are broken down into more explicit and distinct behaviours. As a result of the breakdown, 21 separate categories of behaviour are defined in his taxonomy. A summary of the major categories of Bloom's Taxonomy (Bloom 1956:201-207) is given below.

1. Knowledge: This category represents the lowest level of learning outcome in the cognitive domain. It refers to the processes involved in remembering previously learned material. This may involve the ability to recall a wide variety of material, from terminology and specific facts (terms, facts, rules)

to principles and other generalisations. However, all that is required is bringing to mind the appropriate information.

2. Comprehension: The learning outcomes in this category go one step further beyond the simple remembering of material, and represent the lowest level of understanding. Comprehension involves understanding a given content well enough without necessarily being transferred elsewhere. Comprehension involves the ability to translate material from one form to another (words to numbers), to interpret information (explaining it) or to justify methods and procedures.

3. Application: This refers to the ability to use material that is learned in new and concrete situations. It involves the ability to use rules, methods, procedures, principles, theories, etc. to produce or give reasons for certain consequences. Both knowledge and comprehension are necessary in this category.

4. Analysis: This refers to the ability to break down or analyse material into its component parts so that its organisational structures may be understood. This may involve the ability to identify parts, analyse relationships between parts and recognise the organisational principles involved. Analysis presupposes the categories of knowledge, comprehension and application.

5. Synthesis: This refers to the ability to put parts or elements together so as to form a whole. This may involve the ability to create unique verbal or non-verbal communication (theme or speech), a plan or procedure for accomplishing a particular task (design a simple experiment, research

proposal), or producing a set of abstract relations (scheme for classifying information). Learning outcomes in this category stress creative behaviour.

6. Evaluation: This category is concerned with the ability to judge the value of ideas, people, products and methods for a specific purpose and to state valid reasons for this judgement. The judgements are to be based on definite criteria. Evaluation may be made on the basis of internal criteria (organisation) or external criteria (relevance to the purpose). Learning outcomes in this area are highest in the cognitive hierarchy because they contain elements of all the other categories.

The most important aspect of Bloom's work is that the objectives within the cognitive domain can be modified to suit individual disciplines. Since most behavioural outcomes in mathematics seem to have cognitive origins (Wilson 1971:648; Wood 1972:31), Bloom's classification has special significance for mathematics. Consequently, several authors and researchers have used this taxonomy, or a modification/elaboration of it, in mathematics teaching and evaluation. A discussion on some of the classifications relevant to this study is presented below.

2.6.3 Some classifications of educational objectives (cognitive domain) in respect of mathematics

2.6.3.1 Classification of Avital and Shettleworth

The taxonomy suggested by Avital and Shettleworth (1968:18-19) for mathematics has three categories (all cognitive), called "levels of thinking", as indicated below.

C. Open search — Productive thought

↑

B. Algorithmic thinking, generalisations of varying degrees — Reproductive thought

↑

A. Recognition, recall

As can be observed from the above, the lowest level of the Avital-Shettleworth classification scheme describes thinking which involves only recognition or recall of learned material.

The second level of thinking process is referred to as algorithmic thinking and generalisations. According to the Mathematics Research Project (1978:29), this level includes all routine applications of learned procedures: computation with algorithms, generation of slightly different examples, translation among verbal, numerical and graphic forms; word problems with only slight differences from those done in class; interpreting a scale drawing very similar to one in the book; etc. Since the thinking required at the first two levels is so close to the original learning, Avital and Shettleworth (1968:19) call these levels reproductive thinking.

The third level - open search - involves productive thinking. Open search requires the non-routine manipulation of previously learned material and, at a higher level, discovery of relationships among previously learned material and, at a still higher level, discovery of relationships among previously unrelated concepts and propositions. Hence, thinking at this level is more complex and demanding. Working on open search problems utilises lower levels and higher levels of thinking (Mathematics Research Project 1978:29).

The researcher observes that the classification of Avital and Shettleworth (1968) has been used as a basis by Norton (1983) to develop his taxonomy which classifies test items according to levels of thinking (discussed in paragraph 2.6.4.2).

2.6.3.2 Classification devised by the National Longitudinal Study of Mathematical Abilities (NLSMA)

Work completed by a group of American researchers (which included Wilson) belonging to the School Mathematics Study Group (SMSG) and called the "National Longitudinal Study of Mathematical Abilities" is another example of a cognitive taxonomy written for secondary school mathematics. Wilson (1971: 646-647) lists the following categories which define areas for cognitive objectives.

Computation	:	Knowledge of specific facts.
		Knowledge of terminology.
		Ability to carry out algorithms.
Comprehension	:	Knowledge of concepts.
		Knowledge of principles, rules and generalisations.
		Knowledge of mathematical structures.
		Ability to transform problem elements from one mode to another.
		Ability to follow a line of reasoning.
		Ability to read and interpret a problem.
Application	:	Ability to solve routine problems.
		Ability to make comparisons.

		Ability to analyse data.
		Ability to recognise patterns, isomorphisms and symmetries.
Analysis	:	Ability to solve non-routine problems.
		Ability to discover relationships.
		Ability to construct proofs.
		Ability to criticise proofs.
		Ability to formulate and validate generalisations.

The above classification, amongst others, has been used by the researcher as a basis for identifying the objectives inherent in the cognitive aims under investigation in this study (discussed in Chapter 3).

2.6.4 Some classifications of cognitive objectives for mathematics, developed/used in the RSA

2.6.4.1 Classifications developed by Moodley (1975 & 1981)

Using Bloom's taxonomy of educational objectives as a basis, Moodley (1975:58-59) devised a scheme of objectives for mathematics learning for the senior secondary phase. The major categories of this scheme, devised in 1975, are as follows: knowledge, skills, comprehension, selection - application, and analysis-synthesis.

Subsequently, Moodley (1981) concluded a study on the relationship between attitudes and attainment in mathematics amongst senior secondary pupils.

However, since the nature of his study was such that it did not require a refined classification of his items in terms of all five categories developed in 1975, Moodley (1981:35) devised the following scheme, comprising three broad levels of mental processes (mathematical abilities), for the classification of test items.

Level 1:	Lower level (LL) -	Knowledge and skills
Level 2:	Middle level (ML) -	Comprehension
Level 3:	Higher level (HL) -	Selection-application and analysis-synthesis

It is to be noted that the above classification accords closely with the classification used by the researcher in this study (see paragraph 2.6.5).

2.6.4.2 Classifications developed by Norton

Using the taxonomy of Avital and Shettleworth as a basis, Norton (1983) developed the following 3-category taxonomy of objectives which could be used for the classification of test items in mathematics, according to levels of thinking.

- KS (Knowledge simple process)

  The question tests knowledge, i.e., the pupils should have covered this type of question before and it uses simple algorithmic processes.
- KC (Knowledge complex process)

  The question tests knowledge , i.e., the pupils should have covered this type of question before but it uses complex algorithmic processes.
- N (New-situation)

  The question tests abilities which require the application of knowledge to a new situation, i.e., it is likely that the pupils would not have seen aspects of the question before.

However, after further investigation, Norton (1983) found that his above taxonomy could be elaborated as follows (Norton 1991a:102-104; Norton 1991b: Appendices 2a-d):

* KS theory : Knowledge question in learned theory.
* KS process : Question requiring use of simple known algorithmic process.
* KS/KC : Question using simple algorithmic process, but usually also requiring use of two concepts or interpretation of sketch.
* KC process : Question requiring use of more complex known algorithmic process.
* KC two concepts: Question using more complex algorithmic processes but also requiring use of two concepts in their solution.
* N Type (KC/N or N): Question of type considered new to candidate. More than two concepts must be brought into play for their solution.

It has been found that the different categories in the above classification of Norton (1991a) can be regrouped into a three-category classification, to accord with the classification used in this study (discussed in Chapter 3).

2.6.4.3 Taxonomy used by the HOD

It is observed that the HOD has adopted Moodley's five-category taxonomy for the writing of the secondary mathematics syllabus in terms of educational objectives

(HOD 1983). However, the following four-category classification (modification of the above taxonomy)is used in the teaching and evaluation of mathematics in HOD schools.

A. Knowledge and skills

B. Comprehension/Understanding

C. Application

D. Higher Abilities — Analysis, Synthesis, Evaluation

2.6.5 <u>Classification of cognitive objectives used in this study</u>

From the foregoing discussion, it can be observed that while Bloom (1956) made a major contribution towards the classification of cognitive objectives, recent classifications of objectives for mathematics teaching and learning indicate a move away from Bloom's six-category taxonomy. Also, as mentioned in paragraph 1.5.2, current literature now tends to report on students' achievement in terms of broad categories of mathematical abilities. In view of this and in keeping with the theoretical framework of this study (discussed in paragraph 2.2.2.4 and Chapter 3), the following three - category classification of cognitive objectives has been used in this study.

A.	Lower level abilities :	Knowledge and skills
B.	Middle level abilities :	Comprehension/understanding
C.	Higher level abilities :	Application, analysis, synthesis and evaluation/creative

Attention is now focused on the affective objectives, which are also under consideration in this study.

2.6.6 Affective objectives

Objectives in the affective domain are also important in mathematics teaching and learning. According to Bell (1978:182), nearly all school systems have both cognitive and affective goals; however, most school activities are designed to emphasise student mastery of cognitive objectives. He adds that most testing and evaluation procedures measure, to a large extent, cognitive learning and there is a tendency to evaluate affective learning subjectively, if at all. Authors like Wilson (1971:663), Bell (1978:162), Mathematics Research Project (1978:30) and Gay (1991:74) mention that affective objectives are generally more difficult to promote and to measure. Despite the above, the researcher cautions that the affective domain should not be ignored as objectives in this domain are necessary.

2.6.6.1 Classification of objectives in the affective domain

The taxonomy of affective educational objectives, which was prepared by Krathwohl, Bloom and Masia (1964), is an ordered classification system of interest, appreciation, attitude, value and adjustment objectives. This taxonomy comprises five major affective categories with each category containing two or three affective levels, as indicated below (Bell 1978:182-183) .

Affective Objectives

1. Receiving
    - 1.1. Awareness.
    - 1.2. Willingness to receive.

1.3. Controlled or selected attention.

2. Responding

    2.1 Acquiescence in responding.

    2.2 Willingness to respond.

    2.3 Satisfaction in responding.

3. Valuing

    3.1 Accepting a value.

    3.2 Preferring a value.

    3.3 Commitment to a value.

4. Organisation

    4.1 Conceptualisation of a value.

    4.2 Organisation of a system of values.

5. Characterisation of a value or value complex

    5.1 Generalised set.

    5.2 Characterisation.

With regard to the above, Gay (1991:74) points out that the categories represent a hierarchy of acceptance which ranges from willingness to receive or attend to characterisation by a value. As one moves up the hierarchy, an internalisation process is evident, a process that begins with total rejection of an entity and culminates in total acceptance. However, Gay (1991:75) adds that rarely, if ever, are the sights of educational objectives set to the "characterisation" level of the affective taxonomy.

2.6.6.2 Affective objectives for mathematics education

Concerning the affective outcomes for mathematics, Wilson (1971:647) recognises the importance of only two categories of affective objectives, namely:

- Attitude and interest (comprising attitude, interest, motivation, anxiety and self concept); and
- Appreciation (extrinsic, intrinsic and operational).

From a scrutiny of the list of aims of the senior certificate mathematics curriculum adopted by the HOD (see paragraph 2.5.1.2), it can be observed that only one aim falls into the affective domain, which is concerned with the development of interest in and a positive attitude towards mathematics. In view of this, an explicit account of only these two aspects is presented below.

a) Attitude and Interest

According to Wilson (1971:662-663), any affective construct has an object associated with it; one does not just have an attitude, or interest, but an attitude towards or an interest in something - in this case mathematics. He adds that an affect tends to have the following three components:

- the object (in this case, mathematics);
- a feeling or emotion, which has direction (value) and strength; and
- a tendency to act on the object according to the value and strength of the feeling.

(i) Attitude

The concept of attitude is defined by Aiken (1970:551) as "a learned predisposition or tendency on the part of an individual to respond positively or negatively to some object, situation, concept or another person". Messick (1979: 284-285) also notes that attitudes are always directed favourably or unfavourably. It is this positivity and negativity that renders attitudes towards some social object measurable.

As there are several kinds of attitudes towards mathematics, a teacher should take care to describe the measure of attitude. This could be done by one or more of the following ways (Wilson 1971:685-686):

- ascertain how well a student likes mathematics;
- ascertain how important a student considers mathematics in relation to other school subjects;
- assess the pleasure or boredom a student experiences with regard to mathematics; and
- assess the ease or difficulty which a student associates with mathematics performance.

(ii) Interest

Interest is the deliberate, voluntary focus of attention, concern and activity on a person, object, event or sphere. Interest has a subjective dimension in that it is related to a person's values and the intensity of the interest is indicative of the need for self-actualization in the direction of the interest (Van den Aardweg & Van den Aardweg 1988:120).

Items to asses interest in mathematics give the student an opportunity to express a preference for mathematics activities over other choices. This experience may have a rather low level of feeling or emotion associated with it (Wilson 1971:687).

Teachers should strive to cultivate healthy attitudes in pupils. It is generally observed that a pupil who is interested in a subject learns more, far more easily, than a pupil for whom the subject holds no attraction.

2.6.7 General note on cognitive and affective objectives

Cognitive and affective objectives are interrelated. In this regard,Bell (1978:189-190) points out that one can never receive, respond to, value, organise, nor characterise a vacuous phenomenon, but rather in terms of something which in mathematics classes may be facts, skills, concepts, principles, processes and structures of mathematics. He adds that the students tend to learn (in the cognitive sense) better if they have specific cognitive learning objectives, and they tend to develop and maintain good attitudes (in the affective sense) towards mathematics if they know what is expected of them and how they will be evaluated.

As this study is concerned with the evaluation of the attainment of aims and objectives, it now becomes necessary to examine some of the aspects of educational evaluation.

2.7 EDUCATIONAL EVALUATION

2.7.1 What is evaluation?

Throughout the teaching-learning situation, the teacher is required to make judgements and decisions. Such a view is supported by Gronlund and Linn (1990:5) who state that the teaching-learning process involves a continuous and interrelated series of instructional decisions concerning ways to enhance pupils' learning. They add that the effectiveness of the instruction depends to a large extent on the nature and quality of the information on which the instruction is based. Hence we see that making judgements and decisions are possible if certain information is known. After examining pertinent literature, the researcher notes that most of the definitions of evaluation fall into one of the following two categories:

- Evaluation is a systematic process of collecting and analysing data in order to determine whether, and to what degree, objectives have been, or are being, achieved (e.g., Hartung 1961:22; Gronlund & Linn 1990:2; Avenant 1990:217); and
- Evaluation is the systematic process of collecting and analysing data in order to make decisions (e.g., Ten Brink 1974:10).

A distinction must be drawn between evaluation of the curriculum and evaluation of the success of the teaching and learning activity. Dreckmeyr (1989:88) points out that the evaluation of the latter can take place quite independently of the curriculum, when the intention is only to decide on progress with, or the success of, the teaching activity. It can also be used along with other techniques to determine the success of the curriculum. The researcher draws attention that in the context of this study, evaluation is concerned with the success of the teaching and learning activity.

As the terms "measurement", "assessment" and "evaluation" are used in the context of education, it becomes necessary at this point to examine the distinctions between these terms.

2.7.2 The distinctions between measurement, assessment and evaluation

Measurement is essentially a quantitative process. It is defined as the process of quantifying the degree to which someone or something possess a given trait or characteristic (Gronlund & Linn 1990:5; Gay 1991:8). It is quantitative by nature and involves the allocation of numerical values to certain characteristics according to specific predetermined rules (Dreckmeyr 1989:88; Gronlund & Linn 1990:5; Avenant 1990:217). According to Gay (1991:8), measurement permits more objective descriptions concerning traits and facilitates comparisons. Tests and examinations are organised procedures for measuring the behaviour (traits) of pupils and they represent just one type of measurement.

Although measurement is characterised by the demands of objectivity, reliability and validity, it must be emphasised that measurement in education is not infallible. If for example, the measuring instrument does not match the requirements of reliability and validity, the measurement itself will be inaccurate and misleading.

Assessment may be formal or informal. According to Deale (1975:22), assessment is an all embracing term, covering any of the situations in which some aspects of pupils' education are, in some sense, measured by the teacher, or another person. Satterly (1981:3-4) holds a similar view and asserts that assessment includes all the processes and products which describe the nature and extent of children's learning. He adds that, in making an assessment of a pupil, information may be acquired by formal and systematic methods (such as regular testing) and by impromptu means (such as teachers' judgements of pupils' performance in classroom observation). The NCTM (1989:203) avers that assessment must be more than testing; it must be continuous, dynamic, and often informal process. It adds that assessment should produce a "biography" of students' learning, a basis for improving the quality of instruction.

Evaluation is a more comprehensive and inclusive term than measurement and assessment. Evaluation follows upon measurement and may include qualitative descriptions of pupils. Furthermore, evaluation always includes value judgements concerning the desirability of the results (Hartung 1961:23; Gronlund & Linn 1990:6). Hence we see that evaluation is a qualitative process.

The researcher is of the opinion that a distinction should be made between the terms assessment and evaluation although they are sometimes used to designate overlapping processes. He prefers to associate the term assessment with what the pupil has accomplished and the term evaluation with what has been accomplished in an episode of teaching which involves not only the students but also the teacher.

2.7.3 The aim of evaluation

The evaluation of student performance serves many purposes. For the student, assessment aids learning and measures mathematical knowledge and power. For the teacher, evaluation provides information about how instruction should be modified and paced. For the administrator, it charts the effectiveness of a programme (NCTM 1989:199).

Avenant (1990:219) mentions that the teacher more specifically uses evaluation to achieve the following:

- To determine whether he/she has succeeded in his/her goal, i.e., whether or not all the pupils understood the subject matter.
- To determine whether his/her methods and techniques of presentation (teaching strategies) have been successful.
- To discover possible learning backlogs, concepts vacuums and misunderstanding timeously.
- To ascertain his/her success as a teacher in maintaining standards.
- To demonstrate the standards obtained by pupils.
- To motivate pupils.

2.7.4 The general principles of evaluation

Evaluation should be seen as an integrated process for determining the nature and extent of pupil learning. This process, according to Gronlund and Linn (1990:6-8) will be most effective when the following principles are taken into consideration.

- Clearly specifying what is to be evaluated (which has priority in the evaluation process).
- An evaluation technique should be selected in terms of its relevance to the characteristic or performance to be measured.

- Comprehensive evaluation requires a variety of evaluation techniques.
- Proper use of evaluation techniques requires an awareness of their limitations.
- Evaluation is a means to an end and not an end in itself.

2.7.5 Evaluation and the instructional process

Generally, the main purpose of classroom instruction is to help pupils achieve a set of intended learning outcomes. Viewed in this light, evaluation becomes an integral part of the teaching-learning process. Gronlund and Linn (1990:8) intimate that the intended learning outcomes are established by the instructional objectives, the desired changes in pupils are brought about by the planned learning activities, and pupils' learning progress is periodically evaluated by tests and other evaluation devices. Hence, Dreckmeyr (1989:89) asserts that true teaching is therefore always accompanied by evaluation which must always be planned and used as a teaching aid. According to Gronlund and Linn (1990:10), evaluation procedures can contribute to improvements in the teaching-learning process itself, as well as contribute directly to improved pupil learning.

At this point, it becomes necessary to discuss some of the functional roles of evaluation. While several roles can be identified, two types of evaluation, namely formative and summative evaluation, are discussed hereunder.

2.7.6 Formative and summative evaluation

2.7.6.1 Formative Evaluation

Formative evaluation is more directly associated with the learning situation itself and is used to monitor learning progress during instruction. Its aim is to identify strengths and weaknesses in the teaching and learning activity so that both teachers and pupils can take the necessary remedial steps (Shipman 1987:142; Dreckmeyr 1989:89; Thorndike et al. 1991:51). Feedback to the teachers provides information for modifying strategies, reformulating teaching objectives and instituting remedial programmes. Feedback to pupils provides reinforcements of successful learning and identifies specific learning error. Gronlund and Linn (1990:13) point out that mastery tests are generally used in formative evaluation and such tests are most frequently teacher-made. Since formative evaluation is directed towards improving learning and instruction, the results are typically not used for assigning course grades.

2.7.6.2 Summative evaluation

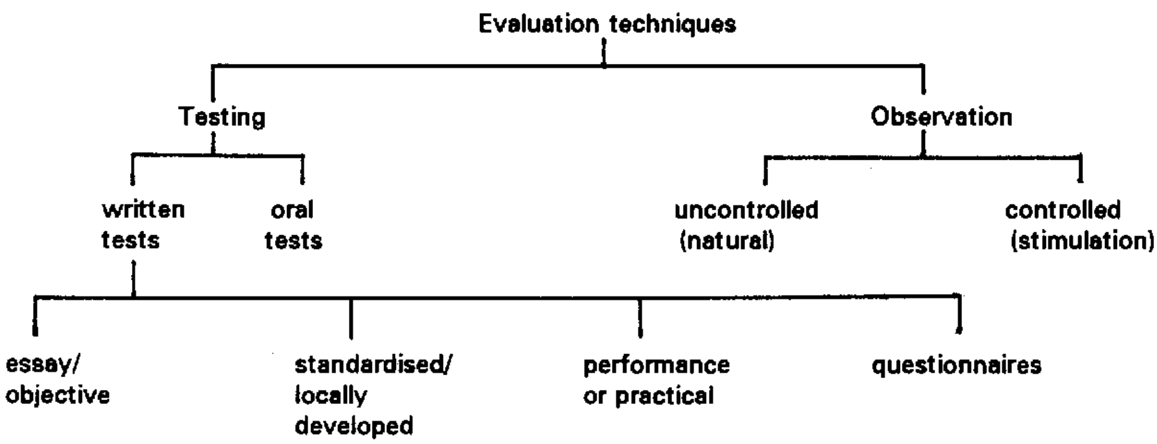
Summative evaluation typically comes at the end of a course (or unit) of instruction or at the end of the year. It focuses on the determination of pupil achievement in terms of the objectives and goals of the course/subject (Dreckmeyr 1989:89). According to Gronlund and Linn (1990:13), the techniques used in summative evaluation are determined by the instructional objectives, but they generally include teacher-made achievement tests, ratings on various types of performance (e.g., laboratory, oral reports) and evaluation of products (e.g., themes, drawings, research report).

It is to be noted that the evaluation of the attainment of the cognitive aims and objectives undertaken in this study is based on a summative evaluation, namely, pupils' attainment in mathematics in the senior certificate examination which is written at the end of the standard 10 year.

Effective evaluation is dependent on the use of the most appropriate technique(s). A closer look at evaluation techniques therefore becomes imperative.

2.7.7 Evaluation techniques

The different techniques which teachers can use to evaluate their pupils' progress can be presented schematically as follows:



(Adapted from Avenant 1990:223 and Gay 1991:108)

As this study is concerned with evaluation in mathematics, only those techniques relevant to mathematics assessment are discussed below.

2.7.8 Evaluation techniques used in mathematics

2.7.8.1 Oral assessment

An oral assessment commands a place in the range of evaluation techniques because it measures skills which may not be measured by written tests or examinations. While there is scope for oral assessment in mathematics at all levels of schooling, it is confined generally to the primary classes. It is interesting to note that in its new recommendations, the NCTM (1989) emphasises the use of oral assessments in all grades of schooling. Thorndike et al. (1991:162) state that oral tests, when administered individually, can also be used for the diagnosis of learning problems. For example, a teacher could ask a pupil a series of two-digit addition problems and ask the pupil to solve them aloud, that is, to talk through the procedures that were used to arrive at the answer. These realisations can provide clues about the misconceptions that underlie not only errors but also sometimes even correct responses.

Pellery (1991:880) points out that the use of oral examinations to evaluate the results of mathematical learning seems to be on the decline. Insofar as this study is concerned, the researcher notes that oral testing does not form part of the pupils' assessment in mathematics in the senior certificate examination.

2.7.8.2 Written tests or examinations

The most common form of assessment is the written test or examination. Written tests are essentially paper-and-pencil tests in which pupils are required to make written responses to questions. In most cases, the questions are also in written form. Two major types of written tests are identified, namely, standardised tests and teacher-made tests.

a) Standardised tests

A standardised test is one that is developed by subject-matter and measurement specialists, field tested under uniform administration procedures, revised to meet certain criteria, and scored and interpreted using uniform procedures and standards (Gay 1991:108). A study of American literature reveals that standardised achievement tests, which are mainly produced commercially and comprise objective-type questions, dominate the field of educational testing in that country. Thorndike et al. (1991:161) note that standardised achievement tests focus on objectives that are common to schools throughout the USA, and for this reason, the tests emphasise general skills. Also, such tests are unlikely to assess objectives related to specific content areas emphasised in the classroom.

On the other hand, limited use of standardised tests to measure achievement in mathematics is made in the RSA. The most well-known local standardised tests are those of the HSRC which include, amongst others, scholastic achievement and diagnostic tests in mathematics. Although these tests are strictly controlled and not freely accessible to the class teacher, they are however made available to educational agencies, e.g., education departments and SAFCERT, to be used for purposes of monitoring standards in particular subjects at certain levels.

b) Teacher-made (locally - developed) tests

Perhaps the most popular form of test used in assessing mathematics in the RSA is the teacher-made achievement test. Thorndike et al. (1991:162) mention that objectives that call for knowledge about a particular subject area (e.g., mathematics) and the capacity to use that knowledge to solve problems can be most reliably and validly appraised by teacher-made paper-and-pencil tests. In the HOD, classroom tests are constructed and administered by the teachers themselves. However in the case of "major " tests and examinations, tests set by teachers are moderated by the Head of Department or a senior teacher in the subject.

The teacher-made mathematics achievement tests in the HOD comprise both objective and non-objective type questions, with greater emphasis being placed on the latter type. However, in the senior certificate examination in mathematics, limited use is made of objective type items, with the vast majority of questions being of the open-ended problem-solving type. This, as viewed by the writer, is in alignment with the NCTM's recommendation that assessments should also yield evidence of students' use of strategies and problem-solving techniques and on their ability to verify and interpret results (NCTM 1989:209).

In view of the importance of tests and examinations in mathematics assessment, it is important that they are good measuring instruments. An examination therefore needs to possess certain characteristics in order that it may be considered a good examination. Some of the characteristics are discussed hereunder.

2.7.9 Characteristics of a "good" test or examination

A good test or examination must be valid, reliable and have a good discriminating power. However, two of the more important characteristics, namely, validity and reliability are discussed below.

2.7.9.1 Validity

A very important quality of an examination or test is its validity. Validity concerns the degree to which an examination measures what it is designed to measure. Gronlund and Linn (1990:48) state that validity ought to be seen as a matter of degree as it does not exist on an all-or-none basis.

Hence, the results of an examination should not be seen as valid or invalid, but rather in terms of categories that specify the degree of validity, e.g., high validity, moderate validity and low validity. In addition to the above, Gay (1991:157) notes that validity is specific to both the purpose and situation ("for what and for whom") for which the test or examination is used.

Different types of validity have been defined and these are indicated below.

a) Face validity: Face validity of an examination is concerned with whether or not it appears to measure what it purports to measure (Gay 1991:159). Stated in another way, Thorndike et al. (1991:126) refer to face validity as the "appearance of reasonableness" of a test or examination.

b) Content validity: This type of validity concerns itself with the effectiveness with which the examination tests the content and objectives of the course or subject. According to Tittle and Miller (1977:41), content validity is essentially established by definition: the teacher or examiner defines the universe of content which the test or examination is to sample and the content validity is established by logical, subjective examination of the questions in the instrument and their relationship to the original definition of the universe. Content validity is determined by expert judgement (Nuttal & Willmott 1972:160; Gay 1991:160) and requires a detailed specification of the course contents and objectives in the form of a blueprint or table of specifications (Nuttal and Willmott 1972:160; Gronlund & Linn 1990:54).

A good examination paper is one that adequately samples both content and objectives. Hence, content validity is of prime importance for achievement tests.

c) Construct validity: The construct validity of a test or examination may be determined by its correlation (relationship in terms of rank order) with other tests that measure the same construct (characteristic). The HOD (then Division of Indian Education) investigated the relationship between the IQ scores of candidates and their performance in mathematics in its senior certificate examination held in 1979. It was found that the mathematics papers set in 1979 by the HOD had a moderate construct validity ($r = 0{,}61$) (HOD 1980:5).

d) Criterion-related validity: The criterion-related validity of a test is generally based on the relationship between the scores on a test and some outside measure. Such a validity can be either concurrent or predictive (Gay 1991:161-162). When the test scores are correlated with some other measure, e.g., teacher's rating, or scores in another test taken at the same time, the criterion-related validity is referred to as concurrent. However, when the test scores are correlated with some criteria in the future, then the validity is referred to as predictive.

In order to determine the concurrent criterion-related validity of some of its senior certificate examination papers, the HOD administered standard 10 Item Bank tests (of the HSRC) in four subjects to candidates, immediately after the senior certificate examination papers were written in the respective subjects. The investigation revealed that the mathematics HG and SG papers set by the HOD in 1981 had a high concurrent criterion-related validity ($r = 0{,}81$ for HG papers and $r = 0{,}83$ for SG) (HOD 1982:4).

2.7.9.2 Reliability

Reliability refers to the consistency or dependability of the results of a test or examination. For the results of a test to be reliable, we expect them to be consistent with what might have been the result if the test had been given on another day; or if it had been marked on another day; and even if the results are not similar, it is expected that the rank order of the candidates should be similar.

Reliability is a necessary condition for validity, but not a sufficient one; that is to say, a test can be reliable without being valid, but to be valid, a test must be reliable and also satisfy other requirements of validity (Gronlund & Linn 1990:167; Thorndike et al. 1991:91; Gay 1991:79).

From a marking point of view, objective tests are perfectly reliable since any marker would award the same mark. However, the same is not true of non-objective or essay examinations or tests. Notwithstanding the above, a higher degree of reliability of the scores in a mathematics test or examination which comprises open-ended questions can be attained if a fairly detailed marking scheme (which indicates the break-down of marks for the various steps) is used and/or if one marker marks the same question throughout.

The validity and reliability coefficients of the test used in this study will be computed and presented in Chapter 5.

2.8 MEASUREMENT OF THE ATTAINMENT OF THE COGNITIVE OBJECTIVES

2.8.1 Model for measuring mathematics achievement

Whilst both cognitive and affective outcomes in mathematics are recognised, Wilson (1971:648) mentions that the first concern of evaluation in mathematics learning has been, and will continue to be, cognitive outcomes or achievement. He points out that mathematics achievement has many facets, that is, it is not a unitary trait. Therefore, a strategy that ensures many different measures of mathematics achievement is necessary.

A study of literature reveals that a useful and successful strategy to evaluate mathematics achievement is to use a matrix which combines two dimensions, that of mathematical content and that of cognitive outcomes. One such double classification scheme was devised and used by the NLSMA and this model is illustrated in the table below (Wilson, 1971:648-649).

TABLE 2.2 MODEL FOR MEASURING MATHEMATICS ACHIEVEMENT

	COMPUTATION	COMPREHENSION	APPLICATION	ANALYSIS
NUMBER SYSTEMS				
ALGEBRA				
GEOMETRY				

In the above model, the categories of mathematical content are number systems, algebra and geometry. The levels of cognitive behaviours are computation, comprehension, application and analysis.

The International Study of Achievement in Mathematics (Husén 1967a) utilised a matrix of three dimensions. The first dimension specifies the behaviours that the student is acquiring. Pellery (1991:878) notes that the ten cognitive objectives used, drawn from five major categories of cognitive behaviours, are as follows:

* Ability to recall and to reproduce definitions, notations, operations and concepts.
* Ability in rapid and accurate computation and symbols manipulation.
* Ability in translating data into symbols.
* Ability in interpreting data appearing in symbolic form.
* Ability in following a line of reasoning or proof.
* Ability in constructing a proof.
* Ability in applying concepts to mathematical problems.
* Ability in applying concepts to non-mathematical problems.
* Ability in analysing problems and determining the operations that may be applied.
* Ability in finding mathematical generalizations.

The second dimension concerns mathematics topics, some of which are listed below.

Arithmetic
Algebra
Plane geometry, solid geometry, analytical geometry
Trigonometry
Probability, calculus
General aspects

The third dimension concerns the use of the knowledge or of the skills acquired.

The above models illustrate that the attainment of the cognitive outcomes of mathematics instruction can best be measured by the use of some sort of organisational framework, or model or table of specifications. Wilson (1971:648) asserts that the process of building the table of specifications may be more important than the table of specifications itself. In view of the above assertion, and since tests or examinations are generally used to measure the attainment of the cognitive outcomes, it becomes necessary to take a closer look at procedures adopted in the setting of tests or examinations, of which the specifications table is an important component.

2.8.2 Procedures adopted in the setting of a test or examination

Evaluation has to be pre-planned and a systematic approach needs to be followed in the setting of a test. That is, to be an effective instrument for evaluating academic achievement, a test has to be structured according to a format decided upon in advance. The NCTM (1989: 193-194) asserts that for assessment to be properly aligned, the set of tasks on the assessment instrument must reflect the goals, objectives and breadth of topics specified in the curriculum. Shrivastava (1979:32-36) identifies the following steps as necessary for setting a good test or examination.

2.8.2.1 Preparation of a design for a test

Design is considered as the first and most important step in setting a test as it lays down the chief dimensions of the test. This includes:

* <u>Weighting among objectives</u>. This implies selecting the learning objectives to be tested, and allotting marks to each in terms of their relative importance.

* <u>Weighting among different areas of content</u>. This involves an analysis of the syllabus and delimiting the scope of each topic/area. This is followed by the allotment of marks to each of the areas in terms of their relative importance. Thorndike <u>et al</u>. (1991:206) mention that the sampling (specifying) of the content is important because it is the vehicle through which the objectives are mastered and measured.

* <u>Weighting for different types of questions</u>. In this regard, consideration is given to the type of question that is most suitable to test a particular objective and content, and to the marks to be allotted to each type of question set in the test.

* <u>Scheme of options</u>. The design may also indicate the pattern of options. There may be options between questions or within questions. If choice is to be allowed, then the questions should be comparable in respect of the learning objectives to be tested, the major area of content covered, and the form and difficulty level of the question.

* <u>Sections in the test</u>. Questions of the same type or those testing the same content area may be put together in sections. With regard to the above, it is observed that Curzon (1990:348-349) places greater emphasis on the first three dimensions, in the design of a test.

2.8.2.2 Preparation of a table of specifications (or test "blueprint")

The test or examination "blueprint" is a detailed plan based on the design for preparing a question paper. Hudson (1973:17) asserts that such a specification is undoubtedly the most difficult, yet the most crucial stage, in the construction of a test or examination. He adds that subsequent decisions, such as the form of the test and the types of questions to be used, flow from it.

A test blueprint (table of specifications) is essentially a two-way grid with the content outlined along the vertical axis and the expected behavioural changes (objectives) along the horizontal axis. These two axes, however, are interchangeable. The cells within the table are used to indicate the number of test items to be prepared for each outcome of instruction. A simplified version of such a table for a final examination in Mathematics (Higher Grade) is presented in Table 2.3 (see next page).

In Table 2.3, the content dimension is defined in terms of 19 mathematics topics and the objectives dimension in terms of 4 categories of outcomes. The result is a 19 - row by 4 - column matrix containing 76 cells which most probably define the universe of test items in which the examiner is interested. The specifications grid indicates the percentage of marks (or the number of items) allocated to the various content areas of the syllabus and also the distribution of these marks/items amongst the various abilities to be tested. In terms of the latter, it is noted that application is given the greatest weighting (40%) and the higher abilities are given the least weighting (15%). In the main body of the grid, it would be noticed that 4% is allocated, for example, to circles in the application category.

TABLE 2.3. TABLE OF SPECIFICATIONS FOR A FINAL EXAMINATION IN MATHEMATICS (HIGHER GRADE)

ABILITIES / COURSE CONTENT	KNOWLEDGE 20%	COMPREHENSION 25%	APPLICATION 40%	HIGHER ABILITIES 15%	NO. OF ITEMS/ PERCENTAGE MARKS
1. ALGEBRA a. Absolute Value	-	1	1	-	2
b. Functions	1	1	2	-	4
c. Linear programming	1	1	2	1	5
f. Quadratic equations and inequalities	1	2	2	-	5
g. Remainder and factor theorems	1	1	1	1	4
h. Equations	1	1	2	1	5
i. Exponents	1	2	4	1	8
j. Logarithms	1	1	2	-	4
k. Sequences and series	1	2	3	1	7
2. DIFFERENTIAL CALCULUS	2	1	2	1	6
3 TRIGONOMETRY a. Identities	1	2	2	-	5
b. Formula for triangles	1	1	2	1	5
c. Trigonometric graphs	1	1	2	1	5
d. Compound and double angles	1	2	2	-	5
4. GEOMETRY - EUCLIDEAN a. Circles	1	1	4	3	9
b. Concurrency	1	1	1	-	3
c. Proportional intercepts	1	1	1	-	3
d. Similarity	1	1	1	2	5
5. GEOMETRY - ANALYTICAL	2	2	4	2	10
TOTAL	20	25	40	15	100

Once the table of specifications has been constructed, a decision needs to be taken on the assessment technique(s) and the type of questions to be used to evaluate each cell in the table. In deciding upon the assessment techniques for any particular syllabus, it is essential to choose the techniques most appropriate to the subject matter and the variety of skills involved.

2.8.2.3. Preparation of questions based on the blueprint

After the specifications of all the questions have been included in the blueprint, the next step is the preparation of the questions themselves. Shrivastava (1979:33-34) states that while writing or selecting questions, the examiner or test-constructor must keep in mind that:

1) each question is based on a well-defined objective;
2) it relates to a well-defined objective;
3) it is written in the form required by the blueprint;
4) it is at the desired level of difficulty;
5) it is so worded that it is well within the comprehension of the pupils, and that it clearly indicates the scope and length of the answer;
6) it is so worded that its scope, meaning and difficulty level will not change in the process of translation; and
7) it has a good discrimination value.

2.8.2.4 Editing the test or examination question paper

Editing the question paper consists of the following procedures:

1) Assembling the questions into sections is done on the basis of their

type or content area they test. Shrivastava (1979:34) states that, whether there are subsections or not, the questions should be organised in a graded order of difficulty, from easy to difficult. He adds that such a grading gives examinees confidence in their ability to attempt the questions, and reflects the complexity of the mental abilities required.

2) The instructions to examinees need to be clear, specific and pointed. General instructions should be given at the beginning of the paper, and specific instructions relating to each section should appear at the top of the corresponding section.

3) Implications for administration. In cases where a question paper is divided into sections, it may be necessary to give a specific time limit to a section, for example, a section containing objective-type questions in a mathematics or science examination.

2.8.2.5 Preparation of the scoring key and/or the marking scheme

It is essential to frame the scoring key and/or the marking scheme concurrently with the framing of the question paper. A scoring key is prepared for objective-type questions while a marking scheme is prepared for the short-answer and open-ended questions. The marking scheme gives the expected outlines of the answers and the marks that each point or aspect (step) of the answer deserves. As far as possible, all the answer-points that may be relevant to the question should be listed in the marking scheme, irrespective of the number asked for in the question. Detailed instructions for marking will also need to be worked out and issued to the markers.

2.8.2.6 Preparation of the question-wise analysis

According to Shrivastava (1979:36), the question-wise analysis will enable examiners or test-constructors to make a qualitative analysis of the question paper. It acts as a check in comparing the question paper with the design and the blueprint. Each question will be analysed in order to determine whether or not all the requirements indicated in the test or examination specifications table have been complied with.

2.8.2.7 Moderation of the test or examination question paper

Whilst this step is not included in the procedures advocated by Shrivastava (1979), the moderation of the question paper is considered as an important and final step in the construction of a test or examination. Noting the procedures used by many examination bodies in the setting of their papers, the researcher intimates that the final draft question paper should first be submitted to an assessor (internal moderator). The moderated (modified) paper should then be presented to an external moderator or moderating committee. He adds that the importance of having the draft question paper checked by persons (specialists) other than the examiner(s) cannot be over-emphasised.

The researcher also acknowledges the importance of the abovementioned procedures necessary in the construction of tests and examinations. He avers that strict adherence to these procedures would result in the compilation of high quality measuring instruments, a goal that is desired by all educators.

As use has been made in this study of the HOD's senior certificate mathematics question papers and answer scripts, a brief discussion on the procedures adopted by the HOD in the setting of its examination papers is considered necessary at this point.

2.8.3 The standard 10 mathematics examination (test) of the HOD

The senior certificate examination papers, including those in mathematics, set by the HOD can be classified as locally developed achievement tests. However, they are subjected to some of the (initial) procedures used in the construction of standardised tests. In order to minimise subjectivity, panels of three examiners (subject experts) each are used to set the question papers in mathematics. Examiners have to comply with several requirements which include, amongst others, the use of a specifications table so as to ensure a fair sampling of the content areas and the objectives that are to be tested. Once set, the draft question papers are scrutinised firstly by an internal moderator, and then by an external moderator. Moderators are expected to ensure that each question is measuring one of the stated objective of the course; that the questions are fair and reasonable for the candidates for whom the test is intended; that the questions are clearly and unambiguously worded and that they can be satisfactorily answered in the time allowed. The moderated papers are finally edited by professional planners before the examination papers are printed and administered.

With regard to the marking of the answer scripts, whilst the HOD appoints panels of markers, one marker is used to mark the same question(s) throughout in order to ensure uniformity/consistency in standards.

In conclusion, it is to be noted that, in this study, pupil's attainment of the cognitive objectives of the senior certificate mathematics curriculum is based largely on their achievement in mathematics in the senior certificate examination. However, qualitative information with respect to the above will also be obtained from a teacher questionnaire, details of which are presented in Chapter 3.

Consideration is now given to the measurement of the attainment of the affective objectives.

2.9 MEASUREMENT OF ATTITUDE TOWARDS MATHEMATICS

Corcoran and Gibb (1961:121) assert that if attitude objectives are accepted as appropriate for school mathematics, then some provision for their testing should be made. However, Aiken (1979:208) observes that the many devices developed to elicit information on attitudes of individuals and groups range from the more direct self report questionnaire and scales to the highly indirect projective techniques. The above observation accords with the view of Corcoran and Gibb (1961:107) who consider that, generally, the various techniques for measuring attitudes fall into the following three major categories:

(a) Self report methods such as questionnaires, attitude scales, projective techniques and content analysis of essays. In such instances, the subject reports his/her own attitude by responding to an attitude scale or describes his/her feelings by writing an essay;

(b) Observable methods, when another individual observes and records behaviour which is indicative of evidence of an attitude or interest; and

(c) Interviews, where another individual interviews and notes down responses using a structured or unstructured questionnaire.

Gay (1991:122-123) mentions that at the classroom level, objectives related to attitude and the assessment of their achievement may be less formal. Teachers are more likely to assess attitudes by observation as well as with informal instruments and sociometric techniques. However, a more valid appraisal of attitudes requires the use of standardised attitude scales. According to Aiken (1979:211), the attitude scale has emerged as the most popular device for determining attitudes, including pupils' attitude towards mathematics.

There are four basic types of attitude scales, namely:

(1) Thurstone's equal appearing interval scale;
(2) Likert's method of summative ratings;
(3) Guttman's scalogram analysis; and
(4) Osgood's semantic differential.

With regard to the suitability of three of the above scales, the following comments are pertinent: Although the Thurstone method of attitude scaling has been widely used since its inception, some reservations have been expressed about the assumptions underlying the model (Nunnally 1978); the Guttman scale is less frequently used (Aiken 1979); and Osgood's semantic differential has been used in a limited way to assess attitudes towards mathematics (McCallon & Brown 1971). In view of the above, these three scales are discussed no further here. However, it has been found that the Likert-type scales have become increasingly popular among workers in the field of attitude measurement.

A Likert scale requires an individual to respond to a series of statements by indicating whether he/she strongly agrees (SA), agrees(A), is undecided(U), disagrees (D) or strongly disagrees (SD) with each statement. However, Corcoran and Gibb (1961:109) note that very often a three-point scale - agree, undecided, and disagree - is used.

Each response is associated with a point value (score). For example, positive statements may be assigned the following point values: SA=5, A=4, U=3, D=2 and SD=1. In the case of negative statements, the point values are reversed, i.e., SA=1, A=2, etc. An example of a positive statement could be "mathematics is enjoyable and stimulating to me". A respondent's total score is obtained by summing across all his or her item scores. Gay (1991:123) mentions that a high point value on a positively stated item would indicate a positive attitude towards that item, and a high total score on the entire questionnaire would be indicative of a (general) positive attitude.

Research indicates that relatively definite attitudes about mathematics have been developed by the time pupils are in the junior secondary classes (Schofield 1980:111; Schofield 1982:280-284). Likewise, research studies indicate that there is a positive relationship (correlation) between mathematics achievement and attitudes towards mathematics amongst junior secondary pupils (Cheung 1988:211) as well as amongst senior secondary pupils (Moodley 1981:176).

In this study, use has been made of the attitude scale developed by Moodley (1981) to appraise the attitude of senior certificate pupils towards mathematics. Details regarding the scale are presented in Chapter 3.

2.10 MEASUREMENT OF INTEREST IN MATHEMATICS

According to Gay (1991:125), interest can be assessed informally in the classroom in the same ways that attitudes can, through the use of informal, teacher-made instruments (checklists and rating scales) and through observation. However, he emphasizes that an ideal testing programme should include a standardised inventory.

A standardised interest inventory requests individuals to indicate personal likes and dislikes, such as the kinds of activities they prefer to engage in (Gay 1991:125).

As mentioned in paragraph 2.6.6.2, items to assess interest in mathematics provide the pupils with an opportunity to express a preference for mathematics or mathematics activities over other choices.

It is observed that the Likert scale, including adaptations of it, is generally used for responding to the statements included in interest scales. Two examples of items that assess interest in mathematics are cited below.

(1) I would like to be a mathematics teacher.

- (a) Strongly agree
- (b) Agree
- (c) Undecided
- (d) Disagree
- (e) Strongly disagree

(2) I would like to study about the life of Pythagoras.

(a) not at all

(b) a little

(c) a lot

In order to appraise senior certificate candidates' interest in mathematics, use has been made of the attitude scale devised by Husén (1967a & 1967b), details of which are presented in Chapter 3.

2.11 CONCLUSION

In this chapter, after placing the study in juxtaposition within a subject didactical perspective and theoretical framework, detailed expositions have been provided in respect of aims and objectives as well as of some classifications of objectives in mathematics teaching and learning. After discussing education evaluation in general, due emphasis has also been given to the procedures and instruments that could be used to measure the attainment of the cognitive and the affective objectives in mathematics. Senior certificate candidates' achievement in mathematics in general and their attainment of the different aims and objectives in particular, have important implications for their future. In order to determine the attainment of the cognitive and affective aims and objectives by senior certificate candidates, criteria (norms) had to be developed and appropriate instruments had to be identified or devised. A discussion of these follows in the next chapter.

CHAPTER 3

RESEARCH AND EMPIRICAL METHODS APPLIED

3.1 INTRODUCTION

Although specific aims are more explicit about the end in view than general aims, specific aims are still somewhat broad statements of intent. Rowntree (1977:26) asserts that to be operationally useful, course (specific) aims must be translated into objectives. In view of the above, and as this study is concerned with the evaluation of aims and objectives, the first step in the empirical methods used necessitated an analysis and interpretation of the cognitive aims of the senior certificate mathematics curriculum, in order to identify the cognitive objectives inherent in each of them. These objectives had to be compared with the cognitive objectives utilised and tested by the HOD. With regard to the attainment of the cognitive objectives, the examination (test) in mathematics set by the HOD in its 1991 senior certificate examination was identified as a suitable instrument. However, since no norms were available for attainment in respect of the different levels or categories of cognitive abilities for mathematics, the second step in the empirical method required the development of norms (or criteria) against which pupils' attainment of the cognitive abilities could be compared. In order to obtain a more comprehensive picture of pupils' attainment of the cognitive objectives, qualitative information in their regard had to be obtained. This necessitated the development of a suitable questionnaire, for completion by teachers. With regard to appraising the affective aims and objectives (attitude towards and interest in mathematics), it was possible to identify suitable standardised scales.

What follows, then, is a discussion on the empirical methods adopted and the research instruments identified, that were used in this study.

3.2 CLASSIFICATION OF THE AIMS OF THE SENIOR CERTIFICATE MATHEMATICS CURRICULUM INTO CATEGORIES

A list of the seven aims of the senior certificate mathematics curriculum currently in use in the RSA has been presented in paragraph 2.5.1.2 of Chapter 2. In terms of their nature and intent, it is observed that these seven aims can be grouped into three broad categories as follows:

a) Cognitive aims

1. To enable pupils to gain mathematical knowledge and proficiency.
2. To develop clarity of thought and the ability to make logical deductions.
3. To develop accuracy and mathematical insight.
4. To instil in pupils the habit to estimate answers where applicable, and where possible to verify answers.

b) Affective aims

5. To develop a love for, an interest in and a positive attitude towards mathematics, by presenting the subject meaningfully.

c) Utilitarian aims

6. To develop the ability of pupils to use mathematical knowledge and methods in other subjects and in their daily life.

7. To provide basic training for future study and careers.

Mention has already been made in paragraph 1.6.3.1 of Chapter 1 that this study is restricted to the attainment of certain aims concerned with the cognitive and affective domains. In view of this, the utilitarian aims are not discussed any further here.

A closer look is now taken at the cognitive aims.

3.3. ANALYSIS OF THE COGNITIVE AIMS OF THE SENIOR CERTIFICATE MATHEMATICS CURRICULUM

3.3.1 Separation of the cognitive aims

A close scrutiny of the cognitive aims reveals that two of the aims (i.e., aims numbered 1 and 3 in the above paragraph) refer to more than one independent mathematical activity. Hence, it was found that each of these aims could be stated in terms of two separate aims. On the other hand, while the remaining two cognitive aims are compound statements of intent, they refer to activities that function in complement with one another. Therefore, these two aims were treated as individual aims, for purposes of analysis. In view of the above, the four cognitive aims were separated to read as follows:

1. To enable pupils to gain mathematical knowledge.
2. To enable pupils to gain mathematical proficiency.

3. To develop clarity of thought and the ability to make logical deductions.
4. To develop mathematical insight.
5. To develop accuracy.
6. To instil in pupils the habit to estimate answers where applicable, and where possible to verify answers.

Each of the above aims is analysed below, with a view to ascertaining the objectives inherent in them.

3.3.2 Analysis and interpretation of the cognitive aims

3.3.2.1 To enable pupils to gain mathematical knowledge

The acquisition of knowledge is learning at its most basic level. Mathematics is organised in a sequence of topics and activities that are associated with appropriate levels of maturity and ability of the students (Sueltz 1961:8). Hence in the first instance, pupils encounter new basic mathematical content prescribed for the standard or course. However, a study of literature indicates that mathematical knowledge is more than mere content. For example, Wilson (1971:660) and Bell (1978:169) state that basic mathematical knowledge comprises knowledge of specific facts, of concepts and of rules and principles. Van den Aardweg and Van den Aardweg (1988:32) mention that mathematical knowledge includes appropriate materials, methods and structures such as specific terms and symbols, methods and techniques, relevant facts and theories and ways of presenting materials. According to Popkewitz (1988:235), knowledge in schooling is conceptualized as specific qualities of learning, steps or stages of problem solving, or formal mathematics equations or concepts.

In the light of the discussion present above, the following objectives (mathematical abilities) were identified in this first aim.

1. To recall specific facts.
2. To recall concepts.
3. To recall rules and principles.
4. To recall skills, techniques and methods learnt.

3.3.2.2 <u>To enable pupils to gain mathematical proficiency</u>

While in the study of mathematics a pupil is required to learn facts, develop concepts, use symbols and master processes and procedures, a major reason for studying mathematics is to gain mastery (proficiency) in problem solving. The significance of this statement is evident in the following assertions: problem solving is the heart of school mathematics (Newell 1983:4); problem solving can be considered as the real essence of mathematics (Orton 1987:35); and, learning to solve problems is the principal reason for studying mathematics (McCoy 1990:48). According to Bell (1978: 311-312), problem solving is a fundamental process in mathematics and constitutes a considerable portion of the work of mathematicians. He adds that it is an important and appropriate activity in school mathematics because the learning objectives which are met by solving problems and learning problem-solving procedures are of significant importance to society.

Arising out of a study of the Standards (1989), Romberg (1990: 474 - 475) observes that problem solving in mathematics requires pupils to make and evaluate conjectures, follow and judge valid arguments, deduce, induce, construct proofs, verify and interpret results , and generalise solutions. Pertinent in this regard is the assertion made by Hirsch (1990:495) who states that pupils should have frequent opportunities to represent problem-situations verbally, numerically, graphically and symbolically.

The reader is also referred to paragraph 2.2.1.4. of Chapter 2 wherein a detailed discussion on problem solving and problem solving strategies for mathematics learning was presented. Although the information is not repeated here, it is to be noted that the information referred to is also of relevance to this section.

From the foregoing, it can be observed that besides attaining proficiency in knowledge and skills, pupils need to attain proficiency in problem solving. Furthermore, proficiency in the former aspects is a pre-requisite for proficiency in the latter. The researcher asserts that proficiency in problem solving can be interpreted as acquiring proficiency in all the mental abilities involved in mathematics.

In terms of the discussion presented above, the following objectives were identified in this aim concerned with mathematical proficiency.

1. To recall and apply appropriate fundamental mathematical knowledge (specific facts, concepts, rules and principles, skills, techniques and methods).
2. To compute and manipulate rapidly and accurately (detailed objectives are presented in paragraph 3.3.2.5).
3. To interpret symbolic and pictorial data.
4. To put data into symbols or geometric form.

5. To interpret a problem.
6. To analyse a problem.
7. To determine the operations that may be applied to solve a problem.
8. To construct proofs.
9. To check the accuracy or reasonableness of proofs constructed.
10. To invent mathematical generalisations.
11. To solve routine problems.
12. To solve non-routine (novel) problems.

3.3.2.3 To develop clarity of thought and the ability to make logical deductions

Thought refers to the process or power of thinking and reasoning (McCoy 1990:53). Hence it is noted that thought is a higher mental activity. Theories on the development and functioning of thought have been presented by educational psychologists like Piaget (1952), Guilford (1967) and Bruner (1966). In the structure of the Intellect Model of Guilford (1967), the activities of the intellect identified are memory, cognition, evaluation, convergent production and divergent production which, according to Van den Aardweg and Van den Aardweg (1988:231), correspond to thought elements such as differentiation, integration, induction, deduction, generalisation, memorisation, problem solving and creative thinking. These authors add that Guilford's (1967) productive thought can possibly be problem solving (convergent thought) and creativity (divergent thought).

Human (1975: 33-34) mentions that mathematical thought is necessarily of an abstract and logical deductive nature and that development of mathematical potential inevitably implies the development of mathematical abstract and logically deductive thinking. He adds that mathematical thinking is a particular form of abstract thinking and the development of this thinking will therefore to a large extent be accompanied by language acquisition. Several mathematics didacticians, for example Polya (1954) and Van Zyl (1942) point out that formal-logical thinking as illustrated in mathematics is not an isolated activity, but only obtains functionally in conjunction with intuitive, inductive hypothesis-stating thinking.

Taking into consideration the points discussed above, the following objectives were identified in this aim.

1. To be able to analyse data.
2. To make comparisons and to perceive relationships.
3. To draw conclusions and make interferences.
4. To be able to support statements (in proofs and solutions) with well substantiated reasons.
5. To solve problems.

3.3.2.4 To develop mathematical insight

Mathematical insight refers to the global and penetrating understanding of mathematics which is associated with the higher intellectual abilities. Insight is the sudden discovery of a solution which the learner has mulled over or has been involved in (Van den Aardweg and Van den Aardweg 1988:166). According to these authors, the learner with his/her experience and knowledge looks at the problem as a whole, structures its components, sees relationships and connections in such a way that a pattern emerges and then insight dawns. They add that insight is a process of problem solving opposed to mere trial-and-error methods, and implies a set that is orientated towards hypothesis formation and testing for the purpose of understanding a particular problem. Hence learning through insight saves time and effort as no repetition is necessary. Sueltz (1961:71) avers that learning with insight possibly has transfer value.

In terms of the above, it is apparent that mental abilities (objectives) such as the following are implied in this aim.

1. To be able to separate the components of a problem, i.e., to analyse the problem.
2. To perceive relationships and connections.
3. To perceive patterns.
4. To formulate and test hypotheses.
5. To make inferences and draw conclusions.
6. To solve problems.

As is the case with the abovementioned aim, it can be observed that insight as illustrated in mathematics is not an isolated activity, but functions in conjunction with other mental abilities and activities.

3.3.2.5 <u>To develop accuracy</u>

In the mathematical context, accuracy is related to and desired in aspects such as computational and manipulative skills as well as in mathematical expression.

Accuracy in computation refers to the carrying out of algorithms speedily and accurately. Computational skills include abilities such as: to carry out numerical calculations, to factorise, to solve, to substitute, to change the subject of a formula. On the other hand, manipulative skills include abilities such as: ability to sketch graphs, read tables, handle mathematical instruments and calculators.

With regard to accuracy in mathematical expression, Cockcroft (1982:3) states that mathematics is a subject in which one has to use appropriate words and terms. He adds that mathematics is a language that is powerful, concise and unambiguous. Hence, by the study of mathematics, the habits of accuracy, clarity, brevity, precision, conciseness and certainty in expression are formed and strengthened. Therefore, it is important that pupils are provided with opportunities to verbalise their learning experiences and observations.

From the foregoing discussion, the following objectives were observed as being involved in this aim, namely, to develop accuracy.

1. To compute rapidly and accurately.
2. To manipulate rapidly and accurately.
3. To express mathematical information precisely and concisely.
4. To express mathematical information in own words.

3.3.2.6 To instil in pupils the habit to estimate answers where applicable, and where possible to verify their answers

Estimating answers can be more appropriately applied to numerical calculations. If pupils master such an ability, their computed answers can be checked. Trafton (1987:199) states that despite the fact that estimation is most useful in applied situations, often it is studied in school only as a tool for checking the reasonableness of written calculations. The verification of answers can also apply to mathematical solutions and proofs, whereby their accuracy or reasonableness could be checked. The NCTM (1989:8) asserts that estimation can, and should, be used in conjunction with procedures yielding exact answers to foreshadow any calculation and to judge the reasonableness of results. Furthermore, it (NCTM 1989:36) mentions that instruction should emphasise the development of an estimation mind-set. Children should come to know what is meant by an estimate and when it is appropriate to estimate, and how close an estimate is required in a given situation.

In terms of the above, the following objectives were derived from this aim.

1. To estimate answers involving numerical calculations.
2. To check the accuracy of computations.
3. To check the accuracy or reasonableness of solutions, and proofs constructed.

In order to verify the researcher's analysis and interpretation of the above six cognitive aims of the senior certificate mathematics curriculum, a list reflecting the aims and the corresponding objectives identified was drawn up and submitted to ten mathematics experts in the HOD (2 subject advisers and 8 senior educators of mathematics) for their comments. Except for certain minor amendments recommended, it emerged that there was general consensus amongst the experts with regard to the translation of the aims into appropriate objectives. They also expressed general concurrence that the various objectives identified constituted a good representation of the objectives pursued by senior certificate mathematics teachers in the classroom.

3.3.3 Classification of the cognitive objectives identified in terms of the taxonomy used in this study

As stated in paragraph 2.6.5. of Chapter 2, for purposes of this study, it was decided to use a three-category classification of cognitive objectives, which regroups the six levels in Bloom's Taxonomy (1956) as follows:

		Bloom's levels
1.	Lower level abilities (LL)............	(knowledge and skills)
2.	Middle level abilities (ML)	(comprehension/understanding)
3.	Higher level abilities (HL)	(application, analysis, synthesis and evaluation)

The motivations influencing the use of the above three-category classification are as follows:

* Current literature tends to report on pupils' mathematical achievement in terms of broad categories of mathematical abilities, e.g., higher order thinking skills, lower mental process, etc. (already stated in paragraph 1.5.2 of Chapter 1);
* Educators find it generally easier to classify items, with a greater degree of accuracy, in terms of fewer and broader categories of objectives;
* Fewer categories of objectives ensure a reasonable number of items falling into a category;
* A similar classification of objectives is used by senior certificate mathematics examiners in the HOD;
* The cognitive behaviours (objectives) inherent in the aims which refer to the

development of the higher mental abilities (clarity of thought, logical deductions, mathematical insight) do not function independently, but rather in conjunction with one another and other higher mental process (discussed in paragraphs 3.3.2.3 and 3.3.2.4), as evidenced in pupil's ability to solve non-routine or novel problems; and

* The cognitive aims of the senior certificate mathematics curriculum can also be expressed in terms of these three broad categories of abilities.

Once the various objectives in the six cognitive aims had been identified, attention was given to grouping these objectives in terms of the three-category classification. Taking into consideration the descriptions presented in the classifications of objectives of Bloom (1956) and Wilson (1971), discussed in Chapter 2, each objective was closely examined in order to determine its level of specificity, and hence its appropriate placement in the classification. The outcome of this exercise is presented below.

<u>Objectives identified as falling into the LOWER LEVEL ABILITIES category</u>

1. To recall fundamental mathematical knowledge, i.e.,

1.1 To recall specific facts;

1.2 To recall concepts;

1.3 To recall rules and principles.

2. To recall skills, techniques and methods learnt, for solutions and proofs.

3. To compute rapidly and accurately.

4. To manipulate rapidly and accurately.

Objectives identified as falling into the MIDDLE LEVEL ABILITIES category

5. To interpret symbolic and pictorial data.
6. To put data into symbols or geometric form.
7. To express mathematical information in own words.
8. To estimate answers involving numerical calculations.
9. To check the accuracy or reasonableness of computations and solutions, and proofs constructed.
10. To follow proofs.
11. To solve routine problems.

Objectives identified as falling into the HIGHER LEVEL ABILITIES category

12. To apply fundamental knowledge to mathematical problems.
13. To interpret a problem.
14. To analyse a problem, i.e.,

14.1 To be able to separate the components of a problem;

14.2 To make comparisons and perceive relationships and connections;

14.3 To formulate and test hypotheses; and

14.4 To determine the operations that may be applied to solve a problem.

15. To construct proofs.
16. To be able to support statements (in proofs and solutions) with well substantiated reasons.
17. To perceive patterns.
18. To invent mathematical generalisations.
19. To solve non-routine (novel) problems.

The objectives identified above were included, amongst others, in a teacher questionnaire that was developed in order to obtain qualitative information on pupils' attainment of the cognitive objectives (discussed further in paragraph 3.7).

3.3.4 Cognitive aims expressed in terms of the taxonomy of cognitive objectives used in this study

With the above classification of the cognitive objectives into the broad categories having been completed, it was found that each of the cognitive aims could also be expressed in terms of these categories of objectives (LL, ML and HL). A summary of the categories of objectives identified in each aim is presented in Table 3.1.

TABLE 3.1 COGNITIVE AIMS EXPRESSED IN TERMS OF THREE CATEGORIES OF COGNITIVE ABILITY LEVELS (LL, ML, AND HL)

AIMS/AIMS ELABORATED		CATEGORY OF COGNITIVE LEVEL
1.	To gain mathematical knowledge.	LL
2.	To gain mathematical proficiency, i.e., with respect to:	LL, ML and HL
2.1	knowledge and skills;	LL
2.2	solving routine problems;	ML
2.3	solving non-routine (novel) problems.	HL
3.	To develop clarity of thought and to make logical deductions.	ML and HL
4.	To develop mathematical insight.	HL
5.	To develop accuracy, i.e.,	LL and ML
5.1	in calculations and skills;	LL
5.2	in mathematical expression.	ML
6.	To estimate and verify answers.	ML

The representation of the cognitive aims in terms of the above format was necessary in order to evaluate their attainment. This aspect is discussed in paragraph 3.6 of this chapter.

3.4 THE DEVELOPMENT OF NORMS FOR MATHEMATICS ATTAINMENT IN TERMS OF THREE CATEGORIES OF COGNITIVE ABILITIES

It was realised at the outset that an evaluation of the efficacy of the cognitive objectives of the senior certificate mathematics curriculum required pre-determined norms (criteria) against which the attainment of these objectives (and hence the aims) could be judged. A comprehensive survey of both local and overseas literature revealed that no norms for mathematics attainment in terms of different cognitive levels, for standard ten pupils, were reported upon. Hence it became necessary for the researcher to develop the relevant norms in order to undertake the evaluation of the attainment of the cognitive objectives.

However, in the survey of literature, the researcher was able to identify four local studies concerning the performance (achievement) of pupils in mathematics at the standard ten level. While two of these studies reported incidentally on pupil attainment in terms of different cognitive levels, the other two studies contained adequate statistical data for such an analysis to be carried out. The information contained in these four studies (Moodley 1981; HOD 1988; HOD 1989; Norton 1991a & 1991b) was used as a basis for the development of the norms for the three broad categories of cognitive abilities.

The discussion that follows, then, is an exposition of the procedures that were adopted and which led to the establishment of the required norms or criteria.

3.4.1 Hierarchical nature of cognitive behaviours

3.4.1.1 The theoretical formulation of Bloom's taxonomic classification of

objectives is based on a hierarchical order, with the arrangement of behaviours from the simple (knowledge) to the complex (evaluation). By studying the performance of individuals in comprehensive examinations, Bloom (1956:18) found evidence to support the above hypothesis. He therefore asserts that it is common to find that individuals attain low scores on complex problems and high scores on the less complex problems, rather than the reverse (Bloom 1956:19). Orlosky and Smith (1978: 430; 434) report that a similar pattern in the attainment of objectives was observed when they made a study of pupil performance in certain international studies that were conducted, namely, in Mathematics (I.E.A. 1967), in Science (Comber & Keeves 1973), in Reading Comprehension (Thorndike 1973), and in Literature (Purves 1973). According to the said authors, it was found that in each subject, in almost every country, pupils performed best on the lower mental processes involving knowledge, performed less well on items involving some interpretation or comprehension, and performed least well on test problems requiring application, higher mental processes and complex inferences.

3.4.1.2 With regard to performance in mathematics, the findings of several other research studies have demonstrated consistency with the above pattern of results (Stoker & Kropp 1964; Smith 1968; Schools Council 1970; Moodley 1975). While these authors acknowledge that an inverse relationship between complexity of behaviour and facility of problem-solving exists in mathematics performance, only one of these authors, namely, Moodley (1975) provided numerical indexes pertaining to the performance of individuals in respect of the lower and higher mental abilities.

3.4.2 Analyses of four research studies involving standard 10 pupils' performance in mathematics

3.4.2.1 Study conducted by Moodley

The survey of literature revealed that Moodley (1981) conducted a research study entitled "A study of achievement in Mathematics with special reference to the relationship between attitudes and attainment". Among the various analyses carried out and reported upon, the author also analysed the mathematics attainment of 680 standards 9 and 10 Indian pupils in terms of the following three levels of behaviours: lower level mathematical abilities (which included knowledge and skills), middle level mathematical abilities (comprehension) and higher level mathematical abilities (which included the behaviours of selection, application, analysis and synthesis). It was observed that for purposes of his study, Moodley constructed, amongst others, a mathematics achievement test comprising 24 multi-choice items (8 items on knowledge/skills, 8 items on comprehension, and 8 items on higher mental abilities) which were drawn from tests utilised in the International Study of Achievement in Mathematics (Husén 1967a and 1967b). The results obtained by Moodley (1981: 123; 134), in terms of difficulty of items shown by mean percentage of sample responding correctly to each of the sets of items for the three levels are reflected in Table 3.2. While only the raw scores (in the form of percentages) were provided by the above author, these scores were converted by the researcher to z-scores using the mean and the standard deviation of the test. The z-scores were then transformed to T-scores, which use a mean of 50 and a standard deviation of 10. In this way, decimal and negative values are eliminated (Noll, Scannel & Craig 1979:61). The major advantages of using standard scores (z-scores and T-scores) is that they allow scores from different tests or from sub-tests to be compared (Gay 1991:346). N.B. The method/formula used to transform raw scores to z-scores and T-scores is reflected in Appendix 8.

TABLE 3.2 MATHEMATICS PERFORMANCE OF STANDARDS 9 AND 10 HG AND SG PUPILS, IN TERMS OF 3 COGNITIVE LEVELS (MOODLEY 1981)

	LOWER LEVEL ABILITIES	MIDDLE LEVEL ABILITIES	HIGHER LEVEL ABILITIES	MEAN	SD	N
Mean Attainment	58,5%	39,0%	29,75%	42,42%	18,0%	680
Mean z-score	0,89	-0,19	-0,71	0	1	
Mean T-score	59	48	43	50	10	

It was noted that through further statistical analysis, Moodley (1981: 134-135) found the difference between the mean scores for the 3 levels of abilities to be significant at the 0,001 level of confidence. A similar significance was also observed at each level of objectives for higher grade and standard grade, with the SG mean scores being significantly lower than the HG scores.

In addition to the above, it was observed that the abovementioned study also reported on the mean difficulty values (mean - scores) for the 3 levels of abilities, separately for standard 9 and for standard 10 (Moodley 1981:138). As the latter is of concern to this study, the mean scores obtained by standard 10 pupils are reflected in Table 3.3. N.B. Computed z-scores and T-scores are also indicated.

TABLE 3.3 MATHEMATICS PERFORMANCE OF STANDARD 10 HG AND SG PUPILS, IN TERMS OF 3 COGNITIVE LEVELS (MOODLEY 1981)

	LOWER LEVEL ABILITIES	MIDDLE LEVEL ABILITIES	HIGHER LEVEL ABILITIES	MEAN	SD	N
Mean Attainment	64,0%	43,75%	34,50%	47,42%	19,0%	297
Mean z-score	0,87	-0,19	-0,68	0	1	
Mean T-score	59	48	43	50	10	

3.4.2.2 Studies carried out by the HOD

In 1988, the HOD carried out an investigation entitled: An analysis of questions set in Mathematics HG and SG in the 1986 S.C. Examination (HOD 1988). The HOD (1989) conducted a similar second study in 1989, in respect of the mathematics papers set in its 1987 S.C. Examination. A study of the reports on these projects revealed pertinent information that could be further analysed so as to obtain pupil attainment in terms of the 3 categories of cognitive levels indicated above.

(A) Analysis of mathematics HG and SG performance in the 1986 S.C. Examination of the HOD

This study of the HOD (1988) involved the analysis of the mathematics performance of 1140 pupils (575 HG and 565 SG) from a sample of 41 secondary schools of the HOD, from Natal, Transvaal and the Cape Province. For purposes of the analysis, the raw marks obtained by pupils in each of the sub-questions set in the 4 papers (papers 1 and 2, HG and SG) in the HOD's 1986 S.C. Examination were used. As the facility index, amongst other statistics, had been computed for each question, information on pupils' mean performance became readily available. It is to be noted that, by definition in the HOD's report, the facility index represented the mean percentage score obtained by all pupils in each sub-question (Nuttal & Willmott 1972:22).

However, it was observed that the sub-questions were already classified as follows: L1 (knowledge and skills), L2 (comprehension), L3 (application) and L4 (analysis, synthesis and evaluation). Sub-questions belonging to the latter two levels were combined in order to obtain a three-category classification that was similar to the one used in this study. Once this was completed, the mean raw scores for each of the categories were computed and then converted to z-scores and T-scores. The findings arising out of the indepth analysis are summarised in Table 3.4.

TABLE 3.4 MATHEMATICS PERFORMANCE OF STANDARD 10 HG AND SG PUPILS, IN TERMS OF 3 COGNITIVE LEVELS, IN THE 1986 S.C. EXAMINATION OF THE HOD

	LOWER LEVEL ABILITIES	MIDDLE LEVEL ABILITIES	HIGHER LEVEL ABILITIES	MEAN	SD	N
Mean Attainment	70,0%	51,00%	36,84%	49,1%	18,46%	1140
Mean z-score	1,13	0,1	-0,66	0	1	
Mean T-score	61	51	43	50	10	

(B) Analysis of mathematics HG and SG performance in the 1987 S.C. Examination of the HOD

As stated earlier, an analysis similar to the abovementioned was undertaken by the HOD, also in respect of mathematics HG and SG in its 1987 S.C. Examination. The sample used in the second study (HOD 1989) comprised 510 HG pupils and 452 SG pupils, drawn from 34 HOD schools from different parts of the Republic of South africa. Since the procedures employed by the researcher in analysing the information contained in this study of the HOD were similar to those used in the above analysis, they are not repeated here.

The findings from the HOD's 1989 study are summarised in Table 3.5.

TABLE 3.5 MATHEMATICS PERFORMANCE OF STANDARD 10 HG AND SG PUPILS, IN TERMS OF 3 COGNITIVE LEVELS, IN THE 1987 S.C. EXAMINATION OF THE HOD

	LOWER LEVEL ABILITIES	MIDDLE LEVEL ABILITIES	HIGHER LEVEL ABILITIES	MEAN	SD	N
Mean Attainment	60,7%	48,7%	28,4%	46,9%	17,81%	962
Mean z-score	0,78	0,1	-1,04	0	1	
Mean T-score	58	51	40	50	10	

3.4.2.3 Study carried out by the Cape Education Department (CED)

During mid-1991, the Cape Education Department released a report on "An investigation into the results obtained by pupils in the S.C. Mathematics Examination", which was undertaken by its Research Section (Norton, 1991a). A scrutiny of the report revealed that various analyses and comparisons were made on aspects relating to standard 10 HG and SG pupils' performance in mathematics in the 1987 and 1988 S.C. Examination of the CED. It was observed, however, that detailed scores attained by pupils, in terms of the different levels of objectives, were provided for all 4 papers (2 HG and 2 SG) in respect of the CED's 1987 Senior Certificate Examination in mathematics.

For purposes of the 1987 study, the CED obtained information from 1597 pupils (415 mathematics HG, 635 mathematics SG, and 547 non-mathematics) drawn from a sample of 30 schools. However, as regards the detailed analysis of examination scripts, the CED utilised only 62 HG scripts (15%) and 73 SG scripts (11,5%), drawn on a random basis, from the 30 sample schools. Norton (1991a:13) compared the symbol distribution of the HG and SG sample with the respective distributions for the total of the 30 schools. Using the chi-square statistic, he found no significant differences between the sample distribution and the total distributions, which led him to conclude that the samples were deemed to be representative.

As regards the classification of the sub-questions set in mathematics in the 1987 S.C. Examination, in terms of the behavioural objective each was testing, the CED utilised the classification developed by Norton (1983), a description of which has been presented in paragraph 2.6.4.2 of Chapter 2.

Since the performance of pupils in the CED had to be analysed in terms of 3 category levels of behaviour (lower level abilities, middle level abilities and higher level abilities), for comparison purposes, the abovementioned classification was submitted to 5 mathematics specialists in the HOD (4 Heads of Department (Mathematics) and one Superintendent for Mathematics)). These educators were requested to group the different levels of objectives in Norton's (1983) classification to accord with the 3 categories of objectives used in this study. All these 5 specialists concurred in their decision that Norton's classification of objectives could be grouped in terms of the 3 broad cognitive levels used in this study as follows:

Lower level abilities	:	KS theory and KS process
Middle level abilities	:	KS/KC and KC process
Higher level abilities	:	KC two concepts and N type

The scores obtained by the pupils in the CED's 1987 Senior Certificate Examination in mathematics were then grouped and analysed in terms of the abovementioned 3 cognitive levels. It was noted that although symbol distributions for mathematics HG and SG for the CED's 1987 Senior Certificate Examination were included in the report, the mean and standard deviation for the combined performance were not provided. In order to obtain these statistics, the cumulative symbol distributions for HG and SG were first combined and then averaged. These percentages were then plotted on probability graph paper and the relevant statistics were determined.

The results obtained, in terms of the three ability levels, by the mathematics HG and SG pupils who wrote the 1987 S.C. Examination of the CED are summarised in Table 3.6

TABLE 3.6 MATHEMATICS PERFORMANCE OF STANDARD 10 HG AND SG PUPILS, IN TERMS OF 3 COGNITIVE LEVELS, IN THE 1987 S.C. EXAMINATION OF THE CED

	LOWER LEVEL ABILITIES	MIDDLE LEVEL ABILITIES	HIGHER LEVEL ABILITIES	MEAN	SD	N
Mean Attainment	71,5%	58,0%	40,4%	56,0%	19,5%	135
Mean z-score	0,80	0,10	-0,80	0	1	
Mean T-score	58	51	42	50	10	

3.4.3 Norms (criteria) established for mathematics attainment in respect of three categories of cognitive levels

In the foregoing section, the mathematics attainment of standard 10 HG and SG pupils in four studies was analysed with a view to establishing norms, in respect of the three categories of cognitive levels (LL, ML and HL) , against which the efficacy of the cognitive objectives, and hence the cognitive aims, of the senior certificate mathematics curriculum could be evaluated. In order to facilitate comparisons and the identification of emerging trends, the mathematics results (mean T-scores), for the three categories of cognitive levels, obtained in the abovementioned four studies are summarised in Table 3.7.

TABLE 3.7 MEAN T-SCORES OBTAINED IN THE FOUR STUDIES BY STANDARD 10 HG AND SG PUPILS IN MATHEMATICS, IN TERMS OF 3 COGNITIVE LEVELS

STUDY	LOWER LEVEL ABILITIES	MIDDLE LEVEL ABILITIES	HIGHER LEVEL ABILITIES	N
Moodley (1981)	59	48	43	680
HOD (1986 S.C. Exam.)	61	51	43	1 140
HOD (1987 S.C. Exam.)	58	51	40	962
CED (1987 S.C. Exam.)	58	51	42	135
Average of 4 studies	59	50,3	42	729

An examination of the above table reveals that, in the first instance, the mean T-scores in respect of the three categories of cognitive levels (LL, ML and HL) obtained in each of the studies reviewed are more or less similar. This trend is evident especially when the mean T-scores of each study are compared with the respective overall averages for the four studies combined. This observation is particularly significant for the following reasons: The test designed and used by Moodley (1981) in his study did not consider the contents per se of the senior certificate mathematics curriculum in use at the time. Yet the results obtained from his study compare favourably with those obtained in the other studies, which were undertaken later. Pupils who wrote the HOD's senior certificate examination in mathematics in 1986 and in 1987 were examined on different curricula. It is to be noted that a new senior certificate mathematics curriculum (with revised aims and contents), which was introduced in standard 9 in 1986, became operative at the standard 10 level as from 1987. Despite the changes in curricula, it is observed that the results obtained, in respect of the three categories of mathematical abilities, in the HOD in 1986 and in 1987 are also comparable. The CED is an independent examining authority and, as such, sets its own senior certificate examination papers. Despite different examinations in mathematics being set by the CED and the HOD in 1987, it is observed that the results obtained from the CED's study, in terms of the three cognitive levels, accord more or less with those obtained in the HOD in the same year as well as with the results obtained in each of the other studies reviewed.

Returning to Table 3.7, in addition to observing that the results of the 4 studies are more or less similar, it is observed that the computed overall averages for the three categories of mathematical abilities (LL, ML and HL) are 59, 50,3 and 42 respectively. Since averages smoothen out certain inconsistencies, it can be concluded that the mean T-scores of LL = 59, ML = 50,3 and HL = 42 represent the group norms of senior certificate mathematics pupils' attainment (HG and SG combined) in terms of these three broad categories of mathematical abilities. An interesting observation is that the above T-scores are approximately equal to z-scores of +1, 0 and -1 respectively.

In summary, then, the norms developed for mathematical attainment in respect of three categories of cognitive levels can be stated as follows:

In any senior certificate mathematics examination, which comprise questions testing the lower, middle and higher levels of mathematical abilities, the expected mean attainment of standard 10 HG and SG pupils in terms of these three cognitive levels should be as follows:

1) <u>For the lower level abilities (knowledge and skills)</u>

Pupils should attain a minimum mean T-score of 59.

2) For the middle level abilities (comprehension/understanding)

Pupils should attain a minimum mean T-score of 50,3.

3) For the higher level abilities (application, analysis , synthesis and evaluation/creative)

Pupils should attain a minimum T-score of 42.

(NB: T-scores are computed using the mean and standard deviation of the overall test (examination))

3.5 METHOD TO BE USED FOR DETERMINING THE ATTAINMENT OF THE COGNITIVE OBJECTIVES

It is interesting to note that the same mean norm values for the three cognitive levels (LL=59, ML=50,3 and HL=42) are also obtained when the mean percentages obtained in the studies (tests) are averaged and then converted to standard scores, as can be observed in Table 3.8.

TABLE 3.8 MEAN PERCENTAGES AND CONVERTED STANDARD SCORES OBTAINED IN THE FOUR STUDIES BY STANDARD 10 HG AND SG PUPILS, IN TERMS OF 3 COGNITIVE LEVELS

STUDY	LOWER LEVEL ABILITIES	MIDDLE LEVEL ABILITIES	HIGHER LEVEL ABILITIES	$\overline{M}$ (%)	SD(%)	N
Moodley (1981)	64,0	43,75	34,50	47,42	19,0	297
HOD (1986 S.C. Exam.)	70,0	51,00	36,84	49,10	18,46	1 140
HOD (1987 S.C. Exam.)	60,7	48,70	28,40	46,90	17,81	962
CED (1987 S.C. Exam.)	71,5	58,00	40,40	56,0	19,50	135
Average of 4 tests	66,6	50,40	35,04	49,90	18,70	729
Mean z-score	0,9	0,03	-0,80			
Mean T-score	59	50,3	42			

From the above table, it can also be observed that the norm (average) values for the three cognitive levels have been derived from what can be regarded as a single test (representing the average of the scores obtained from four tests) with the following statistics: $\overline{M}=49{,}9\%$, SD=18,7% and N=729. The computed standard error of the mean of the above single test is 0,69.

However, in this study, the hypotheses in respect of the cognitive objectives are to be tested at the 0,01 level of significance. Taking this into consideration, the standard error of mean (discussed in Appendix 8) yields a confidence interval for the mean as follows:

$49{,}9 \pm (2{,}58)\ (0{,}69)\ \%$ $\qquad (p < 0{,}01)$

i.e. $49{,}9 \pm 1{,}8\ \%$

i.e. (51,7% to 48,1%)

The computation of the relevant standard scores using the above two mean values yields the following confidence intervals (at the 0,01 level of significance) for each of the norm categories:

LL : T-scores of 59 $\pm$ 1 (58 to 60) ($p < 0,01$)

ML : T-scores of 50,3 $\pm$ 1 (49,3 to 51,3) ($p < 0,01$)

HL : T-scores of 42 $\pm$ 1 (41 to 43) ($p < 0,01$)

The minimum T-scores of LL=58, ML=49,3 and HL=41 would be taken as the critical norm values for these categories in order to determine the attainment of the cognitive objectives in terms of pupils' overall performance as well as in terms of certain variables. For example, should it be found that the computed mean T-score for the lower level abilities category, obtained by standard 10 HG and SG pupils in the 1991 senior certificate examination of the HOD, is equal to or greater than the critical norm value for that category (58), then it would be concluded that senior certificate candidates are adequately attaining the lower level objectives, namely, the knowledge and the skills objectives of the senior certificate mathematics curriculum. Conversely, if the computed mean T-score is less than the critical norm T-score for the relevant category of cognitive objectives, then it would be concluded that senior certificate pupils are <u>not</u> attaining the said category of cognitive objectives to the desired (expected) level. The successful attainment of a category of cognitive abilities would be interpreted as all the various levels of objectives included in the category having been attained.

It is to be noted that the above norm values can be applied to HG pupils and SG pupils as separate groups, except that the T-scores must be calculated separately for each grade, using its respective mean and standard deviation.

The above expositions have been used as theoretical constructs in this study.

3.6 METHOD TO BE USED FOR DETERMINING THE ATTAINMENT OF THE COGNITIVE AIMS

As reflected in paragraph 3.3.4, each of the cognitive aims (and/or their sub-parts) of the senior certificate mathematics curriculum can also be expressed, according to the cognitive behaviours inherent in them, in terms of the same three categories of cognitive ability levels used to classify the cognitive objectives, namely, LL, ML and HL. In view of this, pupils' attainment in terms of these three categories of cognitive objectives, in the senior certificate examination can therefore be related to their attainment of the aims of the senior certificate mathematics curriculum.

Educators are generally in agreement that since specific aims are somewhat broad and general, the attainment of such aims <u>per se</u> cannot be measured quantitatively. This implies that one therefore has to examine the attainment of the objectives in order to gain information that would assist in extrapolating and, hence, making judgements on the attainment of the specific aims. Such an approach becomes meaningful when seen in relation to the assertion made by Moodley (1975:14) who states that there exists a two way approach in terms of derivation of objectives and attainment of aims:

1. the derivation of objectives points to a movement from aims to objectives; and
2. the attainment of aims points to a movement from attainment of objectives to the attainment of the aims.

A similar view is expressed by Gunter (1990:102) who mentions that every objective serves as a necessary step or means towards the achievement of a wider specific aim, which in turn contributes towards the attainment of the final goal. The above assertions imply that an increase in the attainment of the number or different levels of objectives comprising an aim brings one closer to the realisation of the said aim.

From Table 3.1 in paragraph 3.3.4, it can be observed that some of the aims (those numbered 1, 4 and 6) are each concerned with a single category of cognitive abilities while the remainder comprises cognitive behaviours falling into more than one of the three categories of cognitive ability levels. If, in this study, it is found that senior certificate candidates are adequately attaining, say, the LL and ML categories of cognitive objectives, then it would be concluded that all the aims or sub-parts identified as falling into the LL and/or the ML categories are being adequately realised by the pupils. On the other hand, if pupils' attainment in, say, the HL category of cognitive objectives is found to be significantly lower than the norm score, then such a finding would also be taken to indicate that the cognitive aims or sub-parts classified HL are not being adequately realised. In general, then, in order to draw the conclusion that a cognitive aim is being adequately realised, pupils' achievement in the senior certificate mathematics examination must reflect a successful attainment of the various categories of cognitive abilities comprising the aim. **The above procedure for inferring the attainment of the cognitive aims represents another theoretical construct used in this study.**

From the foregoing discussions, it can be observed that for purposes of this study, the evaluation of the cognitive objectives and aims of the senior certificate mathematics curriculum is to be based mainly on pupils' achievement in mathematics in the 1991 senior certificate examination. The decision to use pupils' achievement in mathematics in the senior certificate examination is as follows:

* The senior certificate examination is a public examination. Hence, all standard 10 pupils in the HOD write common mathematics papers (tests) in the respective grades.

* The above tests are considered to possess a fairly high degree of content validity and reliability. (These aspects have been discussed in paragraphs 2.7.9.1. and 2.7.9.2 of Chapter 2).

While this study aims to ascertain pupils' attainment in terms of three broad categories of cognitive ability levels (LL, ML and HL), it was realised that a more comprehensive picture regarding the attainment of the individual levels of objectives may be required, especially if it is found that one or more of the three broad categories of cognitive abilities are not being attained to the desired or expected level of performance. This necessitated the construction and use of a teacher questionnaire, details of which are discussed hereunder. It needs to be emphasised that the teacher questionnaire was used to provide supplementary information to the quantitative analysis undertaken in this study.

3.7 DEVELOPMENT OF THE TEACHER QUESTIONNAIRE

As additional information was required in respect of pupils' attainment of the various cognitive objectives, in order to provide an indepth interpretation of the findings from the quantitative analysis, it was decided to elicit responses from teachers of senior certificate mathematics classes, by way of a questionnaire. Behr (1973:150) states that, if properly administered, the questionnaire continues to be the best available instrument for obtaining information from widely spread sources.

3.7.1 Construction of the teacher questionnaire

Popham (1981) avers that a questionnaire should be constructed according to certain principles. Taking cognisance of the above, the construction of the teacher questionnaire used in this study was guided largely by the general principles proposed by Mouly (1970), Moser and Kalton (1971) and Cohen and Manion (1980), the more important principles being as follows: the length of individual questions, the number of response options, and the format and wording of questions (question content, question format, question order, question type, question formulation and question validity).

A great deal of time and thought was spent on the construction of the teacher questionnaire. The researcher undertook to make the questionnaire as simple and straightforward as possible in order to be easily understood. Care was taken to ensure that the questions were free from ambiguity, vagueness and technical defects in the language used. The answering of the questionnaire was facilitated by the use of mostly closed-ended questions, which used a 5-point Likert scale or 3-point scale and which required respondents to indicate their responses by means of crosses.

The items relating to the attainment of the cognitive outcomes included in the teacher questionnaire were drawn from the list of cognitive outcomes identified after analysing the cognitive aims of the senior certificate mathematics curriculum (see paragraph 3.3.3). After the initial questionnaire was compiled, the researcher submitted this questionnaire to three of his colleagues with a research background. Their few but pertinent and useful comments were used to improve the questionnaire. Thereafter, the questionnaire was submitted to five mathematics experts (one Superintendent of mathematics and four Heads of Department of mathematics (senior teachers of standard 10 classes)) who were requested to, amongst others, assess the items and to check whether the cognitive objectives included in the questionnaire reflected a fair representation of the cognitive abilities pursued in the standard 10 classes. The positive comments received from these experts confirmed the appropriateness of the instrument. The researcher finally discussed the questionnaire with his promoter who recommended the inclusion of examples in certain of the questions, for purposes of greater clarity. These recommendations, which were incorporated into the questionnaire, helped to further refine the teacher questionnaire.

In summary, then, on the basis of discussions/consultations with three research colleagues, five experts in mathematics and the promoter, the final teacher questionnaire was generated (see Appendix 2). The questionnaire used in this study comprised 52 questions. While questions of a demographic nature were included in the questionnaire, in order to determine some of the characteristics of the group canvassed, the major emphasis in the questionnaire focused on the views of senior certificate mathematics teachers on pupils' attainment of the various cognitive objectives and aims as well as on the affective aims.

The teacher questionnaire was sub-divided into eight sections (see Appendix 2) as follows:

- A. General: Name of school and number of HG/SG pupils offering mathematics (Items A: 1 and 2).

- B. Personal Information: Biographical data of respondents (Items B: 3-12).

- C. General guidelines and definitions.

- D. Standard 10 mathematics HG/SG pupils' abilities in terms of objectives (Items D1: 1 to D3: 18).

- E. Standard 10 mathematics HG/SG pupils' abilities in terms of:

  * three groups of cognitive outcomes (Items E1: 19-21); and
  * four content areas (Items E2: 22 to 25).

- F. Standard 10 mathematics HG/SG pupils' attitude towards and interest in mathematics (items F: 26 and 27).

- G. Standard 10 mathematics HG/SG pupils' attainment of certain aims of the senior certificate mathematics curriculum (Items G: 28-35).

- H. Information pertaining to the senior certificate mathematics HG/SG syllabus. (Items H: 36-38).

The items in section H were the only open-ended questions included in the questionnaire. Responses in their regard were considered to be essential for the indepth interpretation of the quantitative analysis as well as for making recommendations in respect of curriculum development in mathematics for the future.

3.7.2 Pilot study

The pilot study, also referred to as pilot testing, is the preliminary or "trial run" investigation that is carried out before the execution of the main investigation. While Cohen and Manion (1980:70) list several purposes that a pilot study serves, it is clear that two important purposes of a pilot study is to determine how the design of the main study can be improved as well as to identify shortcomings in the instrument.

The researcher conducted a pilot testing of the questionnaire by administering it to 8 teachers of senior certificate mathematics classes from 8 different secondary schools (which were not included in the sample used in the main study). Pertinent in this regard is the assertion made by Nisbet and Entwistle (1970:39) who mention that the pilot testing is done with a sample which is similar to the group from which the sample will be selected. Space was provided at the end of the pilot questionnaire for respondents to make comments on any difficulties experienced in completing the questionnaire. Almost all of the respondents tendered a return of "no comments" which suggested that the majority of the respondents experienced no problems with the instructions as well as with the questions. Hence, through the use of the pilot study, the researcher was satisfied that the questions asked in the questionnaire were generally meaningful and unambiguous, since clear responses were received from the respondents.

3.7.3 Reliability and validity of the teacher questionnaire

No attempt was made by the researcher to produce item analysis data for the questions included in the teacher questionnaire. However, the researcher argues that the validity and the reliability of the questionnaire must be judged from the several considerations and painstaking procedures adopted in compiling the final questionnaire as well as in its administration (see Chapter 4). As stated in paragraph 2.7.9.1 of Chapter 2, three types of validity can be distinguished, namely, content validity, criterion-related validity and construct validity. With regard to the teacher questionnaire used in this study, only content validity was found to be applicable. Content validity for the teacher questionnaire was established by 5 experts in mathematics who assessed the items in the questionnaire for the theoretical aspects of this study (see paragraph 3.7.1). Furthermore, since the respondents were assured of their anonymity, a high degree of honesty was expected in their responses.

3.7.4 Method to be used in interpreting data obtained from the teacher questionnaire

As teachers would be rating pupils' proficiency in a greater number of cognitive abilities (objectives), the mean responses could be scrutinised with a view to identifying those objectives that are being attained to a higher degree than the others. Several research studies have demonstrated that teachers' qualitative assessment of pupils' achievement are fairly reliable. For example, Norton (1991a:65) observed a high correlation between teachers' assessment of pupils' overall attainment in mathematics and the latter's performance in the S.C. examination. Similarly, high correlations were recorded by Moodley (1981:216) when he compared teachers' rating of pupils and pupils' achievement in 5 different levels of objectives. In this regard, Moodley (1981) also observed that the mean responses of teachers, on a 5 - point scale, for the 5 levels of objectives were as follows: (NB: The mean responses in terms of the 3 categories - LL, ML and HL - were computed by the researcher).

LEVEL OF OBJECTIVE	MEAN RESPONSE	SD	COMPUTED MEAN RESPONSE I.T.O. 3 CATEGORIES
Knowledge Skills	2,51 2,34	0,89 0,89	LL - 2,43 (2,57)*
Comprehension	2,19	0,88	ML - 2,19 (2,81)*
Selection - Application Analysis - Synthesis	2,06 1,90	0,91 0,87	HL - 1,98 (3,02)*

***Values if the scale used ranges from 1 for "excellent" to 5 for "poor".**

The main purpose of obtaining inputs from teachers, regarding pupils' attainment of the objectives in mathematics, is to provide indepth information in respect of each of the categories identified in the aims. However, a comparison would be made between the teachers' mean responses obtained in this study with those obtained by Moodley, in terms of the 3 broad categories of cognitive ability levels, in order to evaluate teachers' rating of S.C. pupils' attainment of the cognitive aims and objectives.

3.8. <u>RESEARCH INSTRUMENT TO BE USED TO MEASURE STANDARD 10 PUPILS' ATTITUDE TOWARDS MATHEMATICS</u>

3.8.1 The survey of literature also revealed that Moodley (1981) developed and used an attitude scale in mathematics for standard 9 and 10 pupils. The scale, which comprised 6 sub-scales (with the items interdispersed) aims to investigate pupils' attitude towards mathematics in terms of the following 6 aspects:

1. Mathematics teaching
2. School learning
3. Difficulties in learning mathematics
4. Place or importance of mathematics in society
5. School and life in general
6. Enjoyment of mathematics

The scale, which is a Likert-type scale, comprises a total of 48 items (8 items per sub-scale). Pupils are required to respond to each item by indicating whether they agree, disagree or are uncertain. Each item is scored on a 3-point scale (2,1,0) with unfavourable or negative items being scored in the reverse. Thus the maximum total score of the scale is 96. Moodley (1981) provides the following 3 levels, based on total scores on the scale, to describe pupils' attitude:

77 - 96	High
67 - 76	Middle (moderate)
0 - 66	Low

The other important statistics regarding the scale are:

Reliability (r)	:	0,67
Spearman - Bowman Correction - r	:	0,80
Alpha Coeff. (reliability)	:	0,804

Criterion Validity : 0,70

3.8.2 For purposes of this study, the abovementioned scale (see Appendix 3) has been used to measure the attitude of standard 10 pupils towards mathematics. **A minimum mean score of 77 would be taken as the norm score to assess S.C. pupils' adequate attainment of this aim.**

3.9 RESEARCH INSTRUMENT TO BE USED TO MEASURE STANDARD 10 PUPILS' INTEREST IN MATHEMATICS

3.9.1 A survey of literature revealed that, in the absence of an appropriate interest scale, Husén (1967b:102) in his IEA study in mathematics, measured "interest in mathematics" by deriving a composite score (index) using the following variables (items) contained in his pupil questionnaire (Husén 1967a:212 and 292-4).

1. Desired occupation - scientific vs non-scientific.
   -What occupation would you like to enter?

2. Wishes to take additional mathematics courses.
   -Would you like to take more mathematics courses after this year? (check one) YES/NO.

3. Best liked subjects.
   -Which two school subjects have you liked most?

4. Least liked subjects.

   -Which 2 school subjects have you liked least?

5. Best subjects - highest grades.

   -In which 2 school subjects do you do (perform) best?

6. Worst subjects - lowest grades.

   -In which two school subjects do you do (perform) worst?

Husén (1967a:212-213) derived the numerical index to represent interest in mathematics in the following way:

1. If the "desired occupation" is mathematics related, score one point for level of interest in mathematics, otherwise, no point is given.

2. If pupil indicates he "wishes to take additional mathematics courses", score one point (+1) for level of interest in mathematics, otherwise, score no points.

3. If the "best liked subjects" are mathematics courses, score one point (+1) for each mathematics course cited, otherwise score no points.

4. If the "least liked subjects" are mathematics courses, score minus one point (-1) for each mathematics course cited, otherwise score no points.

5. If the "best subjects - highest grades" are mathematics courses, score one point for each mathematics course cited, otherwise no points.

6. If the "worst subjects - lowest grades" are mathematics courses, score minus one point (-1) for each mathematics course cited, otherwise, no points.

7. Total the number of points - positive and negatives. Add a constant of 5 (to eliminate negative scores).
   This score is the index of level of interest in mathematics .

NB: The interest scale has a maximum of 10 points.

In his study, Husén (1967a) found that the national mean index of level of interest in mathematics, using the above scale, was 6,2 in the case of standard 10 pupils.

3.9.2 The abovementioned scale has been used in this study to obtain an index of level of interest in mathematics of standard 10 pupils. The scale was included in Section C in Appendix 3. The index 6,2 would be used as a norm value to assess senior certificate pupils' adequate interest in mathematics. **A mean interest index of 6,2 or greater would indicate that the aim concerned with the development of interest is being adequately realised by senior certificate pupils.**

3.10 CONCLUSION

In this chapter, attention has been given to the development/identification of research instruments and norms that are to be used in the study. Once these had been finalised, the samples (schools, pupils and teachers) had to be chosen. Also, consideration had to be given to the administration of the research instruments. A discussion of these aspects is presented in the next chapter.

CHAPTER 4

SELECTION OF SAMPLES, ADMINISTRATION OF TEST AND RESEARCH INSTRUMENTS AND HANDLING OF DATA

4.1 SAMPLING

4.1.1 Definition of target population

The target population in this study was defined as: All senior certificate (standard 10) pupils offering mathematics higher grade and standard grade in Indian secondary schools in the Republic of South Africa.

Pupils at the standard 10 level were specifically chosen as the senior certificate mathematics curriculum, which spans two years, namely standards 9 and 10, culminates in the standard 10 year when pupils write an external examination (the Senior Certificate Examination) in the various subjects, including mathematics.

4.1.2 Selection of samples

In view of the unequal distribution of the Indian population, and hence Indian schools in South Africa, it was decided that the most appropriate technique to ensure a representative sample in this study was stratified random sampling. In this regard, Behr (1973:9) mentions that stratified sampling, which is based on a number of controls, involves applying sampling ratios to sub-groups within the population under investigation. The choice of the stratified sampling technique also ensured that, amongst others, any generalisations drawn from the study would have wider implications and applicability.

4.1.2.1 Selection of schools

Altogether, 129 secondary schools of the HOD, distributed over three provinces (Natal, Transvaal, Cape) were identified to have offered mathematics in the senior certificate examination in 1991. At the outset, it was decided to use approximately 25% of the schools to form the sample. After the schools were stratified in terms of provinces as well as in terms of regions/areas within each province, 31 secondary schools were selected on a random basis to constitute the sample group. Table 4.1 (see next page) reflects the distribution of population and sample schools according to provinces and regions.

The methods used in the selection of the sample schools were as follows:

Step 1: The total number of schools in each of the 3 provinces (Natal, Transvaal and Cape) that offered mathematics at the senior certificate level in 1991 was determined. As a sample size of approximately 25% was to be used, the relevant proportion was calculated for each province which yielded the following numbers: Natal - 23 schools, Transvaal - 6 schools and Cape - 2 schools.

Step 2: Each province was then further stratified into regions/areas as follows: Natal - 7 regions, Transvaal - 4 regions and Cape 2 - regions. The division of Natal into regions was generally based on the geographic demarcations. However, Phoenix and Chatsworth and their surrounding areas (situated in the Greater Durban Area) as well as Pietermaritzburg (situated in the Midlands of Natal) were identified as separate regions because of their high concentration of Indians.

TABLE 4.1 DISTRIBUTION OF POPULATION AND SAMPLE SCHOOLS ACCORDING TO PROVINCES AND REGIONS/AREAS

			Situation of Schools	
Province and Region/Area	No. of Sec. Schools	No. of Schools chosen	URBAN	RURAL/PERI URBAN
NATAL				
1. Natal South Coast	8	2	-	2
2. Merebank, Chatsworth, Shallcross, Marianhill	23	5	5	-
3. Durban Central	8	2	2	-
4. Phoenix and surrounding area	23	6	6	-
5. Natal North Coast	14	3	1	2
6. Pietermaritzburg	8	2	2	-
7. Northern Natal and the rest of Natal Midlands	13	3	-	3
Sub Total (Natal)	97	23	16	7
TRANSVAAL				
8. Johannesburg - Lenasia	6	1	1	-
9. Pretoria - Laudium	5	1	1	-
10. Benoni - Springs	3	1	1	-
11. Rest of Transvaal	13	3	-	3
Sub Total (Transvaal)	27	6	3	3
CAPE PROVINCE				
12. Western Cape	2	1	1	-
13. Eastern Cape	3	1	1	-
Sub Total (Cape)	5	2	2	-
TOTAL (RSA)	**129**	**31**	**21**	**10**

In the Transvaal, the Indian population is concentrated mainly in 3 regions: Johannesburg - Lenasia, Pretoria - Laudium and Benoni - Springs. The remainder is distributed sparsely over the rest of the Transvaal. Hence the Transvaal was divided into 4 regions according to the above. In the Cape, small populations of Indians are concentrated in Cape Town (Western Cape) and East London and Port Elizabeth (Eastern Cape). Hence two regions were identified in the Cape.

Step 3: The total number of secondary schools in each of the 13 regions was determined. By random sampling, the sample schools from each of the regions were selected until the desired proportion for each region, and consequently the province as a whole, was reached.

As a result of the above, 31 secondary schools were identified to form the sample group, which comprised 21 urban and 10 rural schools. As can be observed from the above and from Table 4.1, the distribution of the sample schools indicates a reasonably good geographic coverage of HOD schools in all 3 provinces and hence was considered as adequately representative of the target population defined in this study.

4.1.2.2. Selection of pupils for determining the attainment of the cognitive objectives and aims

The quantitative analysis of the pupils' performance in terms of the different cognitive abilities formed the major portion of this investigation. In view of this, the total population of standard 10 mathematics HG and SG pupils from the 31 sample schools who wrote the 1991 Senior Certificate Examination of the HOD was included in this part of the study. This number totalled 2 091, comprising 760 HG pupils and 1 331 SG pupils.

4.1.2.3 Selection of pupils for determining the attainment of the affective aims and objectives

For purposes of evaluating the attainment of the affective aims, a sample of standard 10 pupils from the abovementioned 31 schools who offered mathematics at the senior certificate level in 1992 was used. According to Professor Swanepoel, Senior Professor at the Institute for Educational Research at UNISA, the use of separate samples in a study of this nature, which does not aim at establishing relationships between variables, is acceptable. Initially it was decided to use about 20 pupils, selected on a random basis, from the total number of pupils who offered mathematics HG and SG at each of the 31 sample schools. However, it was realised that this could disrupt all the standard 10 mathematics classes at the schools, a situation which previous experience had revealed is not favoured by principals and teachers. The other available alternative was selection by class units. After determining the number of standard 10 mathematics HG and SG class units at each of the schools, from the staff-returns submitted by schools to the HOD, and after confirming these numbers telephonically with the principals concerned, one standard 10 mathematics class unit was randomly selected from each of the schools. This resulted in 200 HG pupils and 288 SG pupils being chosen, yielding a total of 488 pupils. According to Fowler (1984:40), the vast majority of survey samples involve very small fractions of populations and that small increments in the fraction of the population included in a sample will have no effect on the ability of a researcher to generalise from a sample to a population. He adds that a sample of 150 people will describe the population of 15 000 or 15 million with virtually the same degree of accuracy, assuming all other aspects of the sample design and sampling procedures were the same (Fowler 1984:41).

The final sample of pupils (n=488) selected for this part of the investigation represented 19,9% of the total population of standard 10 mathematics HG and SG pupils at the 31 schools in 1992. Furthermore, it was observed that the ratio HG:SG in the sample (41,0% : 59,0%) was almost equal to that in the total population at the 31 schools (41,8% : 58,2%). The above indicates the consistency in sampling and the representativeness of the sample in terms of grades.

4.1.2.4 Selection of teachers for obtaining qualitative information on the attainment of the cognitive and affective aims and objectives

Altogether, 84 teachers taught mathematics HG and/or SG at the standard 10 level at the 31 sample schools in 1992. It was ascertained that all but 4 of these teachers also taught mathematics at the same schools in 1991. Hence, the 80 teachers who taught mathematics at the standard 10 level at the 31 sample schools in 1991 and 1992 constituted the final sample of teachers in this study. Whilst 36 of these teachers taught HG classes, the remaining 44 taught SG classes.

4.1.2.5 Selection of teachers for determining the cognitive levels of questions set in the 1991 S.C. Examination of the HOD

For purposes of analysing the senior certificate pupils' mathematics results in terms of the different levels of cognitive objectives, it was necessary to obtain the cognitive level of each of the sub-questions set in the 1991 Senior Certificate Examination mathematics papers. This data could not be obtained from the HOD as the draft question papers were disposed of. Attempts to obtain this information from the examiners also proved unsuccessful. Consequently, the researcher had to make alternative arrangements. Hudson (1973:22) states that since the classification of an item to a particular objective is a fairly subjective process, we are consequently obliged to rely on consensus judgement of the experienced teachers in the field - if the majority of the teachers agree that a certain question measures say comprehension, for the majority of the candidates, then we can accept that question as one testing comprehension.

With the help of one of the Superintendents of Education (Mathematics) from the HOD, seven senior mathematics teachers were identified and used for the classification of the items in terms of cognitive abilities. In view of the special expertise that was required for this exercise, the selection of appropriate teachers could not be done strictly on a random basis. However, teachers were drawn from schools spread over a wide area along the coast of Natal, from Port Shepstone in the south and Stanger in the north, including schools in the Greater Durban area.

4.2 ADMINISTRATION OF TEST (EXAMINATION)

As stated previously, the researcher did not devise and administer his own test, but made use of an instrument, and the results flowing therefrom, set by the HOD. Hence for purposes of evaluating pupils' attainment of the cognitive objectives, use was made of pupils' mathematics results obtained in the 1991 Senior Certificate Examination of the HOD. The test (examination) was administered during November/December 1991 by the HOD and the answer scripts were marked by senior mathematics teachers from HOD schools. The relevant answer scripts of the 2 091 HG and SG pupils from the 31 sample schools were made available to the researcher during June 1992.

Candidates who offered mathematics HG and SG in the 1991 Senior Certificate Examination of the HOD wrote two papers - HG papers 1 and 2 or SG papers 1 and 2 (see Appendix 4), in keeping with the examination requirements of the HOD (1975:12). Paper 1 consisted of questions on algebra and calculus while paper 2 comprised questions on Euclidean geometry, analytical geometry and trigonometry. Questions testing the different cognitive abilities - ranging from knowledge questions to higher ability questions - were included in both the papers. The HG papers had a maximum mark of 200 each while each of the SG papers carried a maximum mark of 150. The time allocated to answer each paper was three hours.

Ten questions each were set in three of the question papers (HG paper 1 and paper 2 and SG paper 1) while nine questions were set in SG paper 2.

Each question consisted of 2 or more sub-questions and in many cases, the sub-questions were further broken down into sub-parts. In terms of the mark allocation, and for the purposes of the analysis, the following numbers of sub-questions (which include sub-parts of sub-questions) were identified for each of the 4 examination papers.

QUESTION PAPER	NO. OF SUB-QUESTIONS	TOTAL IN RESPECT OF GRADES
Maths HG P1 Maths HG P2	48 42	 90
Maths SG P1 Maths SG P2	49 38	 87

4.3 ADMINISTRATION OF OTHER RESEARCH INSTRUMENTS

After the relevant samples had been chosen and the final questionnaires had been prepared, the next step was to administer the questionnaires. In this study, a teacher questionnaire and a pupil questionnaire (attitude and interest scales) were used to gather information relating to the cognitive and affective objectives. A second teacher questionnaire, referred to as a "teacher schedule", was used to gather information on the cognitive levels of questions. What follows is a brief discussion of the procedures adopted in the administration of the questionnaires and the teacher schedule.

4.3.1 Preliminary arrangements

Since educators and pupils in HOD schools were used in this study, the prior approval of the Chief Executive Director was sought, amongst others, to approach principals of the sample schools with a view to administering the questionnaires. The letter from the HOD, granting approval for the above, is reflected in Appendix 6.

Once the necessary permission had been obtained, the principals of the sample schools were contacted telephonically. After discussing the details regarding the research project, including the participation of pupils and teachers, the co-operation of principals was sought in respect of the administration/handling of the research materials at their schools. At the same time, the numbers of standard 10 mathematics HG and SG teachers at these schools as well as the numbers of pupils in the mathematics class units selected from these schools were ratified. Principals of the sample schools in the Transvaal and the Cape were informed that, in view of the time and cost constraints, the research materials , including a covering letter explaining the nature and scope of the research as well as the procedures to be followed in administering the questionnaires (see Appendix 1), would be posted to them. Principals of the sample schools in Natal were informed of the researcher's intention to visit their schools, and suitable times were mutually agreed upon.

4.3.2 Administration of questionnaires

The principals of the 31 sample schools, or their appointed deputies, assisted with the administration/handling of the pupil and the teacher questionnaires. During the initial telephonic discussion with the principals, it was requested that the pupil questionnaire be administered to the pupils as a group under the supervision of a teacher (either the mathematics teacher or the guidance counsellor of the school). All the principals readily co-operated in this regard.

4.3.2.1 The pupil questionnaire (attitude and interest scales) - (see Appendix 3)

In the covering letter attached to the questionnaire, pupils were urged to complete the questionnaire with great care and to be as frank as possible in their responses. In the preamble of the attitude scale, it was mentioned that the scale consisted of some statements related specifically to mathematics while others were of a general nature. It was emphasised that there were no right or wrong answers to the statements and that they had to respond by indicating how they personally felt about the statements, in terms of the 3-point scale (2 = agree, 1 = uncertain and 0 = disagree).

The interest scale, which comprised 6 open-ended questions, was also included in the pupil questionnaire (see Section C of Appendix 3). Pupils were requested to answer these questions according to the information requested. Judging by the almost 100% returns received, the pupil questionnaire had been successfully administered.

4.3.2.2 The teacher questionnaire - (see Appendix 2)

As stated in paragraph 3.7.1 of Chapter 3, besides the biographical data of teachers, the teacher questionnaire was designed to elicit information on the teachers' assessment of pupils' (1) ability in respect of the different cognitive levels, (2) ability in terms of content areas, (3) overall attitude towards and interest in mathematics, and (4) attainment of the different cognitive aims of the mathematics curriculum. In addition, teachers were requested to indicate the topics/sections that pupils generally had difficulty in understanding, topics in the syllabus they consider unsuitable for senior certificate pupils, and any new topics that ought to be included in the syllabus.

Certain guidelines/explanations were provided in the teacher questionnaire. Teachers were requested to interpret the word "ability" as the general mathematical ability of the majority of the pupils and to take into account pupils' global ability which was to be based on their performance in class exercises, tests, examinations, homework exercises and assignments. In rating the abilities of pupils, teachers were urged to consider not only their present pupils but also the standard 10 pupils they had taught previously, particularly in 1991.

The standard 10 mathematics teachers were required to complete teacher questionnaire HG or SG depending on whether they were teaching mostly higher grade pupils or standard grade pupils. Although only the HG teacher questionnaire is included in the Appendices, it is to be noted that both the HG and SG questionnaires were identical, except that the term "HG" was replaced with "SG", where applicable, in the SG teacher questionnaire.

All 80 teachers included in the sample completed and returned the teacher questionnaire.

4.3.3. Despatch and return of questionnaires

Once the number of pupils in each of the mathematics class unit selected and the number of teachers handling standard 10 mathematics HG and SG classes at each of the sample schools were ratified, the pupil and teacher questionnaires for each school were put into separate jackets, with the appropriate numbers and grade clearly indicated on each jacket. These were then put into envelopes which also contained a copy of the letter of authorisation from the Chief Executive Director and the letter for principals, as well as a stamped, self-addressed envelope for the return of the materials. Included in the principals' letter were, amongst others, guidelines on the handling/administration of the questionnaires, the number of teacher and pupil questionnaires enclosed and the return date for the submission of the research materials.

While it is acknowledged that personal visits to schools elicit a greater degree of co-operation from educators, this was not possible for schools in the Transvaal and Cape, because of time and cost constraints. Hence all the research materials were posted to the principals of the sample schools in these two provinces. Rossi (1983:198) mentions that apart from the fact that low cost makes the mail questionnaire an obvious choice, it is most definitely a good means of collecting data.

Visits were made to all 23 sample schools situated in Natal and the materials were handed personally to the principals. In cases were it was possible and on the recommendation of the principals, the researcher met with the Head of Department /Senior Teacher in charge of mathematics and, in some schools, the entire team of standard 10 mathematics teachers. In order to ensure confidentiality, teachers were requested to seal their completed questionnaires in the envelopes provided, before handing them to the principals. The principals collected all the research materials in respect of their schools and returned them, per post, to the researcher.

The researcher had provided his home and office telephone numbers and principals and teachers were requested to contact the researcher should difficulties be experienced or queries needed to be raised regarding the completion of the teacher and pupil questionnaires. It emerged that no problems were experienced by the educators and pupils in this regard. In general, then, as a result of the co-operation of teachers and principals, the administration of the questionnaires, which proceeded as planned, was considered to be a success.

4.3.4 <u>Teacher schedule in respect of cognitive levels of questions (see Appendix 5)</u>

The teacher schedule in respect of cognitive levels of questions comprised a 4 - column grid reflecting the appropriate numbers of the sub-questions set in each of the 4 examination papers. Next to each sub-question number was a blank block in which the cognitive level the sub-question was testing had to be filled by the teachers.

The teacher schedule and copies of the relevant examination papers as well as a copy of the researcher's classification of educational objectives were handed to the teachers by the Superintendent of Education (Mathematics) of the HOD. The teachers were requested to submit their completed returns to the researcher, in the stamped, self-addressed envelope provided.

4.3.5 Schedule in respect of IQ scores of pupils (see Appendix 1)

As the pupils' results were to be analysed in terms of certain variables, including IQ, the IQ scores which were housed at schools had to be obtained. The HOD had granted permission to ascertain and use scores from the relevant schools provided the pupils were not identified by name.

In view of the above, a schedule in the form of a grid was prepared and used. Using a copy of the schedule for each school, the examination numbers of all the pupils who offered mathematics (HG and SG separately) in the 1991 Senior Certificate Examination, which had been extracted from the pupils' answer scripts, were entered. Two blank blocks were provided next to each examination number for the IQ score and gender of the pupils to be filled in by the schools. The abovementioned schedule was submitted to and returned by schools in the envelope containing the pupil and teacher questionnaires.

4.4 TABULATION AND HANDLING OF DATA

4.4.1 Teacher schedule in respect of cognitive levels of questions

Since only 7 teachers were involved in this exercise, namely, identifying the cognitive level tested by each sub-question, the tabulation of data received from teachers in this regard was handled manually. The sub-questions were classified in terms of the following categories of cognitive abilities: lower level abilities (knowledge and skills), middle level abilities (understanding/comprehension) and higher level abilities (application, analysis, synthesis and evaluation/creative). It is to be noted that determining the cognitive level tested by each sub-question was a pre-requisite for the analysis of pupils' results.

4.4.2 Pupils' test (examination) results

For purposes of the analysis of pupils' performance, the raw marks obtained by pupils in each of the sub-questions in the mathematics examination were used. As mentioned in paragraph 4.2, the mathematics HG papers comprised 90 sub-questions in all, while the SG papers were made up of a total of 87 sub-questions. Taking into consideration certain statistics that had to be computed (which required the individual marks for each sub-question, the total for each paper and the overall total in both the papers) and certain variables (e.g., IQ scores, sex) in terms of which analyses were to be made, the total number of entries in respect of each of the HG pupils was identified as 97 while 94 entries were identified for the SG pupils. Since the scripts of a very large number of pupils (n = 2091), made up of 760 HG pupils and 1 331 SG pupils, were involved in the analysis, the tabulation of data was done with the aid of the computer. However, certain procedures/entries had to be handled manually, before the data was punched into the computer, namely, the ordering of the pupils' scripts schoolwise; the numbering of the scripts in consecutive order; and the transcribing of the pupils' IQ scores (where they were available), sex and school location (urban/rural) onto the cover of the scripts, which expedited the capture and analysis of data.

Each pupil's entries were first punched into a spreadsheet programme (Quattro-Pro). The data in respect of the entire sample was then analysed using the STATISTICA computer programmes.

The computer was programmed to provide, amongst others, the following information.

* The mean (facility index), standard deviation and discrimination index for each sub-question;
* The mean and standard deviation (percent) for the groups of sub-questions falling into the following categories of cognitive abilities: lower level, middle level and higher level abilities;
* the abovementioned statistics, for sub-groups in terms of certain selected variables (see Chapter 5); and
* Significance between means computed for sub-groups.

4.4.3 <u>The teacher questionnaire</u>

The teacher questionnaire was so designed that the information could be processed by the computer. Since 3 of the 52 questions were open-ended, the responses to these (3) questions were collated manually. Hence 49 entries each from a total of 80 teachers were punched into the computer.

A separate computer programme which was used for the analysis of data from the teacher questionnaire provided the following information:

* The total number and percentage of teachers whose responses were the same for each item in the questionnaire;
* The mean response (score) and the variance for those items where the 3-point or 5-point scale was used;
* The same statistics mentioned above, but in terms of sub-groups which were defined in terms of certain selected variables (see Chapter 5); and
* Significance between means computed for sub-groups.

4.4.4. <u>The pupil questionnaire</u>

The pupil questionnaire consisted of two scales - an attitude scale (comprising 48 statements which were to be rated on a 3-point scale) and an interest scale (comprising 6 open-ended questions).

Since the attitude scale was a standardised scale which was already formatted for processing by computerisation, the computer was used to tabulate the scores of the 488 pupils in the sample. However, certain negative statements had to be scored in the reverse, for example, disagreement with a negative statement would have had a response of 0 which had to be reversed to "2". Such items were identified and the necessary reversed scores for these items were put into the computer, using a programme specifically designed for this purpose. The computer provided the following information:

* The total score obtained by each pupil;

* The mean score obtained by the entire group of pupils;
* The number of pupils whose scores fell above and below 77;
* The mean scores obtained by sub-groups defined in terms of certain variables; and
* Significance between means computed for sub-groups.

With regard to the interest scale, it was found that it was easier to score each of the 6 items and to determine the total scores manually. However, the total score obtained by each pupil in the interest scale was entered into the computer, which provided the following information:

* The mean score for the entire group of pupils;
* The number of pupils whose scores fell above and below 6,2;
* The mean scores obtained by sub-groups defined in terms of certain variables; and
* Significance between means computed for sub-groups.

4.5 CONCLUSION

From the foregoing, it is clear that the use of the computer facilitated the collation and processing of data collected in this research. After the scores obtained in the examination (test) and the responses from the questionnaires were entered into the computer, the necessary analysis as well as inferences and conclusions had to be made. A discussion on the analysis of the data and the interpretation of the results is presented in Chapter 5 while the conclusions and recommendations are discussed in Chapter 6.

CHAPTER 5

STATISTICAL ANALYSIS OF DATA AND DISCUSSION AND INTERPRETATION OF THE RESULTS

5.1 INTRODUCTION

As mentioned in Chapter 4, the pupils' scores in the mathematics test (examination) and the responses of teachers and pupils from the respective questionnaires were punched into the computer, to facilitate the analysis of the results. Care was taken to eliminate any possible errors of transference of scores/responses. For this purpose, certain "verification" programmes were developed and used. For example, for the mathematics test scores, a verification programme summed the scores obtained by a pupil in a paper and checked it with the total score that was punched in for that paper. Another verification programme converted each score to a percentage which assisted in checking the correct order of the entries. Likewise, care was taken to ensure that there were no errors in the entry of responses from the two questionnaires (pupil and teacher) into the computer's memory. For this purpose, the computer programme provided for checking after a set of responses had been entered. Only after the correctness of each set of responses had been established was it possible to proceed with the next set of entries.

The statistical analysis was done by Prof. L. Troskie, professor of Mathematics Statistics at the University of Natal, who used the STATISTICA computer programmes in conjunction with SAS/STAT User's Guide, Release 6.03 Edition. Hence through the use of the computer programmes, it was possible to determine the means, standard deviations, frequency distribution of scores, standard error of means, item analysis data and the reliability and validity of the test used in the study. In addition, by using the means obtained, it was possible to determine the significance of the results attained. Likewise, with the use of an appropriate programme, it was possible to determine certain relevant statistics in respect of pupils' scores in the attitude and interest scales (pupil questionnaire). With regard to the teacher questionnaire, the computer provided information as to the number of teachers whose responses were the same for a particular item, the mean score and the variance. With the aid of the above and the application of the chi-square (x^2) statistic, it became possible to determine the significance of the results.

Hence the focus in this chapter is firstly on the methods used to analyse the results obtained statistically. Thereafter, attention is given to a discussion and interpretation of the findings. As mentioned in paragraph 4.1.2 of Chapter 4, different instruments together with different samples were used in data collection for this study. In view of this, the results obtained from each of the instruments are discussed separately.

5.2 ANALYSIS OF DATA PERTAINING TO THE MATHEMATICS TEST

5.2.1 Distribution of sample

The sample used in this part of the study totalled 2091 S.C. pupils, comprising 760 HG and 1 331 SG pupils. The sample of pupils was analysed with respect to grade (HG/SG) over gender and location of schools (urban/rural) as well as over IQ scores.

5.2.1.1 Distribution of sample according to grade over gender and location

Table 5.1, comprising two sub-tables, shows the distribution of the sample according to grade (HG/SG) over gender and location.

TABLE 5.1 DISTRIBUTION OF SAMPLE ACCORDING TO GRADE OVER GENDER AND LOCATION

1. Grade over gender

	MALE		FEMALE		TOTAL	
	No.	%	No.	%	No.	%
Standard	671	50,4	660	49,6	1 331	63,7
Higher	440	57,9	320	42,1	760	36,4
Total	1 111	53,1	980	46,9	2 091	100,0

2. Grade over location

	URBAN		RURAL		TOTAL	
	No.	%	No.	%	No.	%
Standard	917	68,9	414	31,1	1 331	63,7
Higher	526	69,2	234	30,8	760	36,4
Total	1 443	69,0	648	31,0	2 091	100,0

From the above tables, it is evident that there was a good proportion of male and female pupils as well as of urban and rural pupils. Furthermore, it can be observed that the ratio of SG:HG pupils in the sample is approximately 2:1 which accorded with the ratio found in the population.

5.2.1.2 Distribution of sample according to IQ scores

IQ scores (verbal and non-verbal combined) were obtained from school records for 1 420 (930 SG and 490 HG) or 68 % of the total sample of pupils. It is to be noted that IQ scores were not available at certain schools (the entire S.C. pupils or part of the pupils) as the HOD was unable to complete its IQ testing programme. Notwithstanding the above, it is argued that the sub-sample with IQ scores (1 420 pupils) is a large number, and hence considered representative of the sample (and population) for meaningful inferences to be made. Also, the ratio of SG to HG pupils is approximately 2:1 which is more or less the same as that in the original sample. Furthermore, the frequency distribution of the IQ scores of the 1420 S.C. pupils accords more or less with the normal distribution curve, as can be observed from the figure below.

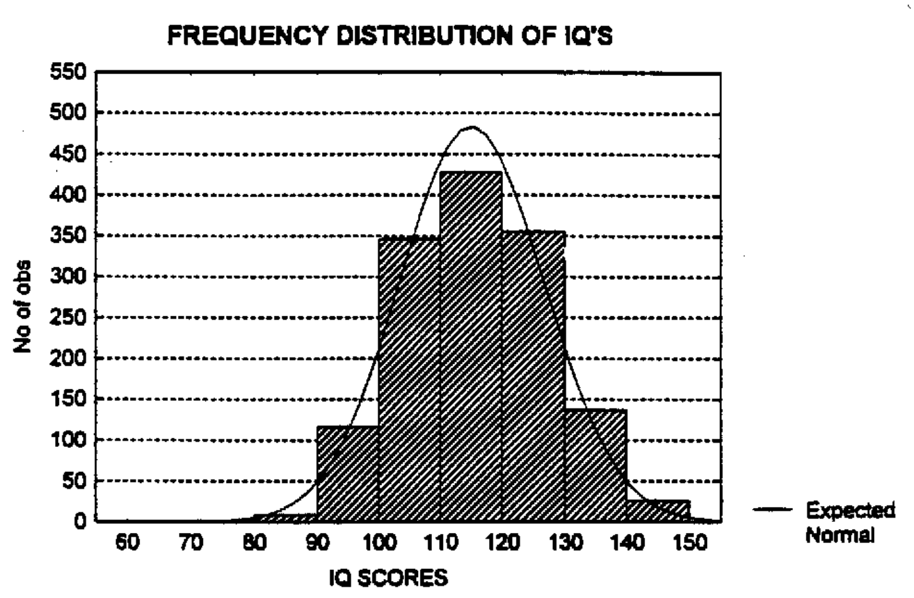


FIGURE 5.1 Frequency distribution of IQ scores of 1 420 S.C. pupils

The mean IQ scores according to grade, gender and location are shown in Table 5.2 As expected, there were no differences in mean IQ between the sexes and school location. However, there were significant differences ($p<0,01$) between HG and SG pupils in respect of IQ, with more able pupils in the higher grade.

TABLE 5.2 MEAN IQ SCORES ACCORDING TO GRADE, GENDER AND LOCATION

SAMPLE	n	$\overline{X}$	SD	Significance
Total	1420	115,1	11,1	
SG	930	111,4	10,5	$p<0,01$
HG	490	122,0	10,7	
Males	747	114,5	11,8	$p>0,01$
Females	673	116,7	11,6	
Urban	1034	115,5	11,6	$p>0,01$
Rural	386	114,0	12,0	

5.2.2 Cognitive levels of questions set in the mathematics test

The mathematics test (examination) comprised 4 papers, HG papers 1 and 2 for HG pupils and SG papers 1 and 2 for SG pupils. The cognitive level (lower level - LL, middle level - ML or higher level - HL) tested by each question in each of the 4 papers is reflected in Tables 7.1 to 7.4 (see Appendix 7). However, an analysis of the cognitive levels, in terms of number and marks allocated is summarised in Tables 5.3 and 5.4 below.

TABLE 5.3 THE TOTAL NUMBER OF QUESTIONS IN THE MATHEMATICS TEST TESTING THE DIFFERENT COGNITIVE LEVELS

	LL	ML	HL	TOTAL
HG paper 1	3	25	20	48
HG paper 2	9	20	13	42
Total	12	45	33	90
Total %	13,3	50,0	36,7	100,0
SG paper 1	5	35	9	49
SG paper 2	5	21	12	38
Total	10	56	21	87
Total %	11,5	64,4	24,1	100,0

TABLE 5.4 THE TOTAL NUMBER OF MARKS ALLOCATED TO QUESTIONS TESTING THE DIFFERENT COGNITIVE LEVELS

	LL	ML	HL	TOTAL
HG paper 1	9	100	91	200
HG paper 2	30	94	76	200
Total Marks	39	194	167	400
Total %	9,7	48,5	41,8	100
SG paper 1	7	110	33	150
SG paper 2	28	65	57	150
Total Marks	35	175	90	300
Total %	11,7	58,3	30	100

The analysis revealed (see Table 5.3) a reasonable spread of questions testing the 3 cognitive levels in both the HG and SG papers. However, more higher level questions were set in the HG papers which met with the examination requirements of the HOD. As can be observed from Table 5.4, 70% of the marks were allocated to LL and ML questions for the SG while approximately 60% of the marks were allocated to these two levels in the HG papers. This was also in keeping with the examination policy requirements of the HOD for SG and HG papers (HOD:1975).

5.2.3 Frequency distribution of test scores (symbolwise)

The frequency distribution of test scores (symbolwise) summarises the number or percentage of pupils who obtained a particular symbol. Tables 5.5 and 5.6 reflect this information for the HG and SG S.C. pupils respectively while Table 5.7 shows the symbol distribution of the test scores of the total sample (HG and SG combined).

TABLE 5.5 FREQUENCY DISTRIBUTION OF TEST SCORES (SYMBOLWISE) OF HG S.C. PUPILS

SYMBOL	PERCENTAGE	% OF PUPILS
H	0-19	1,7
G	20-29	9,9
FF	30-33	5,0
F	33⅓-39	13,0
E	40-49	20,5
D	50-59	19,6
C	60-69	15,5
B	70-79	9,0
A	80-100	5,8
No. of pupils: N = 760 Mean: $\overline{X}$ = 50,9 Standard deviation: SD = 17,2 Standard error of mean: $S\overline{x}$ = 0,62		

TABLE 5.6 FREQUENCY DISTRIBUTION OF TEST SCORES (SYMBOLWISE) OF SG S.C. PUPILS

SYMBOL	PERCENTAGE	% OF PUPILS
H	0-19	23,5
G	20-29	21,9
FF	30-33	5,0
F	33⅓-39	11,3
E	40-49	15,8
D	50-59	10,7
C	60-69	6,9
B	70-79	3,5
A	80-100	1,4
No. of pupils: N = 1331 Mean: $\overline{X}$ = 35,21 Standard deviation: SD = 18,56 Standard error of mean: $S\overline{x}$ = 0,51		

TABLE 5.7 FREQUENCY DISTRIBUTION OF TEST SCORES (SYMBOLWISE) OF THE TOTAL SAMPLE (HG AND SG S.C. PUPILS COMBINED)

SYMBOL	PERCENTAGE	% OF PUPILS
H	0-19	15,6
G	20-29	17,5
FF	30-33	5,0
F	33⅓-39	12,0
E	40-49	17,5
D	50-59	14,0
C	60-69	10,0
B	70-79	5,5
A	80-100	3,0

No. of pupils: $\underline{N} = 2091$
Mean: $\overline{X} = 40{,}91$
Standard deviation: SD = 19,59
Standard error of mean: $S\overline{x} = 0{,}43$

From Table 5.5, it can be observed that the scores of HG pupils showed a good spread over the different symbols. On the other hand, the SG scores (see Table 5.6) revealed that a substantially larger percentage of SG pupils than HG pupils obtained the two lowest symbols. As a result, the frequency distribution of the test scores for the total sample (HG and SG pupils combined) (see Table 5.7) did not accord with the normal curve.

Although the test used in this study was not constructed by the researcher, it was considered necessary to compute certain relevant statistics concerning the mathematics test.

5.2.4 Item analysis data

Item analysis provides quantitative data on the quality of the test items in respect of their difficulty levels and discrimination power. Hence, the two indices used in the evaluation of items/questions are the facility (or difficulty) index and the discrimination index.

5.2.4.1 Difficulty. Nuttal and Willmott (1972:22) state that in the case of non-objective questions, the difficulty (or facility) index is calculated by dividing the average mark obtained by all the candidates by the maximum mark for that question. The difficulty index is converted to a percentage by multiplying the value by 100.

The difficulty (or facility) indices for the questions set in the mathematics test (2 HG papers and 2 SG papers) were computed and are shown in Tables 7.1 to 7.4 (see Appendix 7). A summary of the number of questions falling into the different difficulty index intervals (Moodley 1981) is presented in Tables 5.8 and 5.9.

TABLE 5.8 THE NUMBER OF QUESTIONS IN THE 2 MATHEMATICS HG PAPERS FALLING INTO THE DIFFERENT DIFFICULTY INDEX INTERVALS

Difficulty Index Interval	Difficult 0-19%	Acceptable 20-91%	Easy 92-100%	Total
No. of questions	9	77	4	90
%	10	85,6	4,4	100

TABLE 5.9 THE NUMBER OF QUESTIONS IN THE 2 MATHEMATICS SG PAPERS FALLING INTO THE DIFFERENT DIFFICULTY INDEX INTERVALS

Difficulty Index Interval	Difficult 0-19%	Acceptable 20-91%	Easy 92-100%	Total
No. of questions	23	64	0	87
%	26,4	73,6	0,0	100

It is to be noted that the mathematics examination papers set by the HOD are criterion -referenced tests. Hence no item analysis data are computed beforehand for the questions. However, the analysis revealed that the HG papers constituted a reasonable spread of questions with 86,6 % being found to be acceptable. This had not been the case in the SG papers. From Table 5.9 it can be observed that 26,4% of the SG questions comprised difficult questions, as compared with only 10% in the HG papers, and no questions that were considered to be easy were set in the SG papers. Only 73,6 % of the questions in the SG papers were found to be acceptable. Hence it may be concluded that a slightly more difficult examination had been set by the HOD for SG pupils than for the HG pupils in 1991.

5.2.4.2 Discrimination. According to Nuttal and Willmott (1972:23), the discrimination index of a non-objective question is simply the correlation between the mark on the question and the mark on the whole paper. For this purpose, the Pearson Product Moment correlation coefficient is used. In order to calculate the discrimination indices of the questions set in the test, the formula as suggested by Behr (1973:99) was used (see Appendix 8, paragraph 3).

The discrimination indices for the questions set in the mathematics test (2 HG papers and 2 SG papers) were computed and are shown in Tables 7.1 to 7.4 (see Appendix 7). A summary of the number of questions falling into the different discrimination index intervals (Ebel 1965:363) is presented in Tables 5.10 and 5.11.

TABLE 5.10 THE NUMBER OF QUESTIONS IN THE 2 MATHEMATICS HG PAPERS FALLING INTO THE DIFFERENT DISCRIMINATION INDEX INTERVALS

Discrim. Index Interval	Zero or negative	Low 0,01 - 0,19	Moderate 0,20 - 0,39	High > = 0,40	Total
No. of questions	0	3	20	67	90
%	0,0	3,3	22,2	74,4	100

TABLE 5.11 THE NUMBER OF QUESTIONS IN THE 2 MATHEMATICS SG PAPERS FALLING INTO THE DIFFERENT DISCRIMINATION INDEX INTERVALS

Discrim. Index Interval	Zero or negative	Low 0,01 - 0,19	Moderate 0,20 - 0,39	High > = 0,40	Total
No. of questions	0	1	15	71	87
%	0,0	1,2	17,2	81,6	100

As can be observed from the above tables, the discrimination indices of most of the questions set in both the HG and SG papers fell into the moderate to high categories, with a larger percentage being noted in the high discrimination category. This indicated that the questions in both the grades discriminated fairly well between the more able and less able pupils.

5.2.5 Reliability of the mathematics test

The reliability of a test refers to the consistency of performance in a test from time to time (Nunnally 1972:79: Downie & Heath 1974:236). According to Nuttal and Willmott (1972:32), reliability figures obtained (by any methods) can only be estimates of the reliability of the examination (or test) scores. As difficulty indices had already been calculated, the Kuder-Richardson Formula 20 was used to determine the reliability of the mathematics test (examination) (see Appendix 8, paragraph 4). Using the abovementioned formula, the reliability estimate of the mathematics HG test was found to be 0,91, while that of the SG test was found to be 0,88. It can be observed that fairly high reliability coefficients were obtained in both cases. The HG and SG tests comprised 90 and 87 test items respectively. Downie and Heath (1974:241) mention that an increase in the number of test items tends to increase the reliability.

5.2.6 Validity of the mathematics test

Validity refers to the extent or degree to which a test measures what it is designed to measure. The aspect of validity has been discussed in Chapter 2 under paragraph 2.7.9.1. However, in respect of the mathematics test, two types of validity are focused upon, namely, content validity and construct validity.

5.2.6.1 Content validity

Content validity is determined by definition (Tittle & Miller 1977:41) or by expert judgement (Nuttal & Willmott 1972:160; Gay 1991:160). Since the mathematics HG and SG tests were set by panels of examiners (experts) who took into account the examination requirements of the HOD when setting the papers (e.g., wide sampling of content, attention given to questions testing different cognitive levels, suitable mark allocation to the latter), it is argued that the tests used had a high degree of content validity (Nunnally 1972:30; Downie & Heath 1974:243). Furthermore, the examination (test) was written at the end of the year when S.C. pupils had completed all aspects of the S.C. mathematics syllabus.

5.2.6.2 Construct validity

This is the correlation between the set of scores obtained with those of other tests that measure the same construct (characteristic). In this research, the test scores were correlated with the IQ scores (verbal and non-verbal) of the S.C. pupils. The estimates of validity of the mathematics HG and SG tests were found to be 0,64 and 0,57 respectively. It is clear from the above correlations that there was a substantial relationship between the mathematics test scores and the construct (IQ) scores.

5.2.7 Testing significance of results

One of the main objectives of statistical inference is to enable the researcher to make generalisations from a sample to some larger population as it is not always possible to measure all the members of a given population. In order to establish the significance of results, various statistical methods are used. In this part of the study, the mean attainment of S.C. pupils in three cognitive levels of mathematical ability had to be computed in terms of several variables and then compared. In view of this research design, the multivariate analysis of variance (MANOVA) (see

Appendix 8, paragraph 6) was used to test significance between the various means attained. According to Ferguson (1981:75), the commonly accepted levels of significance are either 0,05 or 0,01. Since the sample used in this part of the study was fairly large (n = 2091), it was decided that all decisions and conclusions would be based on the 0,01 level of significance. It would be recalled that the norm T-scores for the 3 cognitive levels were also established using the 0,01 level of significance (see paragraph 3.5 of Chapter 3).

5.2.8 <u>Analysis of scores obtained from the mathematics test</u>

In order to evaluate the attainment of the cognitive objectives, and hence the cognitive aims, it was required that the mathematical attainment of S.C. pupils (HG and SG combined) be analysed in terms of 3 cognitive levels of mathematical ability, namely, lower level objectives, middle level objectives and higher level objectives (see objective of this study in paragraph 1.5.2.1). However, in view of the stated purposes of this study and in order to obtain more indepth information regarding the attainment of the S.C. pupils, further analyses were carried out in relation to the following dependent variables : gender, location, sections/subjects, IQ and grade (see objective of this study in paragraph 1.5.2.3). In addition, pupils' attainments in certain selected combinations of variables were analysed with a view to investigating the effects of interactions between variables. Consequently, in this part of the research, the following aspects pertaining to S.C. pupils' mathematical attainment were examined in relation to the 3 levels of cognitive objectives.

(1) overall attainment in the 3 cognitive levels - LL, ML and HL (hereafter referred to only as "levels").

(2) levels over gender

(3) levels over location

(4) levels over subjects/sections

(5) levels over gender and subjects

(6) levels over location and subjects

(7) levels over IQ categories

(8) levels over IQ categories and gender

(9) levels over IQ categories and location

(10) levels over IQ categories and subjects

(11) levels over grade

(12) levels over grade and subjects

In each case, the means obtained were subjected to MANOVA, in order to determine the significance of the differences between the means. Thereafter, the mean scores were converted to T-scores and compared with the norm T-scores, in order to determine whether or not each of the 3 levels of cognitive abilities was adequately attained by the S.C. pupils. The results of the various analyses carried out are presented below.

5.2.8.1 <u>Overall attainment of the 3 cognitive levels of mathematical ability</u>

As the questions set in the mathematics test (examination) had already been classified in terms of 3 cognitive levels (see Tables 7.1 to 7.4 in Appendix 7), the scores obtained by the S.C. pupils (HG and SG combined) were firstly grouped in terms of these 3 levels, namely, lower cognitive level (LL), middle cognitive level (ML) and higher cognitive level (HL). Thereafter, the overall mean scores (percent) attained in each of the 3 levels were computed. These mean scores are shown in Table 5.12. The mean scores were statistically analysed to determine whether significant differences exist between the means.

TABLE 5.12 MEAN SCORES (PERCENT) ATTAINED BY S.C. PUPILS IN THE 3 COGNITIVE LEVELS (LL, ML, HL) IN TERMS OF OVERALL MATHEMATICAL PERFORMANCE

	LL	ML	HL	$\overline{M}$	SD	Signif.
Mean %	60,6	44,2	29,1	40,91	19,59	$p < 0,01$

It was found that the mean scores attained by the S.C. pupils in the three cognitive levels differed significantly from one another ($p < 0,01$).

As the norm scores were established in terms of T-scores, the above mean scores were converted to T-scores which are reflected in Table 5.13.

TABLE 5.13 MEAN T-SCORES ATTAINED BY S.C. PUPILS IN THE 3 COGNITIVE LEVELS IN TERMS OF OVERALL MATHEMATICAL PERFORMANCE

	LL	ML	HL
T-Score	60,1	51,7	44,0
Norm T-score	58,0	49,3	41,0

All attained T-scores > norm T-scores

From Table 5.13, it can be observed that the T-scores attained by the S.C. pupils in each of the 3 cognitive levels were significantly ($p < 0,01$) greater that the norm T-scores for the respective levels. This indicated that S.C. pupils as a group (HG and SG combined) adequately attained the lower level, middle level and higher level cognitive objectives of the S.C. mathematics curriculum (see paragraph 1.5.2.1).

5.2.8.2 Levels over gender

For the purpose of this analysis, the S.C. pupils were grouped according to gender. The mean scores attained by male and female S.C. pupils in respect of the 3 levels of cognitive objectives (LL, ML and HL) were calculated (see Table 5.14). The six groups constituted in this way were subjected to MANOVA, in order to determine the significance of the differences.

TABLE 5.14 MEAN SCORES (PERCENT) ATTAINED BY S.C. PUPILS IN THE 3 COGNITIVE LEVELS OVER GENDER

	LL	ML	HL	$\overline{M}$	SD	Signif.
Males	61,2	45,6	30,7	42,2	20,4	$p < 0{,}01$
Females	60,0	42,7	27,3	39,4	18,5	

From the above table, it is clear that male S.C. pupils performed significantly better than female pupils ($p < 0{,}01$). This applied to the overall means as well as to the means for each of the cognitive levels LL, ML, and HL.

The mean scores attained in the 3 levels were transformed to T-scores. These are reflected in Table 5.15.

TABLE 5.15 MEAN T-SCORES ATTAINED BY S.C. PUPILS IN THE 3 COGNITIVE LEVELS OVER GENDER

	LL	ML	HL
Males	60,3	52,4	44,8
Females	59,7	50,9	42,9

All attained T-scores > norm T-scores

It was found that all the T-scores attained by the male and female S.C. pupils were greater than the respective norm scores. This indicated that both male and female S.C. pupils showed adequate proficiency in each of the 3 levels of cognitive objectives.

5.2.8.3 Levels over location

In this case, the S.C. pupils were grouped according to the locality in which the schools were situated - urban and rural. The mean scores of each of these groups with respect to the three levels of cognitive objectives (LL, ML and HL) were calculated (see Table 5.16).

TABLE 5.16 MEAN SCORES (PERCENT) ATTAINED BY S.C. PUPILS IN THE 3 COGNITIVE LEVELS OVER LOCATION

	LL	ML	HL	$\overline{M}$	SD	Signif.
Urban	61,5	45,2	29,6	41,9	19,5	$p > 0,01$
Rural	58,7	42,0	27,9	39,0	19,8	

The application of MANOVA revealed that there were no significant differences between the means attained by urban and by rural S.C. pupils in the 3 cognitive levels.

The above means scores were converted to T-scores, which are reflected in Table 5.17.

TABLE 5.17 MEAN T-SCORES ATTAINED BY S.C. PUPILS IN THE 3 COGNITIVE LEVELS OVER LOCATION

	LL	ML	HL
Urban	60,5	52,2	44,3
Rural	59,1	50,6	43,3

All attained T-scores > norm T-scores

Here again, it can be observed that all the T-scores attained by the urban and rural pupils in each of the 3 levels of cognitive objectives were greater than the respective norm T-scores. Hence, it can be concluded that both the urban and rural S.C. pupils adequately attained the 3 levels of cognitive objectives.

5.2.8.4 Levels over subjects

The mathematics test comprised questions from the following subjects (sections): algebra, trigonometry, analytical geometry and Euclidean geometry. The mean scores attained by S.C. pupils in these four subjects in terms of the 3 levels of cognitive objectives were calculated. These are reflected in Table 5.18 and Figure 5.2. The eight groups constituted in this way, were subjected to MANOVA, in order to determine the significance of the differences between the means.

TABLE 5.18 MEAN SCORES (PERCENT) ATTAINED BY S.C. PUPILS IN THE 3 COGNITIVE LEVELS OVER SUBJECTS

Subjects	LL	ML	HL	$\overline{M}$	SD	Signif.
Alg.	60,7	43,4	28,1	39,4	19,7	$p < 0{,}01$
Trig.	71,1	48,2	36,8	49,9	21,8	
Anal. Geom.	63,3	48,0	26,9	39,2	24,9	
Eucl. Geom.	42,8	44,1	23,1	36,9	24,1	

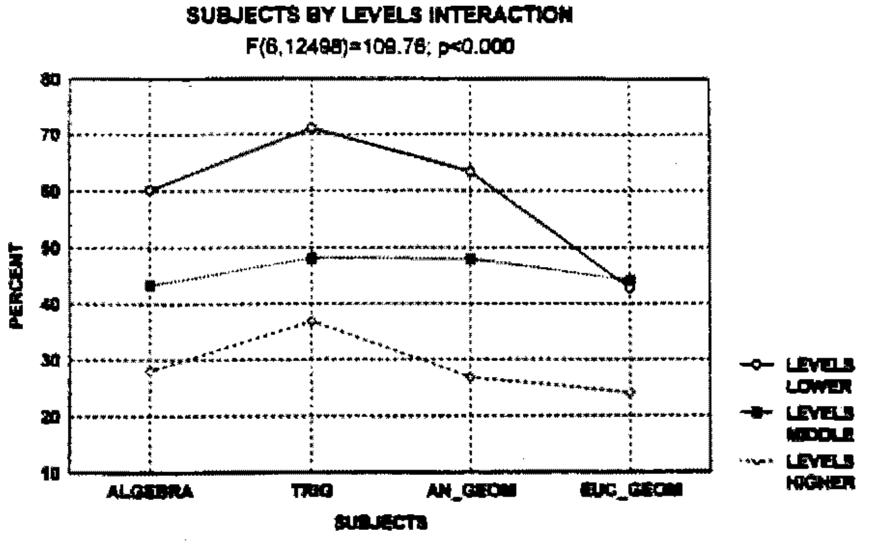


FIGURE 5.2 Graph showing the mean scores (percent) attained by the S.C. pupils in the 3 cognitive levels over subjects.

It was observed that the S.C. pupils' mean performance in the four subjects differed significantly ($p < 0{,}01$). From the above table and graph it can also be observed that the S.C. pupils performed best in trigonometry and worst in Euclidean geometry.

The mean scores attained by S.C. pupils in the four subjects were converted to T-scores. The latter are reflected in Table 5.19.

TABLE 5.19: MEAN T-SCORES ATTAINED BY S.C. PUPILS IN THE 3 COGNITIVE LEVELS OVER SUBJECTS

Subjects	LL	ML	HL
Alg.	59,8	51,3	43,4
Trig.	65,4	53,7	47,9
Anal. Geom.	51,5	53,6	42,8
Eucl. Geom.	51,0*	51,4	40,9*

* Attained T-scores < norm T-scores

From the above table, it can be seen that the S.C. pupils adequately attained all the 3 cognitive levels of objectives in algebra, trigonometry and analytical geometry. However, it was found that the T-scores obtained by the S.C. pupils in the lower and higher cognitive levels in Euclidean geometry were lower than the respective norm scores. This indicated that S.C. pupils did not adequately attain the lower level objectives and the higher level objectives in Euclidean geometry.

5.2.8.5 <u>Levels over subjects and gender</u>

The S.C. pupils' attainment in the different subjects (sections) of the syllabus was investigated over gender. The means attained by male and female S.C. pupils in the different subjects, in terms of the 3 cognitive levels were computed (see Table 5.20). Thereafter, the means were subjected to MANOVA, in order to determine the significance of the differences.

TABLE 5.20 MEAN SCORES (PERCENT) ATTAINED BY S.C. PUPILS IN THE 3 COGNITIVE LEVELS OVER SUBJECTS AND GENDER

MALES

Subjects	LL	ML	HL	$\overline{M}$	SD	Signif.
Alg.	61,2	44,4	28,7	39,9	20,7	$p < 0{,}01$
Trig.	70,9	50,8	39,0	51,8	22,3	
Anal. Geom.	65,1	49,4	29,1	41,0	26,5	
Eucl. Geom.	43,1	46,0	26,4	38,7	24,5	

FEMALES

Subjects	LL	ML	HL	$\overline{M}$	SD	Signif.
Alg.	59,0	42,2	27,4	38,1	18,5	p < 0,01
Trig.	70,3	45,3	34,3	47,8	21,1	
Anal. Geom.	61,4	46,5	24,4	37,1	22,8	
Eucl. Geom.	42,4	42,0	21,5	35,0	23,2	

The above analyses revealed that male S.C. pupils performed significantly better than female S.C. pupils, in each of the 4 subjects in terms of the three cognitive levels ($p < 0{,}01$). Both the male and female pupils performed best in trigonometry and worst in Euclidean geometry.

The above mean scores were converted to T-scores and these are reflected in Table 5.21 below.

TABLE 5.21 MEAN T-SCORES ATTAINED BY S.C. PUPILS IN THE 3 COGNITIVE LEVELS OVER SUBJECTS AND GENDER

MALES

Subjects	LL	ML	HL
Alg.	60,4	51,8	43,8
Trig.	65,3	55,0	49,1
Anal. Geom.	62,3	54,3	44,0
Eucl. Geom.	51,1*	52,6	42,6

FEMALES

Subjects	LL	ML	HL
Alg.	59,2	50,6	43,1
Trig.	65,0	52,2	46,6
Anal. Geom.	60,5	52,8	41,6
Eucl. Geom.	50,8*	50,5	40,1*

*** Attained T-scores < norm T-scores**

From the above table, it can be observed that the male S.C. pupils did not adequately attain the lower level objectives in Euclidean geometry. On the other hand, the female S.C. pupils failed to adequately attain both the lower level and higher level objectives in Euclidean geometry.

5.2.8.6 <u>Levels over subjects and location</u>

The attainment of the S.C. pupils in the different subjects was also investigated over location. For this purpose, the means obtained by urban and rural S.C. pupils in the 4 subjects, in terms of the 3 cognitive levels, were determined. The mean scores are reflected in Table 5.22. The 8 groups constituted in this way were subjected to MANOVA, in order to determine the significance of the differences between the means.

TABLE 5.22 MEAN SCORES (PERCENT) ATTAINED BY S.C. PUPILS IN THE 3 COGNITIVE LEVELS OVER SUBJECTS AND LOCATION

URBAN

Subjects	LL	ML	HL	$\overline{M}$	SD	Signif.
Alg.	60,8	44,6	28,7	40,1	19,5	$p < 0{,}01$
Trig.	71,5	49,1	37,6	50,7	22,0	
Anal. Geom.	64,4	48,8	27,4	39,9	25,0	
Eucl. Geom.	44,1	44,6	24,1	37,5	24,0	

RURAL

Subjects	LL	ML	HL	$\overline{M}$	SD	Signif.
Alg.	58,9	40,6	26,7	36,8	20,0	$p < 0{,}01$
Trig.	70,3	46,2	35,1	48,8	21,0	
Anal. Geom.	60,5	46,2	25,7	37,6	24,7	
Eucl. Geom.	39,7	43,1	23,0	35,8	24,5	

The application of MANOVA showed that the mean scores obtained by urban S.C. pupils in the 4 subjects, in terms of the 3 cognitive levels, were significantly higher than those of the rural S.C. pupils.

The above mean scores were converted to T-scores, for comparison purposes. The computed T-scores are shown in Table 5.23.

TABLE 5.23 MEAN T-SCORES ATTAINED BY S.C. PUPILS IN THE 3 COGNITIVE LEVELS OVER SUBJECTS AND LOCATION

URBAN

Subjects	LL	ML	HL
Alg.	60,2	51,9	43,8
Trig.	65,6	54,2	48,3
Anal. Geom.	62,1	54,0	43,1
Eucl. Geom.	51,6*	51,9	41,6

RURAL

Subjects	LL	ML	HL
Alg.	59,2	49,8	42,7
Trig.	65,0	52,7	47,0
Anal. Geom.	60,0	52,7	42,2
Eucl. Geom.	49,4*	51,1	40,9*

***Attained T-scores < norm T-scores**

It was found that the T-score attained by urban S.C. pupils in the lower level objectives in Euclidean geometry was significantly less than the norm T-score for that category. A similar finding was observed for rural S.C. pupils' attainment in the lower level and higher level objectives in Euclidean geometry. This demonstrated that the urban S.C. pupils failed to adequately attain the lower cognitive level in Euclidean geometry while rural S.C. pupils failed to adequately attain both the lower and higher cognitive levels in this subject.

5.2.8.7 Levels over IQ categories

In order to arrive at as accurate a picture as possible of S.C. pupils' attainment of the different levels of cognitive objectives, IQ was included as a further important variable in the examination of attainment. Several other research studies on achievement in mathematics have included IQ (ability) as a variable (e.g., HOD 1980; Moodley 1981). For the purpose of this analysis, the pupils were grouped according to the following IQ categories, used by Noll et al. (1979:288).

90 - 99	-	normal average
100 - 109	-	normal average
110 - 119	-	high average
120 - 129	-	superior
> = 130	-	superior to very superior

The mean scores attained by S.C. pupils falling into each of the above IQ categories, in terms of the 3 levels of cognitive objectives (LL, ML and HL) were computed (see Table 5.24).

The 15 groups constituted in this way were subjected to MANOVA, in order to determine the significance of the differences.

TABLE 5.24 **MEAN SCORES (PERCENT) ATTAINED BY S.C. PUPILS IN THE 3 COGNITIVE LEVELS OVER IQ CATEGORIES**

IQ	LL	ML	HL	$\overline{M}$	SD	Signif
90-99	45,4	29,6	16,2	27,3	16,2	
100-109	51,8	34,6	20,5	32,2	17,2	
110-119	59,4	48,8	27,2	39,5	17,7	p < 0,01
120-129	66,7	50,8	33,8	46,6	17,9	
> = 130	72,7	56,7	40,6	52,4	17,8	

The application of MANOVA revealed that the differences between the mean scores attained by S.C. pupils falling into the different IQ categories, in terms of LL, ML and HL, were highly significant ($p < 0,01$). From Table 5.24, it can be observed that the mean scores attained by S.C. pupils (overall as well as in the 3 cognitive levels) improved progressively, from the lower IQ categories to the higher IQ categories.

The above mean scores were converted to T-scores and the latter are shown in Table 5.25.

TABLE 5.25 **MEAN T-SCORES ATTAINED BY S.C. PUPILS IN THE 3 COGNITIVE LEVELS OVER IQ CATEGORIES**

IQ	LL	ML	HL
90-99	52,3*	44,2*	37,4*
100-109	55,6*	46,8*	39,6*
110-119	59,4	54,0	43,0
120-129	63,1	55,1	46,4
> = 130	66,2	58,0	49,9

*** Attained T-scores < norm T-scores**

From the above table, it can be observed that the mean T-scores obtained by S.C. pupils with IQ less than 109 (normal average) in the lower level, middle level and higher level cognitive objectives were significantly less than the norm T-scores for these categories. This demonstrated that the S.C. pupils of normal average (IQ < 109) failed to adequately attain all the 3 levels of objectives in their overall mathematical performance. On the other hand, it was found that S.C. pupils with IQ > 110 successfully attained all the 3 levels of cognitive objectives.

5.2.8.8 Levels over IQ categories and gender

The S.C. pupils' attainment in the 3 cognitive levels was also investigated over IQ categories and gender. The statistical analysis revealed that there were no significant differences between the means attained by males and by females, in each of the different categories, in terms of the 3 cognitive levels ($p > 0{,}01$). The mean scores were converted to T-scores which were then compared with the norm T-scores. Here, too, it was observed that both male and female S.C. pupils with IQ less than 110 failed to adequately attain all 3 levels of cognitive objectives - LL, ML and HL.

5.2.8.9 Levels over IQ categories and location

Analyses were also carried out on S.C. pupils' performance in the 3 cognitive levels over IQ categories and location. The respective means were computed and subjected to MANOVA, to determine the significance between the differences. The analyses showed no significant differences in the means obtained by the two groups - urban and rural. The mean scores were converted to T-scores which were then compared with the corresponding norm T-scores. Here again it was found that both urban and rural S.C. pupils with IQ less than 110 did not adequately attain all the 3 levels of cognitive objectives.

5.2.8.10 Levels over IQ categories and subjects

The S.C. pupils' attainment in terms of the 3 cognitive levels over IQ and subjects was also investigated, in order to observe any trends and the effects of the interaction. The respective means attained in the different subjects (sections) over IQ categories were computed. The application of MANOVA revealed that the mean attainment of S.C. pupils differed significantly in terms of the above variables ($p < 0{,}01$).

The respective mean scores were then converted to T-scores. These are reflected in Table 5.26 (see next page).

When compared with the norm T-scores, it was found that S.C. pupils with IQ less than 110 (normal average) failed to adequately attain the 3 cognitive levels (LL, ML and HL) in algebra and Euclidean geometry. However, in trigonometry the same group failed to adequately attain only the middle level objectives. Furthermore, the same group did not adequately attain the middle level and higher level objectives in analytical geometry. The analysis also revealed that S.C. pupils falling in the IQ category 110-109 did not adequately attain the lower level and higher level objectives in Euclidean geometry.

TABLE 5.26 MEAN T-SCORES ATTAINED BY S.C. PUPILS IN THE 3 COGNITIVE LEVELS OVER IQ CATEGORIES AND SUBJECTS

ALGEBRA			
IQ	LL	ML	HL
90- 99	52,1*	44,0*	37,1*
100-109	55,7*	46,3*	39,5*
110-119	59,6	50,1	42,6
120-129	62,9	54,7	45,8
> = 130	64,2	58,5	48,7
TRIGONOMETRY			
90- 99	58,2	45,2*	41,5
100-109	62,1	48,8*	44,3
110-119	65,0	53,4	47,7
120-129	67,3	56,6	49,7
> = 130	69,6	59,1	52,3
ANAL. GEOMETRY			
90- 99	58,1	43,6*	36,4*
100-109	59,7	47,3*	38,5*
110-119	61,5	51,5	41,8
120-129	63,2	57,7	45,0
> = 130	63,7	62,5	49,8
EUCL. GEOMETRY			
90- 99	41,9*	44,2*	33,3*
100-109	44,3*	47,0*	34,9*
110-119	49,9*	53,1	39,2*
120-129	58,8	55,1	45,0
> = 130	61,1	54,6	50,6

***Attained T-scores < norm T-scores**

5.2.8.11 Levels over grade

For the purpose of this analysis, the S.C. pupils were divided into two groups according to the grade on which the pupils offered mathematics, namely, higher grade and standard grade. The means obtained by the HG and SG pupils in the 3 cognitive levels, in terms of their overall mathematical performance, were computed (see Table 5.27). The means were then subjected to MANOVA, in order to determine the significance between the difference.

TABLE 5.27 MEAN SCORES (PERCENT) ATTAINED BY S.C. PUPILS IN THE 3 COGNITIVE LEVELS OVER GRADE

GRADE	LL	ML	HL	$\overline{M}$	SD	Signif.
Higher	73,1	55,3	39,9	50,9	17,2	$p < 0{,}01$
Standard	53,5	37,9	17,9	35,2	18,6	

From the above table, it can be observed that HG pupils performed significantly better than the SG pupils ($p < 0{,}01$). This applied to the overall mean performance as well as the attainment in each of the 3 cognitive levels LL, ML and HL.

The above means attained by the HG and SG pupils were converted to T-scores, using the mean and standard deviations of the HG and the SG tests respectively (see paragraph 3.5). The T-scores attained by HG and SG pupils are shown in Table 5.28.

TABLE 5.28 MEAN T-SCORES ATTAINED BY S.C. PUPILS IN THE 3 COGNITIVE LEVELS OVER GRADE

GRADE	LL	ML	HL
Higher	63,4	52,6	43,6
Standard	59,8	51,4	40,7*

***Attained T-scores < norm T-scores**

It was found that the HG S.C. pupils adequately attained all the 3 levels of cognitive objectives. However, it was observed that the T-score attained by the S.G. S.C. pupils for the higher cognitive level was less than the corresponding norm score. This demonstrated that the SG pupils did not adequately attain the higher level cognitive objectives, in terms of their overall mathematical performance.

5.2.8.12 Levels over grade and subjects

Based on the trends observed in the previous analyses, it was decided to examine the interaction of S.C. pupils' attainment in the 3 cognitive levels only over grade and subjects. For the purpose of this analysis, the pupils were first divided into two groups according to grade. Thereafter each group's attainment (means) in the four subjects in terms of the 3 cognitive levels were computed. The application of MANOVA revealed that the differences in the means obtained by HG and SG pupils in the four subjects, in terms of the 3 cognitive levels, were significant ($p < 0,01$).

The respective means obtained by the two groups were then converted to T-scores which are reflected in Table 5.29.

TABLE 5.29 MEAN T-SCORES ATTAINED BY S.C. PUPILS IN THE 3 COGNITIVE LEVELS OVER GRADE AND SUBJECTS

HIGHER GRADE

SUBJECTS	LL	ML	HL
Alg.	60,8	54,0	42,4
Trig.	65,7	52,8	47,4
Anal. geom.	54,5	58,8	41,5
Eucl. geom.	59,9	50,1	45,2

STANDARD GRADE

SUBJECTS	LL	ML	HL
Alg.	60,6	49,9	42,1
Trig.	63,2	51,7	46,7
Anal. geom.	61,6	50,3	41,5
Eucl. geom.	46,3*	49,1	38,3*

***Attained T-scores < norm T-scores**

From the above table, it can be observed that the HG S.C. pupils adequately attained all 3 cognitive levels in the four subjects (sections) of the syllabus. However, SG S.C. pupils failed to adequately attain the lower level and higher level objectives in Euclidean geometry.

The discussion and interpretation of the findings from the mathematics test are presented in paragraph 5.5.

5.3 ANALYSIS OF DATA PERTAINING TO THE ATTITUDE AND INTEREST SCALES (PUPIL QUESTIONNAIRE)

5.3.1 Distribution of sample

The sample used in this part of the study totalled 488 S.C. pupils, comprising 200 HG and 288 SG pupils. A further breakdown of the sample in terms of gender and location is shown in Tables 5.30 and 5.31 respectively.

TABLE 5.30 DISTRIBUTION OF SAMPLE ACCORDING TO GRADE OVER GENDER

GRADE	MALE		FEMALE		TOTAL	
	No.	%	No.	%	No.	%
Higher	98	36,0	102	47,2	200	41,0
Standard	174	64,0	114	52,8	288	59,0
Total	272	55,7	216	44,3	488	100,0

From the above table, it is evident that there was a good proportion of male and female S.C. pupils. This proportion accorded more or less with the proportion of male and female S.C. pupils used in the analysis of the mathematics test (see Table 5.1). The number of males and females that offered mathematics HG differed significantly from that which offered mathematics SG ($p < 0{,}01$).

TABLE 5.31 **DISTRIBUTION OF SAMPLE ACCORDING TO GRADE OVER LOCATION**

GRADE	URBAN		RURAL		TOTAL	
	No.	%	No.	%	No.	%
Higher	118	59,0	82	41,0	200	41,0
Standard	218	75,7	70	24,3	288	59,0
Total	336	68,9	152	31,1	488	100,0

Here again, it is evident that there was a good proportion of urban and rural S.C. pupils. From Table 5.1, it can be observed that this proportion accorded with that used in the analysis of the mathematics test. The number of urban and rural pupils that offered mathematics HG differed significantly from that which offered mathematics SG ($p < 0{,}01$).

5.3.2 Testing significance of results

As the distribution of the sample was analysed in terms of proportions, use was made of the chi-square (x^2) statistic to test the significance between the observed and expected frequencies (see Appendix 8, paragraph 7). However, in order to assess S.C. pupils' attitude towards and interest in mathematics, means had to be computed and compared. For this purpose, a one-way analysis of variance (ANOVA) was used to test significance between the various means attained (see Appendix 8, paragraph 5).

5.3.3 Analysis of data from the attitude scale

After the scoring for the negative items in the attitude scale was reversed (which was also done by the computer), the total score obtained by each pupil was determined. The total scores obtained by the entire group of S.C. pupils were then analysed in terms of certain variables.

5.3.3.1 Overall attitude of S.C. pupils towards mathematics

According to the attitude scale used in this study, pupils' attitude towards mathematics could be described, depending on the mean score attained, as high (score of 77-96), moderate (score of 67-76) or low (score of 0-66). The mean attitude score obtained by the total sample of S.C. pupils (HG and SG combined) was computed and found to be 73,4 with a standard deviation of 13,4. It is to be noted that this mean falls into the moderate category. However, as a minimum mean score of 77 (high) was set as the norm score to indicate S.C. pupils' successful attainment of this aim, it can be concluded that the group of S.C. pupils did not adequately display a positive attitude towards mathematics (see objective of this study in paragraph 1.5.2.4).

The frequency distribution of the total attitude scores, in terms of the 3 categories, was computed. This is reflected in Table 5.32.

TABLE 5.32 FREQUENCY DISTRIBUTION OF ATTITUDE SCORES OF S.C. PUPILS

CATEGORY	NO.	%
0-66 (low)	144	29,5
67-76 (moderate)	111	22,8
77-96 (high)	233	47,7

From the above table, it can be observed that only some 48% of the S.C. pupils displayed a high positive attitude towards mathematics.

5.3.3.2 S.C. pupils' attitude towards mathematics over gender

After the mean scores attained by male and female S.C. pupils were computed, they were compared and statistically analysed. It was observed that the mean scores of both these groups fell into the moderate category (67-76). The application of ANOVA revealed that there were no significant differences in attitude towards mathematics of male and female pupils ($p > 0{,}01$). It was also observed that both groups did not adequately display a positive attitude towards mathematics.

5.3.3.3 S.C. pupils' attitude towards mathematics over location

The sample of S.C. pupils were divided into two groups - urban and rural. The mean scores attained by these two groups were computed and analysed statistically. The application of ANOVA revealed that there were no significant differences in attitude towards mathematics between urban and rural pupils. The means obtained by both these groups fell into the moderate category (67-76). This indicated that both urban and rural pupils did not adequately attain this aim.

5.3.3.4 S.C. pupils' attitude towards mathematics over grade

The mean scores attained by HG pupils and SG pupils in the attitude scale were computed (see Table 5.33) and analysed statistically.

TABLE 5.33 MEAN ATTITUDE SCORES OF S.C. PUPILS OVER GRADE

GRADE	$\overline{M}$	SD	Signif.
Higher	77,5	11,3	P < 0,01
Standard	70,6	14,0	

From the above table it can be observed that the mean attitude score of HG pupils fell into the high category (>77) whilst that of the SG pupils fell into moderate category. The difference between the means of these two groups was found to be significant ($p < 0,01$). It can therefore be concluded that HG S.C. pupils adequately attained this aim (positive attitude towards mathematics), whilst SG S.C. pupils did not.

5.3.4 Analysis of data from the interest scale

The overall scores obtained by the S.C. pupils (HG and SG combined) in the interest scale were computed and then analysed in terms of certain variables. These are presented below.

5.3.4.1 Overall interest of S.C. pupils in mathematics

As mentioned in paragraph 3.9.2, the index (score) of 6,2 would be used as the norm score to assess S.C. pupils' interest in mathematics. The mean score obtained by the sample of S.C. pupils in the interest scale was calculated and found to be 5,91. Since this score is less than the norm score, it can be concluded that the S.C. pupils as a group did not adequately display a positive interest in mathematics (see objective of this study in paragraph 1.5.2.4).

The number of interest scores falling above and below the norm score was found to be as follows:

Number less than 6,2	:	292 (59,8%)
Number equal to or greater than 6,2	:	196 (40,2%)

When compared with the norm score, it was found that only about 40% of the S.C. pupils adequately displayed a positive interest in mathematics.

5.3.4.2 <u>S.C. pupils' interest in mathematics over gender</u>

The mean interest score attained by male S.C. pupils was 6,2 whilst that of the female pupils was found to be 5,6. The application of ANOVA showed that the difference between the mean scores obtained by males and females was significant ($p < 0{,}01$). However, since the mean score of only the males was equal to the norm score, it can be concluded that only male S.C. pupils displayed an adequate interest in mathematics.

5.3.4.3 <u>S.C. pupils' interest in mathematics over location</u>

The mean interest scores obtained by the two groups, urban and rural pupils, were computed and found to be 5,7 and 6,3 respectively. The application of ANOVA showed a significant difference between the means, with rural S.C. pupils displaying a higher positive interest in mathematics. When compared against the norm score, it can be concluded that urban pupils did not adequately display a positive interest in mathematics, whilst rural pupils did.

5.3.4.4 S.C. pupils' interest in mathematics over grade

The mean scores obtained by HG pupils and SG pupils in the interest scale were computed (see Table 5.34) and analysed statistically.

TABLE 5.34 MEAN INTEREST SCORES OF S.C. PUPILS OVER GRADE

GRADE	$\overline{M}$	SD	Signif.
Higher	6,3	1,9	p < 0,01
Standard	5,4	2,0	

It can be observed from the above table that the mean interest score of HG pupils was above the norm score (6,2), whilst that of the SG pupils was below the norm score. The difference between the mean scores of HG and SG pupils was found to be significant ($p < 0{,}01$). When compared with the norm score, it can be concluded that HG S.C. pupils displayed an adequate positive interest in mathematics, whilst SG pupils did not.

The discussion and interpretation of the above findings from the pupil questionnaire (attitude and interest scales) are presented in paragraph 5.5.

5.4 ANALYSIS OF DATA PERTAINING TO THE TEACHER QUESTIONNAIRE

5.4.1 Background information on mathematics teachers

As pointed out earlier, information on pupils' mathematical performance and attainment of the aims and objectives was elicited from all the mathematics teachers who taught S.C. mathematics pupils at the sample schools in 1991 and 1992 (see objective of this study in paragraph 1.5.2.5). Altogether, a total of 80 teachers taught the standard 10 classes at these schools. It would be useful to consider some of these teachers' biographical data before discussing their responses. This information is summarised under the following headings.

5.4.1.1 Distribution of teachers according to gender, location and grade

Table 5.35 below shows the distribution of the sample of teachers used in study, according to gender, location of schools and grade of pupils taught.

TABLE 5.35 DISTRIBUTION OF TEACHERS ACCORDING TO GENDER, LOCATION AND GRADE

	No.	%	Signif.
GENDER: Male	70	87,5	$p < 0,01$
Female	10	12,5	
LOCATION: Urban	59	73,8	$p < 0,01$
Rural	21	26,2	
GRADE: Higher	36	45,0	$p < 0,01$
Standard	44	55,0	
TOTAL	80	100,0	

From the above table, it is evident that the teaching force involved in this part of the study consisted predominantly of males. The difference between the number of males and females in the sample was significant ($p < 0,01$). Likewise the difference between the numbers of urban and rural teachers as well as HG and SG teachers were also significant ($p < 0,01$).

5.4.1.2 Level of teachers

Table 5.36 shows the distribution of teachers according to levels (status held).

TABLE 5.36 LEVEL (STATUS) OF TEACHERS

LEVEL	NO.	%
Educator level 1	57	71,2
Head of Department (HOD)	22	27,5
Principal	1	1,3
TOTAL	80	100,0

Of the 80 teachers involved in this study, 28,8 % (23) held promotion posts (Head of Department-Mathematics or Principal) and the remaining 71,2 % (57) were level 1 teachers.

5.4.1.3 Teaching experience of teachers

It was observed that some 51% of the teachers had between 7 to 12 years of teaching experience while another 25% were teaching for more than 18 years. Only about 14% of the teachers had entered the profession within the course of the last 6 years.

It was also observed that the teachers' experience in teaching mathematics was more or less the same as their total teaching experience. In other words, almost all the teachers were involved in the teaching of mathematics since joining the profession.

Table 5.37 reflects the experience of teachers in teaching mathematics at the standard 10 level.

TABLE 5.37 EXPERIENCE OF TEACHERS IN TEACHING MATHEMATICS AT THE STANDARD 10 LEVEL

YEARS	NO.	%
0-5	36	45,0
6-10	21	26,2
11-15	9	11,3
+ 15	14	17,5
TOTAL	80	100,0

From the above table, it can be observed that 36 (45%) of the teachers had under 6 years of experience in teaching mathematics at the S.C. level. Approximately 38% of the teachers had between 6 and 15 years of experience in teaching mathematics at this level. A substantial percentage (17,5%) had taught mathematics at the standard 10 level for more than 15 years.

5.4.1.4 Academic and professional background of the mathematics teachers

Table 5.38 shows the academic and professional qualifications of the sample of mathematics teachers used in the study.

TABLE 5.38 ACADEMIC AND PROFESSIONAL QUALIFICATIONS OF THE MATHEMATICS TEACHERS

	NO.	%
Received special training to teach mathematics	77	96,3
Professional qualifications:		
2-year Teachers' Diploma	4	5,0
3-year Teachers' Diploma	50	62,5
1-year post graduate Teachers' Diploma	19	23,8
B.Paed degree	7	8,8
Academic qualifications:		
Highest: Matric or undergraduate	38	47,5
Bachelor's degree	30	37,5
B. Ed	2	2,5
M. Ed	10	12,5
Highest qualification in mathematics		
Matric	7	8,8
Mathematics I or equivalent	41	51,3
Mathematics II	15	18,7
Mathematics III	17	21,2
Mathematics (Hons)	0	0,0

While the sample consisted of all qualified teachers (with 95% having attained a minimum of 3 years professional training), only 3,7% of the teachers had not received any special training to teach mathematics. It was also observed that 52,5% possessed at least a bachelor's degree and 91,2% possessed at least mathematics I or equivalent qualification. From the foregoing, it can be concluded that the teachers in the sample were fairly well qualified and experienced in teaching mathematics at the standard 10 level to express views on the S.C pupils' mathematical performance and their attainment of the aims and objectives of the S.C. mathematics curriculum.

5.4.2 <u>Testing significance of results</u>

As the responses of teachers were analysed in terms of proportions, use was made of the chi-square statistic (see Appendix 8, paragraph 7) to test the significance between the observed and expected frequencies.

5.4.3 <u>Analysis of responses from the teacher questionnaire</u>

As mentioned in Chapter 4, teachers' opinions were to be canvassed in order to provide additional and more indepth information on the attainment of the aims and objectives of the S.C. mathematics curriculum by the S.C. pupils. The teachers involved in this part of the study were required to rate:

1. pupils' attainment of the individual objectives in mathematics learning (questions 1.1 to 18 in the teacher questionnaire - see Appendix 2);
2. pupils' attainment of the 3 groups of objectives - LL, ML and HL (questions 19 to 21);

3. pupils' attainment in mathematics in terms of subjects/sections (questions 22 to 25);
4. pupils' attitude towards mathematics (question 26);
5. pupils' interest in mathematics (question 27); and
6. pupils' attainment of the cognitive aims of the S.C. mathematics curriculum (questions 28-35).

The teachers' responses in respect of the above independent variables (35 questions in all) were analysed in terms of the following dependent variables: location of schools at which teachers taught - urban/rural (location), gender of teacher (gender), teachers' experience in teaching mathematics at the standard 10 level (Exper. TM 10), and grade of pupils taught by teachers - higher/standard (Teachers' grade). The table below illustrates the chi-square and p-value of the dependent variables in terms of teachers' response to the S.C. pupils' attainment of the aims and objectives of the mathematics curriculum. Those variables that were statistically significant have been indicated by an asterisk (*).

TABLE 5.39 THE CHI-SQUARE AND THE P-VALUE OF THE DEPENDENT VARIABLES AGAINST THE INDEPENDENT VARIABLES IN TERMS OF TEACHERS' RESPONSES TO PUPILS' ATTAINMENT OF THE AIMS AND OBJECTIVES OF MATHEMATICS LEARNING

QUESTION	LOCATION		GENDER		EXPER. TM 10		TEACHERS' GRADE	
	x^2	p-value	x^2	p-value	x^2	p-value	x^2	p-value
1.1	3,6	0,46	5,5	0,24	6,7	0,88	29,9	0,00*
1.2	2,5	0,65	4,1	0,39	7,5	0,82	25,7	0,00*
1.3	1.3	0,86	5,6	0,23	17,2	0,14	40,4	0,00*
2	7,4	0,12	2,5	0,64	10,1	0,61	45,5	0,00*
3	12,2	0,02	1,3	0,86	14,5	0,35	29,9	0,00*
4	22.6	0,00*	3,6	0,47	12,2	0,43	30,5	0,00*
5	10,9	0,03	1,5	0,83	9,1	0,69	30,4	0,00*
6	10.0	0,02	0,8	0,84	11,6	0,24	28,9	0,00*
7	3,2	0,37	0.6	0,91	8,1	0,53	18,6	0,00*
8	6,9	0,14	3,1	0,54	13,6	0,33	13,4	0,01*
9	12,4	0,01*	1,5	0,69	7,4	0,60	25,5	0,00*
10	7,2	0,12	8,0	0,09	20,6	0,06	29,8	0,00*
11	3,8	0,44	6,0	0.20	25,2	0,01*	24,5	0,00*
12	4,8	0,31	4,4	0,36	9,4	0,67	32,5	0,00*
13	1,9	0,60	6,1	0,11	5,9	0,75	29,4	0,00*
14	1,6	0,67	3,7	0,30	7,5	0,59	21,3	0,00*
15	2,6	0,63	3,8	0,44	12,0	0,45	27,5	0,00*
16	4,1	0,25	3,2	0,38	6,3	0,71	25,8	0,00*
17	1,5	0,83	2,2	0,71	14,1	0,30	13,2	0,00*
18	3,2	0,36	1,2	0,76	4,0	0,91	14,9	0,00*
19	2,6	0,46	0,5	0,92	4,5	0,88	26,3	0,00*
20	3,6	0,46	2,5	0,64	15,5	0,22	25,9	0,00*
21	1.7	0,64	2,6	0,47	6,1	0,74	33,3	0,00*
22	8,9	0,06	0,9	0,93	12.9	0,38	17,0	0,00*
23	4,9	0,18	5,4	0,14	7,7	0,56	38,7	0,00*
24	1,8	0,77	12,8	0,01*	21,2	0,05	25,5	0,00*
25	9,2	0,06	0,7	0,93	16,1	0,19	24,8	0,01*
26	2,5	0,28	2,8	0.24	7,8	0,25	41,7	0,00*
27	1,9	0,39	1,9	0,38	7,5	0,28	32,2	0,00*
28	1,8	0,62	4,0	0,26	7,1	0,63	33,9	0,00*
29	3,2	0,36	3,2	0,37	6,1	0,73	21,0	0,00*
30	1,9	0,39	1,4	0,85	11,4	0,49	17,2	0,00*
31	1,0	0,91	1,3	0,86	14,8	0,25	6,7	0,16
32	7,2	0,13	1,7	0,80	13,4	0,34	9,0	0,06
33	6,7	0,08	3,1	0.37	13,6	0,14	20,5	0,00*
34	3,4	0,50	3,5	0,48	13,5	0,34	19,7	0,00*
35	2,8	0,58	7,0	0.14	7,1	0,85	16,4	0,00*

A scrutiny of the above table reveals that the differences in the responses were generally not significant when considered in terms of teachers' location, gender and experience in teaching mathematics at the standard 10 level. However, the opinions of teachers in terms of the grade of pupils (HG or SG classes) they taught differed significantly in almost all the items in the questionnaire (independent variables). In view of the above observations, it was decided to provide detailed analyses and discussion of the independent variables in terms of teachers' grade only.

5.4.3.1 Effect of teachers' grade on S.C.pupils' attainment of the individual objectives

The HG and SG teachers of S.C. classes were required to rate their pupils' mathematical ability in respect of twenty individual objectives. As illustrated earlier (see Table 5.39), after cross-tabulation of the dependent variables (individual objectives) and the grade teachers taught and after the application of the chi-square test of significance, all the variables were found to be statistically significant ($p < 0{,}01$). However, the researcher has chosen only one of these variables for detailed analysis, as shown below:

STATEMENT: To apply concepts to mathematical problems.
(Question 12 in Table 5.39)

TABLE 5.40 FREQUENCY DISTRIBUTION OF RESPONSES ACCORDING TO GRADE TEACHERS TAUGHT

T's GRADE	EXCELL.	GOOD	AVERAGE	FAIR	POOR	TOTAL	$\overline{M}$
Higher	0 0,0	8 22,2	23 63,9	4 11,1	1 2,8	36 45,0	2,9
Standard	0 0,0	0 0,0	16 36,3	20 45,5	8 18,2	44 55,0	3,8
TOTAL	0 0,0	8 10,0	39 48,7	24 30,0	9 11,3	80 100,0	3,4

According to Table 5.40, 8 (22,2%) of the HG teachers indicated that their (HG) pupils' ability to apply concepts to mathematical problems was good. Twenty-three (63,9%) indicated that their pupils' ability in this regard was average while 4 (11,1%) indicated that their pupils' ability was fair.

On the other hand, none of the SG teachers considered their (SG) pupils' ability to apply concepts to mathematical problems to be good. Only 16 (36,3%) of the SG teachers considered this ability amongst their pupils to be average, while 20 (45,5%) indicated this ability to be fair and 8 (18,2%) to be poor.

Taken as a combined group of HG and SG teachers, 47 (58,7%) of the teachers rated S.C. pupils' ability to apply concepts to mathematical problems as being average to good. However, the mean response of this group of teachers indicated that the attainment of this objective by the S.C. pupils was about average.

According to the teachers' responses, it is evident that HG pupils' attainment of this objective was better than that of the SG pupils. The difference in the views between the HG and SG teachers in this regard is highly significant: $x^2 = 32,5$; $p < 0,01$.

5.4.3.2 Effect of teachers' grade on S.C. pupils' attainment of the 3 groups of objectives (LL, ML and HL)

Cross tabulations of the independent variables and the dependent variable (grades taught by teachers) and the application of the chi-square revealed a significant difference in the responses of HG and SG teachers with regard to pupils' attainment of the 3 groups of objectives - lower level, middle level and higher level cognitive objectives ($p<0,01$) (see questions 19-21 in Table 5.39).

The mean ratings of the HG teachers, the SG teachers and the combined group of HG and SG teachers for each of the 3 cognitive levels were found to be as follows:

TABLE 5.41 MEAN RATINGS OF TEACHERS FOR S.C. PUPILS' ATTAINMENT OF THE 3 GROUPS (LEVELS) OF COGNITIVE OBJECTIVES

	HG	SG	HG + SG	Signif.
LL	2,1	2,9	2,5	$P < 0,01$
ML	2,5	3,1	2,8	
HL	2,7	3,3	3,0	

The differences between the means were not only significant ($p < 0,01$) but also revealed a strict hierarchical pattern with mean ratings better for lower level objectives and worse for the higher level objectives. Here again, it was evident that, according to the teachers, the HG pupils' attainment in each category of objectives was better than that of SG pupils.

5.4.3.3 Effect of teachers' grade on S.C. pupils' attainment in the different subjects (sections) of the syllabus

The teachers were also requested to rate S.C. pupils' ability in each of the following subjects (sections/content areas) : algebra, trigonometry, analytical geometry and Euclidean geometry. The analysis revealed that there were significant differences in the responses of HG and SG teachers on the pupils' attainment in these sections, with higher ratings being recorded for HG pupils ($p < 0,01$).

In terms of the mean responses, the ability of S.C. pupils in the 4 subjects was rated, in rank order, by the sample of teachers as follows:

TABLE 5.42 MEAN RATINGS OF TEACHERS FOR S.C. PUPILS' ATTAINMENT IN THE 4 SUBJECTS OF THE SYLLABUS

RANK	SUBJECT	$\overline{M}$
1	Algebra	2,4
2	Trigonometry	2,6
3	Analytical geometry	2,9
4	Euclidean geometry	3,4

From Table 5.42, it can be observed that the teachers ranked S.C. pupils' ability as best in algebra and worst in Euclidean geometry.

5.4.3.4 Effect of teachers' grade on S.C. pupils' attitude towards mathematics

The HG and SG teachers were required to rate their pupils' attitude towards mathematics on a 3-point scale. The responses of the teachers are illustrated in Table 5.43 and Figure 5.3.

TABLE 5.43 FREQUENCY DISTRIBUTION OF TEACHERS' RESPONSES ACCORDING TO GRADE FOR S.C. PUPILS' ATTITUDE TOWARDS MATHEMATICS

T'S GRADE	POSITIVE	NEGATIVE	INDIFF.	TOTAL
Higher	32 88,9	0 0,0	4 11,1	36 45,0
Standard	10 22,7	12 27,3	22 50,0	44 55,0
Total	42 52,5	12 15,0	26 32,5	80 100,0

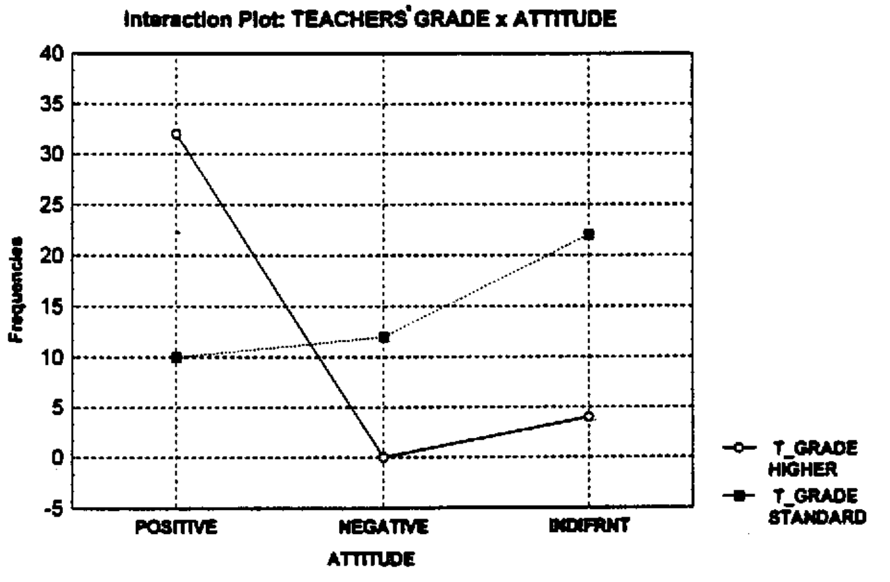


Figure 5.3 Graph showing teachers' responses according to grade for S.C. pupils' attitude towards mathematics

It was observed that 32 (88,9%) of the HG teachers indicated that the attitude towards mathematics of their (HG) pupils was positive. However, only 10 (22,7%) of the SG teachers considered their pupils' attitude towards mathematics as being positive. A large majority of the SG teachers indicated that SG pupils' attitude towards mathematics was negative or indifferent. The difference in responses between HG and SG teachers was highly significant: $x^2 = 41{,}7$; $p < 0{,}01$. It can be concluded that HG pupils were better disposed towards the subject that SG pupils.

Taken as a combined group, just over half of the HG and SG teachers (52,5%) considered the attitude of S.C. pupils towards mathematics as being positive.

5.4.3.5 <u>Effect of teachers' grade on S.C. pupils' interest in mathematics</u>

The HG and SG teachers were also required to rate their pupils' interest in mathematics. The responses of the teachers are shown in Table 5.44.

TABLE 5.44 **FREQUENCY DISTRIBUTION OF TEACHERS' RESPONSES ACCORDING TO GRADE FOR S.C. PUPILS' INTEREST IN MATHEMATICS**

T'S GRADE	POSITIVE	NEGATIVE	INDIFF.	TOTAL
Higher	30 83,3	1 2,8	5 13,9	36 45,0
Standard	10 22,7	12 27,3	22 50,0	44 55,0
Total	40 50,0	13 16,3	27 33,7	80 100,0

The majority of the HG teachers (83,3 %) indicated that the interest in mathematics of their (HG) pupils was positive. However, only 10 (27,7%) of the SG teachers reflected that the interest of their (SG) pupils in mathematics was positive. The remainder of the SG teachers (77,3%) indicated that SG pupils' interest in mathematics was negative or indifferent. The difference in the responses between the HG and SG teachers was highly significant: $x^2 = 32,2$; $p < 0,01$. It can be concluded that HG pupils displayed a more positive interest in mathematics than SG pupils.

According to the group of HG and SG teachers, only 50% of the S.C. pupils displayed a positive interest in mathematics.

5.4.3.6 Effect of teachers' grade on S.C. pupils' attainment of the cognitive aims

The teachers were required to rate the extent to which the cognitive aims of the S.C. mathematics curriculum was being attained by the S.C. pupils. Cross tabulations of the independent variables with the teachers' grade and the application of the chi-square test of significance revealed that the difference between the responses of HG and SG teachers was significant in 6 out of the 8 cases ($p < 0,01$) (see questions 28 - 35 in Table 5.39). The mean responses of HG teachers indicated that the cognitive aims were being attained "to a large extent" by the HG pupils while the mean responses of SG teachers indicated that the cognitive aims were being attained "to a reasonable extent" by SG pupils.

In terms of the mean responses of HG and SG teachers combined, it was found that the attainment of the cognitive aims by S.C. pupils could be ranked in the following order (highest to lowest).

1. The acquisition of mathematical knowledge.
2. The acquisition of mathematical proficiency.
3. The development of accuracy.
4. The ability to estimate answers.
5. The ability to verify answers (where applicable).
6. The development of mathematical insight.
7. The development of clarity of thought.
8. The ability to make logical deductions.

From the above, it can be observed that, in terms of the teachers' responses, the rank order in which the cognitive aims were being attained by S.C. pupils moved hierarchically (highest to lowest) from the lower level aims to the higher level aims. It would be recalled that a similar pattern was observed in respect of S.C. pupils' attainment of the 3 levels of cognitive objectives in the mathematics test (see paragraph 5.2.8.1).

5.4.3.7 Teachers' responses on curriculum development issues

(1) The HG and SG teachers were requested to indicate the topics in the respective mathematics syllabuses that pupils had difficulty in understanding. The following topics were considered by some teachers to be difficult for S.C. pupils.

NB: Only those topics identified by a minimum of 20% of the teachers as being difficult have been listed.

FOR HG PUPILS

1. Calculus: Rate of change (30,5%)
2. Calculus: Maxima and minima values (27,7%)
3. 3-D problems in trigonometry (27,7%)

FOR SG PUPILS

1. Solving non-routine problems in geometry (46%)
2. Calculus (25%)
3. Loci (in analytical geometry) (25%)
4. Proportional intercepts (20,5%)
5. Compound increase and decrease (20,5%)

(2) Approximately 20% of the SG teachers considered the following topics to be unsuitable for SC pupils, and hence recommended that they be dropped from the syllabus.

1. Compound increase and decrease
2. Loci (in analytical geometry)

(3) A little over 20% of the HG teachers indicated that the following topics should be included in the S.C. mathematics syllabus.

1. Vectors
2. Simple integral calculus
3. Statistics

The discussion and interpretation of the findings from the teacher questionnaire are presented in the next section.

5.5 DISCUSSION AND INTERPRETATION OF THE RESULTS

In order to evaluate the S.C. pupils' attainment of the cognitive and affective aims and objectives of the S.C. mathematics curriculum, information was gathered using three research instruments, namely a mathematics test (HOD's S.C. mathematics papers and scripts of 1991), a pupil questionnaire (attitude and interest scales) and a teacher questionnaire. In the preceding sections, detailed statistical analyses of data obtained from these research instruments were presented. In this section, the results of the statistical analyses of data are discussed and interpreted.

5.5.1 S.C. pupils' attainment of the 3 levels of cognitive objectives in terms of overall mathematical performance

The overall mean scores obtained by the S.C. pupils in the 3 levels of cognitive objectives in mathematics (LL, ML, and HL) are reflected in Table 5.12. The differences between the means were not only significant ($p < 0,01$) but they also revealed a strict hierarchical pattern which was expected. This clearly indicated that the classification into cognitive levels of the various questions set in the 1991 S.C. Mathematics Examination of the HOD was fairly accurately done by the expert teachers who assisted with this task. The teachers of S.C. classes were also requested to rate their pupils' ability (attainment) in these 3 levels of cognitive abilities. The mean responses of teachers also revealed a strict hierarchical pattern (see Table 5.41), with mean ratings being better for the lower level objectives and worse for the higher level objectives. The findings also illustrated that S.C.

teachers are adequately differentiating between the different cognitive levels in their teaching of mathematics in the classroom.

As the study utilised norm scores that were established in terms of T-scores (standard scores), the means scores attained by the S.C. pupils were converted to T-scores and compared against the norm scores. It was found that the mean T-scores obtained by the 1991 S.C. pupils (HG and SG combined) in each of the 3 levels of cognitive objectives were greater than the respective norm scores. This clearly demonstrated that the lower level, middle level, and higher level cognitive objectives in mathematics were adequately attained by the S.C. pupils.

Further analyses according to gender and location were carried out, in order to obtain any significant trends in the attainment. The means obtained in the 3 cognitive levels by male and female S.C. pupils in mathematics are shown in Table 5.14. The differences between the means, though small, were found to be significant ($p < 0,01$), with better performance in favour of males. Orton (1987: 118) draws attention that evidence from around the world indicates that sex-related differences in mathematical ability is not consistent. For example, in Russia, Krutetskii (1976) concluded from his studies of the mathematical ability of post-primary boys and girls that there was no clear evidence of any difference. Conclusions based on results from the California State Assessment of Mathematics (1978), as reported by Fennema (1981:96), revealed the following about sex-related differences:

"An analysis of the results by sex showed that girls do consistently better than boys in computations with numbers, fractions and decimals. The girls also outperformed boys in simple one-step word problems. However, the committee found that boys typically scored higher on word problems that were either multiple-step problems or required more reasoning ability". With regard to the fourth NAEP assessment in mathematics held in the United States in 1986, Swafford, Silver and Brown (1989: 99-100) report that few gender-related differences in mathematics achievement were found at ages 9 and 13, but at age 17, there had been small yet significant differences with males scoring higher than females. Differences were particularly evident among 17-year olds at the level of questions testing moderately complex mathematical procedures and reasoning as well as questions involving multi-step problem solving and algebra (higher abilities). However, it is to be noted that in this study it was found that the performance of females was significantly lower than that of males in the knowledge and skills, understanding and higher abilities categories.

Notwithstanding the above, it was found that the mean scores obtained by both males and females were higher than the mean T-scores for each of the 3 cognitive levels. This indicated that both male and female S.C. pupils adequately attained all 3 levels of cognitive objectives in mathematics.

The means obtained by urban and rural S.C. pupils in the 3 levels of cognitive objectives are shown in Table 5.16. The difference between the mean scores obtained by these two groups was found not to be significant ($p > 0,01$). This was a pleasing observation as it indicated that able and less able S.C. pupils were equally distributed in urban and rural areas. This is supported by the fact that the computed mean IQ scores of these two groups were found to be not significant (see Table 5.2). From the pupils' performance, it was evident that senior mathematics teachers were also fairly well distributed over urban and rural schools. It was noted that some 40% of the rural schools had a Head of Department in mathematics.

From the discussions presented in respect of the attainment in the 3 cognitive levels, it is clearly evident that the 1991 S.C. pupils (HG and SG combined), both males and females as well as urban rural pupils, adequately attained the 3 cognitive levels of mathematical abilities - LL, ML and HL (see objective of this study in paragraph 1.5.2.1). The findings negate the views of the HOD mathematics educators that the S.C. pupils' mathematics performance in the S.C. examination is unsatisfactory. Also, the findings negate the assertion made by the HOD mathematics examiners that S.C. pupils are not adequately attaining the higher level mathematical abilities (see paragraph 1.3). Perhaps the examiners' comments in this regard applied only to a limited number of candidates whose (poor) performance has had no significant adverse effect on the performance of S.C. candidates as an entire group. In view of the findings, the null hypotheses that S.C. pupils are not adequately attaining the lower level objectives, the middle level objectives and the higher level objectives are therefore rejected.

5.5.2 S.C. pupils' attainment of the 3 levels of cognitive objectives in the 4 subjects (sections) of the syllabus

The mathematics curriculum offered by S.C. pupils comprised 4 major sections: algebra, trigonometry, analytical geometry and Euclidean geometry. The findings revealed that while all 3 cognitive levels were adequately attained in algebra, trigonometry and analytical geometry, the lower and higher cognitive levels were not adequately attained in Euclidean geometry. Further analyses were carried out over gender and location. It was found that male and urban pupils failed to adequately attain the lower level objectives in Euclidean geometry while females and rural pupils failed to adequately attain both the lower and higher level objectives in this section.

In all 3 cognitive levels, the mean performance of males was significantly better than that of females in the different sections including geometry ($p < 0,01$). Several research studies have noted girls' poor performance in certain aspects of geometry than that of males. For example, the APU Secondary Survey (1982), as reported by Orton (1987:120), noted that girls' performance was lower than males in spatial ability in geometry. Fennema (1981) reports on the findings from California State Assessment of Mathematics in grades 6 and 12 (1978) as follows: In geometry, the girls scored higher than the boys on questions involving recall and identification of geometric shapes, while boys achieved higher than girls on items dealing with spatial relationships and reasoning ability. Likewise, Swafford et al. (1989:100) mention that the 1986 NAEP assessment results showed a significant advantage for males in geometry and measurement at grades 3 and 11. Females tended to outperform males in the area of knowledge and skills while males showed a consistent advantage in the area of higher-level applications. In this study, it was found that boys and girls failed to attain the lower level objectives in (Euclidean) geometry. However, only the girls failed to adequately attain the higher level objectives in geometry (see Table 5.21).

In terms of the mean performance for each section in the mathematics test, S.C. pupils' performance in the different sections can be ranked as follows: trigonometry, algebra, analytical geometry and Euclidean geometry (see Table 5. 18). The differences between the means were found to be significant ($p < 0,01$). From the above, it can be observed that S.C. pupils performed best in the mathematics test in trigonometry and worst in Euclidean geometry. The teachers however rated S.C. pupils' performance as best in algebra and worst in Euclidean geometry (see Table 5.42). The findings revealed that there was concurrence between teachers' rating and pupils' performance in the mathematics test for S.C. pupils' worst subject, namely, Euclidean geometry. It would appear that teachers have certain reservations on S.C. pupils' attainment in geometry. It is clear that some of the S.C. pupils had not adequately studied their "bookwork" and hence were unable to satisfactorily apply the concepts, principles and rules to novel problems which are commonly set in geometry in examinations. It would be noted that some 46% of the SG teachers had indicated that their pupils have difficulty in solving non-routine (novel) problems (see paragraph 5.4.3.7).

In terms of the findings and discussion presented above, it can be concluded that the 1991 S.C. pupils adequately attained all 3 cognitive levels in algebra, trigonometry and analytical geometry. However, some of the pupils, both male and female as well as urban and rural, failed to adequately attain one or both of the lower level and higher level cognitive objectives in Euclidean geometry (see objective of this study in paragraph 1.5.2.3).

5.5.3 S.C. pupils' attainment of the 3 levels of cognitive objectives over IQ categories

As IQ was used as one of the dependent variables, the S.C. pupils were grouped into five IQ categories, in order to investigate each group's attainment of the 3 levels of cognitive objectives. The findings revealed that S.C. pupils who possessed IQ scores of less than 110 failed to adequately attain all 3 levels of cognitive objectives, in their overall mathematical performance (see Table 5.25). Further analyses over gender and location also revealed a similar pattern of attainment. On computing the percentage of S.C. pupils with IQ < 110, it was found that 41,1% of the SG pupils fell into this group, as compared with only 10,6% of HG pupils. It can therefore be concluded that more of the SG pupils failed to adequately attain the 3 levels of cognitive abilities. Noting the poor mathematical performance of S.C. pupils with IQ less than 110, it is possible that such pupils did not offer mathematics by choice, but took the subject because it was part of a package (subject set) offered by the school. A further analysis over subjects showed that S.C. pupils with IQ less than 110 did not adequately attain one or more of the 3 levels of cognitive objectives in the various subjects. However, S.C. pupils with IQ between 110 and 119 also failed to adequately attain the lower and higher level objectives in Euclidean geometry.

Generally, then, the findings have shown that S.C. pupils with IQ greater than 110 (higher average and superior) adequately attained the 3 levels of cognitive objectives, whilst S.C. pupils with IQ less than 110 did not (see objective of this study in paragraph 1.5.2.3). It is clear from the above that S.C. pupils with higher IQ's (>110) have greater potential for success in attaining all the 3 levels of cognitive objectives in their mathematical performance. Several research studies, for example, the HOD (1980) and Moodley (1981), have demonstrated that there is a positive and significant correlation between IQ and attainment in mathematics.

5.5.4 <u>S.C. pupils' attainment of the 3 levels of cognitive objectives over grade</u>

The mean scores attained by HG and SG pupils in the 3 cognitive levels, in the mathematics test, are reflected in Table 5.27. The findings revealed that the differences between the means attained by these two groups were significant ($p < 0,01$). This result was expected as the more able pupils generally offer mathematics on the HG. This is substantiated by the fact that the mean IQ score of the HG group was significantly higher than that of the SG group (see Table 5.2). Notwithstanding the above, the mean T-scores (see Table 5.28) revealed that while

HG pupils adequately attained all the 3 levels of cognitive objectives, the SG pupils failed to attain the higher level objectives. It is apparent that S.C. teachers recognise the ability of HG pupils as more superior than SG pupils. When rating the extent to which the individual and groups of objectives were being attained by the S.C. pupils, in all the cases the mean responses of HG teachers were significantly higher than that of SG teachers.

In view of the trends observed in the other analyses, a further analysis was carried out over subjects. It was found that the HG pupils adequately attained all the cognitive levels in the 4 subjects. However, the findings showed that SG pupils failed to adequately attain the lower level and the higher level objectives in Euclidean geometry (see objective of this study in paragraph 1.5.2.3).

5.5.5 S.C. pupils' attitude towards mathematics

A minimum mean score of 77 on the attitude scale used, was taken as the norm score to reflect S.C. pupils' display of an adequate positive attitude towards mathematics. The mean score attained by the S.C. pupils (as a combined group) was significantly lower than the norm score ($p < 0{,}01$). Hence, it can be concluded that the group of S.C. pupils failed to adequately realise this aim, namely, display a positive attitude towards mathematics (see objective of this study in paragraph 1.5.2.4). The results also revealed that there were no significant differences in attitude over location and gender. In his study, Moodley (1981:249) also found that there was no sex differences in attitude towards mathematics.

However, when considered over grade, the mean scores attained by HG and SG pupils revealed that HG pupils adequately attained this aim, whilst SG pupils did not. Concurrence in this regard was noted from the teachers' responses on their S.C. pupils' attitude towards mathematics. Whilst some 89% of HG teachers indicated that the attitude towards mathematics of HG pupils was positive, only about 23% of the SG teachers considered their SG pupils' attitude towards mathematics as being positive. Several studies (e.g., Moodley 1981:234; Cheung 1988:211; Rambaran 1989:379) have reported that there is a significant positive correlation between positive attitude and attainment in mathematics, showing that the more positive the students' attitude towards mathematics, the higher their achievement in mathematics. In this study, it was found that the mean attainment of HG pupils (who displayed a higher positive attitude towards mathematics) was significantly higher than that of SG pupils ($p < 0,01$) (see Tables 5.27 and 5.28) Worthy of note is the assertion made by Callahan (1971), Schofield (1980) and Schofield (1982) that relatively definite attitudes about mathematics have been developed by the time pupils are in the junior secondary classes.

In view of the findings, the null hypothesis that S.C. pupils did not display an adequate positive attitude towards mathematics is accepted ($p < 0,01$). However, if considered separately over grade, the null hypothesis is rejected in the case of HG pupils ($p < 0,01$).

5.5.6 S.C. pupils' interest in mathematics

The mean score obtained by the group of S.C. pupils in the interest scale was found to be significantly less ($p < 0,01$) than the norm score. In view of this observation, it can be concluded that the group of S.C. pupils failed to display an adequate positive interest in mathematics (see objective of this study in paragraph 1.5.2.4). While the findings showed significant differences in interest in mathematics between males and females and urban and rural pupils ($p < 0,01$), the means of all four groups were also found to be less that the norm score.

However, when considered over grade, the mean score of only the HG pupils was found to be greater than the norm score, reflecting that the HG pupils displayed an adequate positive interest in mathematics. It was found that the S.C. teachers also rated the HG pupils' interest in mathematics as higher than that of SG pupils. Kulm (1990:74) draws attention that the NCTM Standards includes a description of mathematical disposition, which is an important aspect of many theories of problem solving and higher order thinking. He adds that performance on higher order thinking tasks is generally affected by students' <u>interest</u>, confidence and perseverance. In this study, it was observed that SG pupils who were found to be less positively disposed towards mathematics did not adequately attain the higher level cognitive objectives (see Table 5.28). Orton (1987:122) points out that it is important to realise that a decision to study mathematics does not imply a positive liking for the subject. He cites the study carried out by Russel (1983) who found many sixth-form boys studying mathematics who did not like the subject. They had opted for mathematics because they considered it a useful subject.

In view of the findings, the null hypothesis that S.C. pupils did not display an adequate positive interest in mathematics is accepted ($p < 0,01$). However, if considered separately over grade, the null hypothesis is rejected in the case of HG pupils ($p < 0,01$).

5.5.7 <u>S.C. pupils' attainment of the cognitive and the affective aims and objectives of the S.C. mathematics curriculum</u>

The aim of this investigation was to ascertain whether or not the S.C. pupils (as a combined group of HG and SG pupils) were adequately attaining the desired proficiency in the cognitive and affective aims and objectives of the S.C. mathematics curriculum. With regard to the attainment of the cognitive objectives, the S.C. pupils' attainment was to be examined in the 3 levels of cognitive objectives in terms of their overall mathematical performance. In terms of the theoretical construct used in this regard (see paragraph 3.6 of Chapter 3), it was averred that S.C. pupils' successful attainment in each of the 3 levels of cognitive objectives in the S.C.mathematics examination would be taken to indicate that the corresponding cognitive aims (or sub-parts) are also being successfully attained by the pupils. The findings from this study have shown that in terms of their overall mathematical performance, the S.C. pupils (as a combined group of HG and SG pupils) adequately attained the norm T-scores in each of the 3 levels of cognitive objectives, namely, lower level cognitive outcomes (knowledge and skills), the middle level cognitive outcomes (comprehension/understanding) and the higher level cognitive outcomes (application, analysis, synthesis and evaluation/creative). In view of the findings and in terms of the theoretical construct used, it is concluded that the S.C. candidates who wrote the 1991 S.C. Mathematics Examination of the HOD also successfully attained the cognitive aims of the S.C mathematics curriculum (see objective of this study in paragraph 1.5.2.2). The null hypothesis that the cognitive aims of the S.C. mathematics curriculum are not being attained by S.C. pupils is therefore rejected.

With regard to the two affective aims (positive attitude towards and interest in mathematics), from the discussions presented in paragraphs 5.5.5 and 5.5.6, it is evident that S.C. pupils failed to successfully attain the mean norm scores in respect of attitude and interest. This demonstrated that the S.C. pupils failed to attain the affective aims of the S.C. mathematics curriculum (see objective of this study in paragraph 1.5.2.4). The null hypothesis that the affective aims of the S.C. mathematics curriculum are not being attained by S.C. pupils is therefore accepted.

As regards the attainment of the cognitive and affective aims (and objectives), the study showed a close correspondence between the results obtained from the quantitative analysis (scores obtained from the mathematics test) and those obtained from the qualitative analysis (teachers' ratings). In paragraph 3.7.4, it was mentioned that the following values (mean ratings of teachers) would be used as norm values to evaluate teachers' perceptions of the attainment of the 3 levels (categories) of cognitive aims by the S.C. pupils:

For lower level aims (LL): $\overline{M} < = 2{,}57$

For middle level aims (ML): $\overline{M} < = 2{,}81$

For higher level aims (HL): $\overline{M} < = 3{,}02$

The mean ratings of teachers also indicated that the S.C. pupils adequately attained all the 3 categories of cognitive aims, as can be observed from the table below.

TABLE 5.45 **MEAN RATINGS OF TEACHERS FOR S.C. PUPILS' ATTAINMENT OF THE COGNITIVE AIMS OF THE S.C. MATHEMATICS CURRICULUM**

	AIM	LEVEL(S)	MEAN RATING OF TEACHERS
1.	The acquisition of mathematical knowledge	LL	2,50
2.	The acquisition of mathematical proficiency	LL,ML,HL	2,55
3.	The development of accuracy	LL,ML	2,56
4.	The ability to estimate answers	ML	2,76
5.	The ability to verify answers	ML	2,78
6.	The development of mathematical insight	HL	2,80
7.	The development of clarity of thought	ML,HL	2,90
8.	The ability to make logical deductions	HL	3,00

Several research studies (e.g., Moodley 1981:216; Norton 1991a:65) have demonstrated that teachers' qualitative assessment of S.C. pupils' attainment are fairly reliable. When completing the teacher questionnaire, the S.C. teachers were requested to rate the mathematical ability of their pupils taking into account their global performance (in class exercises, tests, examinations, homework exercises, assignments, etc). Furthermore, in conducting this exercise, teachers were requested to consider the mathematical ability of not only their present S.C. pupils, but also the S.C. pupils whom they had taught previously over the years at their schools (see Appendix 2). Noting that the teachers' observation regarding the S.C. pupils' mathematical attainment spanned over several years, it is possible to make a generalisation of the findings of this study, namely, that Indian S.C. pupils in general are adequately attaining the desired proficiency in the cognitive aims and objectives of the S.C. mathematics curriculum, but are not doing so in respect of the affective aims and objectives (positive attitude towards and interest in mathematics).

5.5.8 Information pertaining to curriculum development obtained from the study

It is possible that when carrying out analyses based on overall mathematical performance of pupils, certain pertinent information sometimes become "suppressed". Consequently, in order to investigate this aspect, indepth (finer) analyses were undertaken with respect to the S.C. pupils' attainment in the 3 levels of cognitive objectives in terms of certain selected variables, namely subjects, IQ and grade. The findings have revealed that, generally, while S.C. pupils' performance was satisfactory in algebra, trigonometry and analytical geometry, the pupils' performance was poor in Euclidean geometry. Further analyses revealed that this applied particularly to S.G. pupils, both males and females and urban and rural pupils. On the other hand, S.C. pupils with IQ scores less than 110, the majority of which comprised SG pupils, performed poorly in their overall attainment as well as in the different subjects, especially in algebra and Euclidean geometry. The findings from the teacher questionnaire also indicated that SG pupils' performance was significantly lower than their HG counterparts. In addition, it was noted that only SG teachers (some 46%) indicated that SG pupils experienced difficulty in solving non-routine (novel) problems.

In general, then, the findings from the study have also shown evidence for the need for curriculum development in certain areas, particularly in respect of Euclidean geometry for SG S.C. pupils (see objective of this study in paragraph 1.5.2.3).

5.6 CONCLUSION

With the aid of the 3 research instruments, the aim and the purposes of the study were accomplished. The interpretation of the statistical analyses has revealed that S.C. candidates (as a group of HG and SG pupils) are successfully attaining the cognitive aims and objectives of the S.C. mathematics curriculum. However, the affective aims are not being adequately attained. Indepth analyses have revealed that there are certain shortcomings in the attainment of the lower and higher level cognitive objectives in Euclidean geometry, particularly amongst SG pupils.

In general, several highly significant implications and considerations have emanated from this study. These together with certain conclusions and recommendations are discussed in the final chapter.

CHAPTER 6

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

The previous chapter dealt in detail with the statistical analyses of the results as well as with the hypotheses concerning S.C. pupils' attainment of the cognitive and affective aims and objectives of the S.C. mathematics curriculum. The emphasis in this final chapter is on the following 4 basic aspects:

1. Summary of the project which provides a brief outline of how the research was conducted;
2. Summary of the findings;
3. Recommendations on educational aspects and their implications; and
4. Recommendations for future research.

6.1 SUMMARY OF THE PROJECT

This project, which dealt with an evaluation of the efficacy of the aims and objectives of the S.C. mathematics curriculum, provided the researcher with the opportunity to study pertinent research and literature in respect of mathematics education and evaluation, to develop norms to assess the attainment of the cognitive objectives, and to obtain information (both quantitative and qualitative) on the extent to which the cognitive and affective aims and objectives are being realised by S.C. pupils. The procedure followed in this research project is outlined below.

• As this project was concerned with the evaluation of the attainment of aims and objectives, the study was first placed in the context of a subject didactical perspective, noting the philosophical and didactical shifts in mathematics education since the 1980's. Thereafter, a detailed discussion on the aims and objectives of mathematics education at the secondary school level was presented. Sufficient attention was given to classifications of cognitive objectives, especially recent classifications which utilise fewer groups of objectives. While the aspect of educational measurement was broadly covered, due attention was also given to the methods of assessment for the attainment of cognitive objectives and affective objectives (attitude towards and interest in mathematics).

• It has been observed that current literature and recent research studies tend to report on pupils' mathematical performance in terms of broad categories of cognitive outcomes. The cognitive aims of the S.C. mathematics curriculum were analysed with a view to ascertaining the more important cognitive objectives inherent in them. The objectives were then classified in terms of 3 broad categories of mathematical abilities - lower level, middle level and higher level cognitive outcomes. As no norms for the attainment of the cognitive abilities could be identified, these had to be developed by the researcher. For this purpose, 4 research studies on S.C. pupils' mathematical attainment were identified and analysed, which yielded norms for the 3 categories of cognitive outcomes.

• S.C. pupils' performance in the HOD's 1991 S.C. mathematics examination (test) was used to evaluate the pupils' attainment of the cognitive objectives and hence the cognitive aims. Although the S.C. examination question papers are criterion-referenced tests, it was evident from the item analysis that a test which was fairly reliable and valid, was used to evaluate S.C. pupils' attainment of the cognitive objectives and aims. The answer scripts of a sample of 2091 S.C. pupils (760 HG and 1 331 SG) were used. The pupils were drawn from a sample of 31 Indian secondary schools selected from throughout the RSA.

• In order to evaluate S.C. pupils' attainment of the affective aims and objectives, two standardised instruments, namely, an attitude scale and an interest scale, were identified and administered to a sample of 488 S.C. pupils (200 HG and 288 SG) from the same 31 secondary schools, in 1992. As the instruments were standardised, they provided valid results in respect of S.C. pupils' attitude towards and interest in mathematics respectively.

• More indepth information on S.C. pupils' attainment of the aims and objectives was obtained from a total sample of 80 teachers of S.C. mathematics classes from the same 31 schools, in 1992. As the teacher questionnaire proved to be unambiguous, it successfully elicited responses that assisted the researcher at arriving at conclusions and making generalisations about S.C. pupils' attainment of the cognitive and affective aims and objectives of the S.C. mathematics curriculum.

• Data obtained from the mathematics test, the pupil questionnaire (attitude and interest scales), and the teacher questionnaire were statistically analysed with the aid of the computer. The findings are presented below.

6.2 RÉSUMÉ OF THE MOST IMPORTANT FINDINGS BASED ON THE ADMINISTRATION AND ANALYSES OF THE RESEARCH INSTRUMENTS

The research indicated that while S.C. pupils are attaining the desired proficiency in the cognitive aims and objectives of the S.C. mathematics curriculum, the pupils are not adequately attaining the affective aims and objectives (see aim of this study in paragraph 1.5.1).

• More specifically, it was established that in terms of their overall mathematical performance, S.C. pupils (as a combined group of HG and SG pupils):

6.2.1 are adequately attaining the desired proficiency in the 3 levels of cognitive objectives, namely, lower level objectives (knowledge and skills), middle level objectives (understanding/comprehension) and higher level objectives (application, analysis, synthesis and evaluation/creative) (see objective of this study in paragraph 1.5.2.1);

The same conclusions were arrived at when comparisons were made between males and females as well as urban and rural pupils.

6.2.2 are adequately attaining all the cognitive aims of the S.C. mathematics curriculum (see objective of this study in paragraph 1.5.2.2);

6.2.3 are not displaying an adequate positive attitude towards mathematics;

6.2.4 are not displaying an adequate positive interest in mathematics; and

6.2.5 in view of paragraphs 6.2.3 and 6.2.4, are not adequately attaining the affective aims and objectives of the S.C. mathematics curriculum (see objective of this study in paragraph 1.5.2.4).

• The findings also revealed that there is close concurrence in the ratings of teachers and the observations derived from the quantitative analysis, in respect of S.C. pupils' attainment of the cognitive and affective aims and objectives of the S.C. mathematics curriculum (see objective of this study in paragraph 1.5.2.5).

• In the light of the finer analysis carried out, it was further established that there is a need for curriculum development in certain areas (see objective of this study in paragraph 1.5.2.3), as evidenced by the following findings.

* S.C. pupils with IQ less than 100, both males and females as well as urban and rural pupils, are not adequately attaining all the 3 levels of cognitive objectives;
* SG pupils are not adequately attaining the higher level objectives; and
* SG pupils are not adequately attaining the lower level and higher level objectives in Euclidean geometry. Also, S.C. pupils with IQ less than 110 are not adequately attaining one or more of the cognitive levels in the 4 subjects of the syllabus.

From the foregoing, it is clear that the aim and objectives of this study, as enunciated in paragraphs 1.5.1 and 1.5.2 in Chapter 1, had been accomplished.

6.3 RECOMMENDATIONS AND IMPLICATIONS

Arising out of this study, the following recommendations are made:

6.3.1 Recommendation 1

S.C. pupils, particularly SG pupils, should be encouraged and assisted to learn "bookwork" in Euclidean geometry with understanding.

Motivation

The knowledge questions set in the Euclidean geometry section of the mathematics examination (test) generally comprised the proof of theorems (bookwork). When perusing the examination answer scripts, for purposes of extracting marks, it was observed that the majority of the SG pupils and some HG pupils failed to supply the proofs of the theorems tested. A fair percentage of pupils was able to draw the diagram only. It became apparent that pupils are learning the proofs of theorems without proper understanding and without conceptual development of meaning, as they were unable to recall this knowledge successfully in the examination.

Implications

It is essential that memorisation takes place only after the pupil has fully understood the meaning. In order to promote successful learning of theorems and other concepts, etc., teachers need to give careful thought to the preparation of their lessons and to the strategies (teaching methods) to be used that would promote understanding. Due attention would also have to be given to the formulation of precise objectives and the teaching aids required for lessons. Currently, greater emphasis is being placed on the discovery of concepts meaningfully, by the active participation of pupils. All the theorems lend themselves to discovery by induction.

In teaching theorems on chords and perpendiculars, for example, teachers should emphasise that the concept of congruency is used to prove the theorems. Hence the "constructions" that need to be drawn should complete two triangles. Once the concepts have been meaningfully understood, teachers should revise the theorems by supplying some of the steps in the proof and requesting pupils to provide the missing steps. Frequent oral and written testing of proofs of theorems should be undertaken until pupils have mastered the proofs.

6.3.2 Recommendation 2

SG pupils should be given greater exposure to problem solving, particularly in Euclidean geometry, which involves the use of higher level abilities.

Motivation

The majority of the pupils who take mathematics at the S.C. level offer the subject on the SG. This has been evident also in the case of the HOD where some 60% of the S.C. pupils took the subject on the SG. Noting the above, it is important, therefore, that SG pupils should also gain the necessary proficiency in all 3 levels of cognitive abilities, particularly in the higher cognitive abilities. In view of the poor attainment of SG pupils noted in the higher cognitive abilities, it is apparent that SG pupils are not given enough exposure and training in solving non-routine problems in Euclidean geometry. Alternatively, it would appear that the heuristics in solving problems in geometry are not developed or are under-developed in this group of pupils.

Implications

As Euclidean geometry constitutes a fair part of the S.C. syllabus, it would pay teachers and pupils to increase their efforts in this area. Teachers of SG classes should place greater emphasis on higher level abilities in their instructional programmes. Use should be made of carefully constructed exercises with built-in graded examples which lead pupils from the simple to the more complex abilities. In particular, teachers should emphasise the discovery approach to learning Euclidean geometry. It would be useful for teachers to consider adopting the problem solving or the problem-centred approach, as proposed by the NCTM

(1989), in dealing with non-routine problems in this section as well as in other sections (subjects) of the syllabus.

6.3.3 Recommendation 3

A concerted effort should be made to develop positive attitudes towards and interest in mathematics amongst secondary school pupils.

Motivation

Research studies have shown that pupils who display a positive attitude towards and/or interest in mathematics perform significantly better than pupils who adopt a negative or indifferent attitude and/or interest (see paragraphs 5.5.5 and 5.5.6). Since, in this study, it was found that S.C. pupils are not displaying an adequate positive attitude towards and interest in mathematics , it becomes incumbent on all mathematics educators (teachers, school counsellors, subject advisers, etc.) to embark on a conscious programme that would promote S.C. pupils to develop an adequate positive attitude towards and interest in the subject.

Teachers should emphasise to pupils the need for them to study mathematics and the importance of mathematics at the tertiary or at the post school level. Teachers should help pupils to improve their attitude through individual instruction and simple graded exercises which provide pupils with successful experiences. With regard to the latter, pupils should experience success as often as possible and in many varied situations. By using the above approaches it is expected that pupils' attainment in mathematics would improve which in turn may lead to a better attitude towards and interest in the subject.

Several research studies have also shown that attitude towards and interest in mathematics become developed in the primary or junior secondary level. In view of this, it is also recommended that specific efforts to foster or develop positive attitudes and interest should commence well before the senior secondary level.

Implications

With an improved attitude towards and interest in mathematics, S.C. pupils would be motivated to work harder, thereby gaining greater success in multi-step and non-routine problems (higher cognitive abilities). Consequently, this would result in better results being attained in mathematics. Also pupils would be better equipped to enter tertiary institutions or the work place.

6.3.4 Recommendation 4

The norms established for the 3 levels of cognitive abilities should be used by senior secondary mathematics educators in order to monitor pupils' mathematical attainment in these 3 levels of cognitive abilities.

Motivation

Currently, pupils' mathematics achievement is measured in terms of their overall attainment in a mathematics test or examination. No detailed assessments are made in respect of pupils' attainment in the different categories of cognitive abilities. In order to ensure that all the broad levels of cognitive objectives as well as the aims are being adequately attained, pupils' attainment in each of these 3 levels of cognitive abilities needs to be monitored. Norms assist educators to assess whether the desired proficiency levels are being attained by pupils.

As norms have been established and successfully used in this research study, it is recommended that the norms be used by senior secondary teachers, starting in standard 9, to monitor pupils' attainment in the 3 categories of cognitive abilities. Whilst this may not be possible for all class exercises and class tests, such analyses could be carried out for the major tests and examinations.

Implications

By using the norms established, teachers would be in a better position to assess pupils' attainment in the 3 levels of cognitive objectives. By monitoring the achievements of pupils in these 3 levels on a regular basis, teachers would be able to timeously carry out the necessary remedial work in the required areas. By starting in standard 9, teachers would have sufficient time to improve pupils' performance in all the categories of cognitive abilities.

All the above measures would contribute largely towards ensuring that the cognitive objectives, and hence the cognitive aims, of the S.C. mathematics curriculum are adequately attained by pupils in the S.C. examination.

6.3.5 Recommendation 5

The development of any new curriculum/syllabus for mathematics should be based on considerable and widespread classroom-based research and the syllabus should be trial-tested sufficiently before adoption.

Motivation

As already stated in paragraph 1.2.1, historically, curriculum development in the

RSA has been a "top-down" process with limited representation and classroom research being included in the curriculum development process. Laridon (1990 : 21) notes that "in a pell mell haste to implement a new syllabus in mathematics from 1991 onwards in high schools, no research beyond opinionairing is at the base of the new proposals." More recently, the interim Committee of the Heads of Education Departments has proposed new syllabuses and curricula for mathematics, with implementation from 1995. Here again it has been observed that no field research had been undertaken in compiling the "new" syllabuses.

This research has shown that teachers are not entirely happy with all aspects of the S.C. mathematics curricula/syllabuses. It was reported in paragraph 5.4.3.7 that teachers considered certain topics to be too difficult for S.C. pupils and/or that certain new topics need to be introduced into the S.C. mathematics curriculum. What we see is that teachers' inputs in curriculum development is crucial and necessary. The researcher concurs with Laridon (1990:21) who states that what we need in the RSA is proper curriculum development based on classroom research relevant to this country and all its peoples, resulting in some principles from which to proceed.

Implications

Proper curriculum development carried out on the lines indicated above would ensure that syllabuses developed are suitable for all pupils in the country. It must be acknowledged that teachers, and even pupils, are important role players, amongst others, in the curriculum development process. Teachers' active participation in the curriculum development process would give them (teachers) a feeling of worthiness. This in turn would ensure the successful implementation of syllabuses as it is teachers who are the classroom practitioners who are responsible for pupils' realisation of the aims and objectives of the curriculum.

6.3.6 Recommendation 6

When new curricula/syllabuses in mathematics are adopted, attention should be given to an analysis and interpretation of the new specific aims in terms of the different cognitive abilities inherent in them. Furthermore, the tasks of school mathematics should be classified both by content area and by the different types of learning (cognitive abilities) they seek to attain.

Motivation

Besides reflecting the content areas to be covered, syllabus documents compiled in this country contain only the general aims and specific aims for a subject and standards. Although the specific aims are concise statements of intent, they do not adequately indicate to the ordinary classroom teachers the various objectives which need to be pursued and realised. Such an exercise needs to be carried out by expert teachers in conjunction with the curriculum developers and the results passed on to other teachers. An attempt has been made in this study with regard to an analysis and interpretation of the specific aims of the S.C. mathematics curriculum used by the HOD prior to 1995 (see paragraph 3.3.2 of Chapter 3). As some of the specific aims of the new (1995) S.C. mathematics syllabus are the same as the old syllabus, perhaps the information provided in the respective paragraph in this study could be used as a starting point to this exercise. Once drawn up, the document reflecting the analysis and interpretation of the specific aims should be disseminated to teachers of S.C. mathematics and discussed at scheduled orientation courses.

It would also pay the curriculum developers and/or the expert teachers to organise the mathematics curriculum in terms of both subject matter and types of learning or cognitive outcomes (lower level, middle level and higher level abilities). This would ensure that the necessary emphasis is given by classroom teachers to the relevant levels, particularly the higher level cognitive abilities.

Implications

Adequate and lucid supportive materials to teachers would not only make the work of teachers easier, but would also ensure that the respective content areas are being taught to the required depth. This could lead to the adequate realisation of the cognitive aims of the curriculum. Regular forums provided for teachers to interact with one another would assist in building up the morale and confidence of teachers which could motivate them towards achieving greater success in the classroom. The above provisions would also motivate teachers towards active participation in the curriculum development and evaluation process.

6.3.7 Recommendation 7

Teachers should use as many evaluation techniques as possible to evaluate S.C. pupils' attainment in mathematics. Furthermore, teachers' assessment should be included as part (50%) of the promotion requirements of pupils in mathematics in the S.C. examination.

Motivation

Currently, promotion in mathematics in the S.C. examination is based only on a written examination held at the end of grade 12. The written examination in mathematics focuses mainly on the attainment of the cognitive objectives. However, not all the cognitive abilities (aims and objectives) are measured in the written examination. For example, no direct questions are set in the S.C. examination to measure pupils' ability to estimate answers and to verify answers. The written examination also cannot measure growth of the thinking process or the development of skills amongst pupils. As regards the affective domain, the attainment of the affective aims and objectives is not and cannot be assessed in the written S.C. examination.

In order to provide a true picture of S.C. pupils' attainment in all the aims and objectives of the S.C. mathematics curriculum, it is necessary that pupils' global performance in the different aspects/areas be taken into consideration, for example, pupils' performance in class exercises, class tests, assignments and oral work as well as their interest and attitude towards these aspects. The class teacher who works closely with the pupils, is in a good position to assess pupils' performance in these aspects. This study has shown that teachers' assessments of S.C. pupils' performance are fairly reliable. Consideration could be given by teachers to the use of evaluation techniques used in mathematics which are discussed in paragraphs 2.7.8, 2.9 and 2.10 of Chapter 2 of this study.

<u>Implications</u>

Teachers' assessment of pupils' performance, carried out on a continuous basis, would motivate pupils to work consistently throughout the year. The inclusion of teachers' assessment in pupils' promotion in the S.C. examination would ensure that pupils' global performance in the different aspects of the curriculum is taken into account. Teachers' assessment would not only benefit pupils who suffer from examination anxiety, but would also provide teachers and pupils with the scope to explore the curriculum more fully.

6.4 RECOMMENDATIONS FOR FUTURE RESEARCH

In the preceding section, implications of this research were presented specifically in terms of the different aspects of the research. Arising out of the study, there are implications for future research in general.

6.4.1 RESEARCH INTO THE ESTABLISHMENT OF NORMS FOR THE 3 COGNITIVE LEVELS IN MATHEMATICS FOR OTHER PHASES/STANDARDS

For meaningful reporting on pupils' ability and attainment in mathematics, assessments need to be provided in terms of 3 levels of cognitive abilities - lower level, middle level and high level cognitive abilities. This study has successfully developed and applied norms for mathematics attainment in respect of 3 categories, at the S.C. level. In order to monitor progress of pupils from an early age, it is necessary that research be carried out on the establishment of norms for mathematics attainment, in these 3 categories, for grades that constitute the end of a school phase, particularly the senior primary and junior secondary phases.

6.4.2 RESEARCH INTO THE TIME ALLOCATION FOR MATHEMATICS TEACHING AND LEARNING IN STANDARDS 9 AND 10 (GRADES 11 AND 12)

This research has shown that greater emphasis needs to be placed on the attainment of the higher cognitive abilities (problem-solving) in mathematics amongst S.C. pupils, particularly S.G. pupils. Recent developments in mathematics education in overseas countries have recommended the use of the problem-centred approach. Such an approach is also proposed, directly or indirectly, for mathematics teaching and learning in the RSA. As this approach is activity-based and emphasises investigatory/discovery methods (which have implications on teachers' and pupils' time in the classroom), research needs to be conducted in order to ascertain whether or not the present time allocation for the teaching of mathematics in standards 9 and 10 is sufficient for the successful implementation of the new approach.

6.5 GENERAL

Besides contributing towards the establishment of norms for 3 cognitive levels of mathematical attainment at the senior certificate level, this study has successfully investigated and utilised certain procedures and techniques for the evaluation of the attainment of the cognitive and affective aims and objectives of the S.C. mathematics curriculum. With regard to the latter, the writer is of the view that the abovementioned procedures have the potential for applicability in other S.C. subjects also. The findings of this study and their implications justify continued research into the monitoring of pupil attainment, in the 3 cognitive levels, in mathematics at the different levels of pupils' schooling. The recommendations made in this study, if heeded by mathematics educators and curriculum developers, have the potential for improvements in the teaching and learning of mathematics in our schools. As mathematics has assumed great importance in every person's life, any improvements in mathematics education and mathematics performance would also contribute towards the reconstruction and development of the South African society and the empowerment of its people.

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APPENDIX 1

LETTER TO PRINCIPALS

H. RAMBEHARI
43 Bikaner Road
Merebank
DURBAN
4052

1 September 1992

The Principal

.... Secondary School

Dear Sir/Madam

QUESTIONNAIRES : SENIOR CERTIFICATE MATHEMATICS - AIMS AND OBJECTIVES

1. I have taught mathematics at the senior certificate level for a period of 13 years. As a result of certain observations made, I am carrying out an investigation entitled **AN EVALUATION OF THE EFFICACY OF THE AIMS AND OBJECTIVES OF THE SENIOR CERTIFICATE MATHEMATICS CURRICULUM IN THE RSA,** for purposes of a D. ED Degree. It is envisaged that the research would provide valuable information that would have an important bearing on mathematics teaching and learning in the country in the future.

2. The accompanying questionnaires form part of the research programme which involves both a quantitative and qualitative assessment of Senior Certificate candidates' performance in mathematics. Since the answering of the questionnaires by teachers and pupils is very important to this research, your full co-operation in this regard is humbly requested.

3. Kindly be advised that permission has been obtained from the office of the Chief Executive Director to:

    1. Administer the accompanying questionnaires to current Senior Certificate mathematics teachers and pupils; and

    2. Utilise the IQ scores of all the pupils from your school who offered mathematics HG and mathematics SG in the 1991 S.C. Examination.

    N.B. A copy of the letter of approval is attached.

4. You would observe that the names of teachers and pupils are not required anywhere in the questionnaires. You are assured that teachers' and pupils' responses will be treated as strictly confidential and be used solely for research purposes.

5. The questionnaires are to be handled as follows:

   1. <u>Teacher questionnaires</u>

      Teacher questionnaires are to completed by <u>all</u> teachers of std. 10 mathematics HG and SG at your school. Altogether .... copies of these questionnaires are provided.

      Teacher questionnaire (HG), of which .... is/are provided, is to be completed by the teacher(s) who is/are teaching at least ONE mathematics HG class unit.

      Teacher questionnaire (SG), of which .... is/are provided, is to be completed by the teacher(s) who is/are teaching at least ONE mathematics SG class unit.

   2. <u>Pupil Questionnaire</u>

      Altogether .... copies of this questionnaire are provided. N.B. the sample of std. 10 mathematics pupils from your school has already been selected. Hence the pupil questionnaire is to be completed by all the pupils offering mathematics HIGHER GRADE/STANDARD GRADE from the following class unit: STD. 10 ....

6. As regards the IQ scores of the 1991 candidates, schedules reflecting the <u>1991</u> S.C. Examination numbers of the relevant mathematics HG and mathematics SG candidates from your school are attached. Kindly fill in the IQ scores and sex of each candidate in the appropriate columns next to the examination numbers supplied.

7. After completion, but not later than 11 September 1992, the teacher and pupil questionnaires, as well as the schedules reflecting the IQ scores of the 1991 mathematics candidates are to be returned to the researcher. For this purpose, kindly use the stamped self-addressed envelope provided.

8. Thank you very much for your kind co-operation and the time set aside for this research.

Yours faithfully

....

H. RAMBEHARI

SCHEDULE

IQ SCORES AND SEX OF CANDIDATES WHO OFFERED MATHEMATICS ON THE ________ GRADE IN THE 1991 SENIOR CERTIFICATE EXAMINATION

EXAM NO.	IQ SCORE	SEX M/F	EXAM NO.	IQ SCORE	SEX M/F

APPENDIX 2

TEACHER QUESTIONNAIRE

43 Bikaner Road
MEREBANK
4052

1 September 1992

Dear Mathematics Teacher

I have taught mathematics as the senior certificate level for 13 years. For purposes of a D. Ed Degree, I am carrying out an investigation entitled : **AN EVALUATION OF THE EFFICACY OF THE AIMS AND OBJECTIVES OF THE SENIOR CERTIFICATE MATHEMATICS CURRICULUM IN THE RSA.** The investigation entails, amongst others, the completion of a questionnaire by teachers who are currently teaching mathematics at the senior certificate level.

Kindly note that by responding to the questionnaire, you would not only be making a valuable contribution to the investigation, but also be providing information that may have an important bearing on mathematics learning and teaching in the country in the future.

It would therefore be greatly appreciated if you would complete the attached questionnaire with great care. Answer ALL the questions, and please be as frank as you can possibly be. If you have any difficulty in completing the questionnaire, please do not hesitate to contact me on telephone (031) 3606248 (work) or (031) 484867 (home).

You are assured that your responses will be treated as strictly confidential and will only be used for research purposes.

N.B. YOUR NAME IS NOT REQUIRED ANYWHERE IN THIS QUESTIONNAIRE

Kindly hand your completed questionnaire, sealed in the envelope provided, to your principal as soon as possible, not later than <u>11 September 1992.</u>

Thank you for your co-operation and the time set aside for this research.

Yours faithfully

....
H. RAMBEHARI

TEACHER QUESTIONNAIRE - HIGHER GRADE

(for teachers of Senior Certificate Mathematics HG Classes)

A. GENERAL

1. Name of your school : ______________________ Secondary

2. Number of S.C. pupils offering Mathematics HG at your school :

Boys	
Girls	
Total	

B. PERSONAL INFORMATION

Wherever applicable, place a cross (X) in the relevant block. (Disregard the numbers on the right of the blocks).

3. Sex :

Male		1
Female		2

4. General teaching experience (in years) :

1 - 6		1
7 - 12		2
13 - 18		3
+ 18		4

5. Experience in teaching mathematics (in years) :

1 - 6		1
7 - 12		2
13 - 18		3
+ 18		4

6. Experience in teaching Std 10 mathematics HG (in years) :

0 - 5		1
6 - 10		2
11 - 15		3
+ 15		4

P.T.O.

7. Have you received special training in teaching mathematics ?

Yes	☐	1
No	☐	2

8. Present level :

Educator Level 1	☐	1
H.O.D.	☐	2
D.P.	☐	3
Senior D.P.	☐	4

9. Highest Professional qualification :

2-year Teachers' Diploma	☐	1
3-year Teachers' Diploma	☐	2
1-year post-graduate Teachers' Diploma	☐	3
B. Paed.	☐	4

10. Highest Academic qualification :

Bachelors Degree	☐	1
Honours Degree	☐	2
B. Ed.	☐	3
M. Ed.	☐	4

11. Highest academic qualification in mathematics :

Mathematics I or equivalent	☐	1
Mathematics II	☐	2
Mathematics III	☐	3
Mathematics (Hons)	☐	4

12. Composition of std. 10 mathematics classes you are currently teaching :

HG only	☐	1
HG & SG combined	☐	2

C. GENERAL GUIDELINES

1. In this questionnaire, you are required to rate the mathematical ability of std. 10 HIGHER GRADE pupils in respect of certain aspects/areas of mathematics. In carrying out this exercise, you must :

1.1 take into account HG pupils' global performance in class exercises, tests, examinations, homework exercises, assignments, etc.; and

1.2 consider not only your present pupils, but also std 10 HG pupils whom you taught previously at your school, particularly in 1991.

N.B. The word "mathematical ability" must be interpreted as the general mathematical ability of the majority of the pupils who offer mathematics HG.

2. Answer ALL the questions that follow. Read the explanations/ examples, wherever provided, carefully before indicating your responses.

D. STANDARD 10 MATHEMATICS HG PUPILS' ABILITY I.T.O. OBJECTIVES

Indicate, by placing a cross (x) in the appropriate column, the ability of Std. 10 Mathematics HG pupils in terms of the following objectives. Use the following key to rate :-

1 = EXCELLENT
2 = GOOD
3 = AVERAGE
4 = FAIR
5 = POOR

D1. LOWER LEVEL MATHEMATICAL ABILITIES
(Knowledge and Skills)

	EXCELLENT	GOOD	AVERAGE	FAIR	POOR
1. To recall fundemental mathematical knowledge, i.e. :					
1.1 To recall specific facts. (eg. definition of terms; symbols; formulae)	1	2	3	4	5
1.2 To recall concepts. (eg. functions; locii; derivative)	1	2	3	4	5
1.3 To recall rules and principles. (eg. log. laws; theorems; trig. identities)	1	2	3	4	5

P.T.O.

		EXCELLENT	GOOD	AVERAGE	FAIR	POOR
2.	To recall skills, techniques and methods learnt, for solutions and proofs	1	2	3	4	5
3.	To compute rapidly and accurately. (eg. to carry out numerical calculations; to factorize; to solve; to substitute; to change the subject of a formula)	1	2	3	4	5
4.	To manipulate rapidly and accurately. (eg. ability to sketch graphs, read tables, handle mathematical instruments and calculators) ..	1	2	3	4	5

D2 <u>MIDDLE LEVEL MATHEMATICAL ABILITIES</u>
(Comprehension/Understanding)

5.	To interpret symbolic and pictorial data. (eg. $h^2 = a^2 + b^2$; graphs; diagrams)	1	2	3	4	5
6.	To put data into symbols or geometric form. (eg. write the algebraic equation to a verbal problem; draw the diagram for a geometry rider)	1	2	3	4	5
7.	To express mathematical information in own words...	1	2	3	4	5
8.	To estimate answers involving numerical calculations	1	2	3	4	5
9.	To check the accuracy of computations/proofs constructed.	1	2	3	4	5
10.	To follow proofs................................	1	2	3	4	5

	EXCELLENT	GOOD	AVERAGE	FAIR	POOR
11. To solve routine problems. (i.e. problems that are similar to those encountered during the course of instruction and which involve the direct application of knowledge and skills learnt)..	1	2	3	4	5

D3 HIGHER LEVEL MATHEMATICAL ABILITIES
(Application, Analysis, Synthesis, Creative)

12. To apply concepts to mathematical problems.........	1	2	3	4	5
13. To interpret a problem............................	1	2	3	4	5
14. To analyse a problem..............................	1	2	3	4	5
15. To determine the operations that may be applied to solve a problem.................................	1	2	3	4	5
16. To construct proofs...............................	1	2	3	4	5
17. To invent mathematical generalizations............	1	2	3	4	5
18. To solve non-routine (novel) problems. (i.e. problems that are new and not similar to those encountered before. Their solution requires the transfer of knowledge and methods learnt into a new situation)................................	1	2	3	4	5

E. STANDARD 10 MATHEMATICS HG PUPILS' ABILITY IN TERMS OF 3 GROUPS OF ABILITIES

Using the same 5-point scale as above, indicate by placing a cross in the appropriate column, the ability of std 10 Mathematics HG pupils :

E1 In terms of the following groups of abilities :-

	EXCELLENT	GOOD	AVERAGE	FAIR	POOR
19. Lower level Mathematical abilities. (i.e. knowledge and skills)	1	2	3	4	5
20. Middle Level Mathematical abilities. (i.e. Comprehension/Understanding)...........	1	2	3	4	5
21. Higher Level Mathematical abilities. (i.e. Application, Analysis, Synthesis, Creative)..	1	2	3	4	5

E2 In terms of the following content areas :-

22. Algebra	1	2	3	4	5
23. Euclidian Geometry	1	2	3	4	5
24. Analytical Geometry	1	2	3	4	5
25. Trigonometry	1	2	3	4	5

F. STANDARD 10 MATHEMATICS HG PUPILS' ATTITUDE TOWARDS AND INTEREST IN MATHEMATICS

26. The general attitude of standard 10 Mathematics HG pupils towards Mathematics is : (place a cross in the appropriate block)

Positive		1
Negative		2
Indifferent		3

27. The general interest of standard 10 Mathematics HG pupils in Mathematics is : (place a cross in the appropriate box)

Positive		1
Negative		2
Indifferent		3

P.T.O.

G. <u>STANDARD 10 MATHEMATICS HG PUPILS' ATTAINMENT OF THE AIMS OF THE S.C. MATHEMATICS CURRICULUM</u>

Listed below are some of the aims of the current Senior Certificate Mathematics HG curriculum.

Indicate, by placing a cross in the appropriate column, the <u>extent</u> to which the following aims are attained by S.C. Mathematics HG pupils as a result of studying the S.C. course in Mathematics. Use the following key to rate :

1 = ALMOST ENTIRELY
2 = TO A LARGE EXTENT
3 = TO A REASONABLE EXTENT
4 = TO A LITTLE EXTENT
5 = NOT AT ALL

	ALMOST ENTIRELY	TO A LARGE EXTENT	TO A REASONABLE EXTENT	TO A LITTLE EXTENT	NOT AT ALL
28. The acquisition of mathematical knowledge..........	1	2	3	4	5
29. The acquisition of mathematical proficiency........	1	2	3	4	5
30. The development of clarity of thought.............	1	2	3	4	5
31. The ability to make logical deductions	1	2	3	4	5
32. The development of accuracy.......................	1	2	3	4	5
33. The development of mathematical insight...........	1	2	3	4	5
34. The ability to estimate answers...................	1	2	3	4	5
35. The ability to verify answers (where applicable)...	1	2	3	4	5

H. INFORMATION PERTAINING TO THE S.C. MATHEMATICS HG SYLLABUS

36. List below the topics in the S.C. Mathematics HG syllabus that you observed pupils had difficulty in understanding.

37. In terms of your experience in teaching the S.C. Mathematics HG syllabus, which topics in the said syllabus do you consider to be <u>not</u> suitable for HG pupils, and hence need to be dropped ? Please give reasons.

<u>Topic(s)</u>	<u>Reasons</u>

38. List below, any new topics which you consider should be included in the S.C. Mathematics HG syllabuses. Please give reasons.

<u>Topic(s)</u>	<u>Reasons</u>

THANK YOU FOR YOUR CO-OPERATION.

APPENDIX 3

PUPIL QUESTIONNAIRE

43 Bikaner Road
MEREBANK
4052

1 September 1992

Dear Pupil

I am carrying out an investigation entitled: AN EVALUATION OF THE EFFICACY OF THE AIMS AND OBJECTIVES OF THE SENIOR CERTIFICATE MATHEMATICS CURRICULUM IN THE RSA. The investigation entails, amongst others, the completion of a questionnaire by pupils who are currently offering mathematics at the senior certificate level.

Kindly note that by responding to the questionnaire, you would not only be making a valuable contribution to the investigation, but also be providing information that may have an important bearing on mathematics learning and teaching in the country in the future.

It would therefore be greatly appreciated if you would complete the attached questionnaire with great care. Answer ALL questions, and please be as frank as you can possibly be. As your individual responses are of importance to this investigation, please do not consult with your friends. The questionnaire should take approximately 30 minutes to complete.

You are assured that your responses will be treated confidentially and used for research purposes only.

N.B. YOUR NAME IS NOT REQUIRED IN THIS QUESTIONNAIRE

Kindly hand your completed questionnaire to your principal.

Thank you for your co-operation and the time set aside for this research.

Yours faithfully

....

H. RAMBEHARI

PUPIL QUESTIONNAIRE

(To be completed by pupils offering mathematics at the Senior Certificate Level)

SECTION A : GENERAL

1. Name of your school: ________________ Secondary School

2. Grade on which you are offering mathematics this year:
(Place a cross in the appropriate block). HG ☐ SG ☐

3. Your sex: (Place a cross in the appropriate block) Male ☐ Female ☐

SECTION B

Below are some statements about mathematics, school and life in general. For each of the statements indicate, by means of placing a cross (x) in the appropriate column, whether you agree, disagree or are uncertain. Use the following key:

2 = AGREE
1 = UNCERTAIN
0 = DISAGREE

NB. THERE IS NO RIGHT OR WRONG ANSWER TO ANY STATEMENT - IT IS JUST WHAT YOU SINCERELY FEEL.

	AGREE	UNCERTAIN	DISAGREE
4. My mathematics teacher shows us different ways of solving the same problem.	2	1	0
5. In our school we get a great deal of practice and drill until we are almost perfect in our learning. ...	2	1	0
6. Very few people can learn mathematics.	2	1	0
7. More of the most able people should be encouraged to become mathematicians and mathematics teachers. ..	2	1	0
8. Most school learning has little value for a person. .	2	1	0
9. Mathematics is not a very interesting subject.	2	1	0
10. My mathematics teacher does not like pupils to ask questions after he has given an explanation.	2	1	0

P.T.O.

		AGREE	UNCERTAIN	DISAGREE
11.	The pupils spend most of their class time listening to the teachers and taking notes.	2	1	0
12.	Almost anyone can learn mathematics if he/she is willing to study.	2	1	0
13.	Outside of science and engineering there is little need for mathematics in most jobs.	2	1	0
14.	I am bored most of the time in school.	2	1	0
15.	I have usually enjoyed studying mathematics in school.	2	1	0
16.	My mathematics teacher wants pupils to solve problems only by the procedures he teaches.	2	1	0
17.	Our teachers want us to do most of our learning from the textbook which is used in the course.	2	1	0
18.	Any person of average intelligence can learn to understand a good deal of mathematics.	2	1	0
19.	Mathematics is of great importance to a country's development.	2	1	0
20.	I enjoy most of my school work and want to get as much additional education as possible.	2	1	0
21.	I have seldom liked studying mathematics.	2	1	0
22.	My mathematics teacher expects us to learn how to solve problems by ourselves but helps when we have difficulties.	2	1	0
23.	We are expected to learn and discover many ideas for ourselves at school.	2	1	0
24.	Even complex mathematics can be made understandable and useful to every high school pupil.	2	1	0
25.	Mathematics is not useful for the problems of everyday life.	2	1	0
26.	I find school interesting and challenging.	2	1	0

P.T.O.

		AGREE	UNCERTAIN	DISAGREE
27.	Mathematics is enjoyable and stimulating to me.	2	1	0
28.	My mathematics teacher requires us not only to master the steps in solving problems, but also to understand the reasoning involved.	2	1	0
29.	In our school we are expected to develop a thorough understanding of ideas and not just to memorize information.	2	1	0
30.	Almost all pupils can learn complex mathematics if it is properly taught.	2	1	0
31.	It is important to know mathematics in order to get a good job.	2	1	0
32.	Success depends to a large part on luck and fate. . ..	2	1	0
33.	Mathematics is dull and boring.	2	1	0
34.	My mathematics course requires more thinking about the methods of solving problems than memorization of rules and formulae.	2	1	0
35.	Our teachers believe in strict discipline and each pupil does exactly what he is told to do.	2	1	0
36.	Only people with a very special talent can learn mathematics.	2	1	0
37.	Other subjects are more important to people than mathematics.	2	1	0
38.	By improving industrial and agricultural methods, poverty can be eliminated in the world.	2	1	0
39.	I like trying to solve new problems in mathematics. ..	2	1	0
40.	My mathematics teacher wants us to discover mathematical principles and ideas for ourselves.	2	1	0

P.T.O.

		AGREE	UNCERTAIN	DISAGREE
41.	We do not use just one textbook for most of our subjects. Various sources and books from which we can learn are suggested to us.	2	1	0
42.	I am very calm when studying mathematics.	2	1	0
43.	Mathematics helps to develop the mind and teaches a person to think.	2	1	0
44.	Education can only help people develop their natural abilities; it cannot change people in any fundamental way.	2	1	0
45.	I am not motivated to work very hard on mathematics lessons.	2	1	0
46.	Most of the problems my mathematics teacher assigns are to give us practice in using a particular rule or formulae.	2	1	0
47.	Much of our classroom work is discussing ideas and problems with the teacher and other pupils.	2	1	0
48.	Mathematics is one of my most dreaded subjects.	2	1	0
49.	Mathematics has contributed greatly to the advancement of civilization.	2	1	0
50.	With hard work anyone can succeed.	2	1	0
51.	I plan to take as much mathematics as I can during my education.	2	1	0

P.T.O.

SECTION C

Answer the following questions.

52. What occupation would you like to enter?

    ____

53. Would you like to take more mathematics courses after this year? (Place a cross in the appropriate box)

    YES ☐

    NO ☐

54. Which two school subjects have you liked MOST?

    1. ______________________

    2. ______________________

55. Which two school subjects have you liked LEAST?

    1. ______________________

    2. ______________________

56. In which two school subjects do you do (perform) BEST?

    1. ______________________

    2. ______________________

57. In which two school subjects do you do (perform) WORST?

    1. ______________________

    2. ______________________

THANK YOU FOR YOUR CO-OPERATION

APPENDIX 4

THE MATHEMATICS TEST (EXAMINATION)

SC

M A T H E M A T I C S
HIGHER GRADE
PAPER ONE

15
(P.P. 12)
Page 1

ADMINISTRATION: HOUSE OF DELEGATES
DEPARTMENT OF EDUCATION AND CULTURE
SENIOR CERTIFICATE EXAMINATION

NOVEMBER/DECEMBER 1991

M A T H E M A T I C S
HIGHER GRADE
PAPER ONE

3 HOURS
200 MARKS

INSTRUCTIONS TO CANDIDATES

1. Number your answers exactly as the questions are numbered.

2. Unless otherwise stated,

   (i) All working details must be shown.

   (ii) Calculators may be used, in which case answers must be given to two decimal places.

3. Graph paper is NOT to be used for the sketching of any graph.

P.T.O. .../QUESTION ONE

QUESTION ONE

1.1 Given that $-\frac{3}{2}$ is a root of the equation

$$2x^3 + 11x^2 + kx - 15 = 0 .$$

1.1.1 Show that $k = 2$. (2)

1.1.2 Hence, determine the other roots of the equation. (5)

1.2 Solve for x :

1.2.1 $2x = 3\sqrt{x} + 2$ (5)

1.2.2 $\frac{(x - 3)(x + 1)}{x - 2} \leq 0$ (5)

1.2.3 $5 - |x + 8| \geq 0$ (5)

1.2.4 $5^{x + 1} - 10 = 5^x$ (4)

1.3 Show that the expression

$$\sqrt{\left(\frac{5^x + 5^{-x}}{2}\right)^2 - 1}$$

simplifies to $\frac{1}{2}(5^x - 5^{-x})$. (4)

Total Marks: 30

QUESTION TWO

2.1 Simplify without using a calculator:

$$\log \sqrt{18} + \log \sqrt{40} - \log \sqrt{45} + 2 \log 5 \ . \qquad (5)$$

2.2 Solve for x :

2.2.1 $\log_3 x + \log_3(x + 6) = 3$ (4)

2.2.2 $2 \log_5 x - 3 = - \log_x 5$ (6)

2.2.3 $\log_{\frac{1}{3}} 3x - 1 \geq 0$ (5)

2.3 If $2^{2x - 1} = (\frac{1}{5})^x$ and $\log 2 = a$, prove that

$$x = \frac{a}{a + 1} \ . \qquad (5)$$

Total Marks: 25

P.T.O. .../QUESTION THREE

QUESTION THREE

3.1 Determine the value(s) of m for which

$$x^2 - 2mx + 7m - 12 = 0$$

has equal roots. (4)

3.2 A quadratic function $f(x) = ax^2 + bx + c$ has maximum value of $5\frac{1}{2}$ when $x = -3$.
Further, $f(2) = -7$. Determine the values of a, b and c. (5)

3.3 If $kx^2 - x - k - 1 = 0$, determine the value(s) of k for which the roots of the quadratic equation are

3.3.1 real and unequal. (6)

3.3.2 real and have opposite signs. (5)

Total Marks: 20

P.T.O. .../QUESTION FOUR

QUESTION FOUR

4.1 When $f(x) = 6x^3 - 3x^2 + 5kx - 2$ is divided by $x - 1$ the remainder is $k^2 - 5$. Determine the value(s) of k. (5)

4.2 If $x^2 - 4x - 5$ is a factor of $-x^3 + px^2 - 7x + q$, determine the values of p and q. (7)

4.3 If $p(x) = (3x + 2)^n - 2^n$, $n \in N$, show that x is a factor of $p(x)$. (3)

4.4 $f(x) = 2x^2 + 3px - 3$ and
$g(x) = 2x^2 + (p - 2)x - 1$

have a common factor $(x - r)$.

4.4.1 Prove that $r = \dfrac{1}{p + 1}$ (4)

4.4.2 Hence, or otherwise, determine the numerical value of r. (4)

Total Marks: 23

QUESTION FIVE

5.1 Prove that the sum to n terms (S_n) of an arithmetic series is given by

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

where a is the first term and d the common difference. (5)

5.2 $-16 ; -12 ; -8 ; \ldots$ is an arithmetic sequence consisting of n terms.

5.2.1 What is the 23rd term of the sequence? (3)

5.2.2 For what value of n will the sum of the terms be equal to 72 ? (4)

5.3 A 100 litre cask is filled with insecticide from which 10 litres are drawn out and replaced with water and stirred thoroughly. Thereafter 10 litres of the mixture are drawn out and replaced with water and stirred. This process is continued until 10 drawings and 10 replacements are made.

5.3.1 Determine the quantity of insecticide removed in the 2nd and 3rd drawings. (2)

5.3.2 Hence, determine the total quantity of insecticide removed in the ten drawings. (5)

Total Marks: 19

P.T.O. .../QUESTION SIX

QUESTION SIX

Consider the functions $m = \{(x;y) : xy = -2\}$

and $n = \{(x;y) : y = |x| - 3\}$.

6.1 Sketch the curves of m and n on the same set of axes. (6)

6.2 Explain how you would use your graph to solve the equation

$$|x| = \frac{3x - 2}{x} .$$

Show your method clearly. (3)

6.3 Calculate the value(s) of t for which the equation

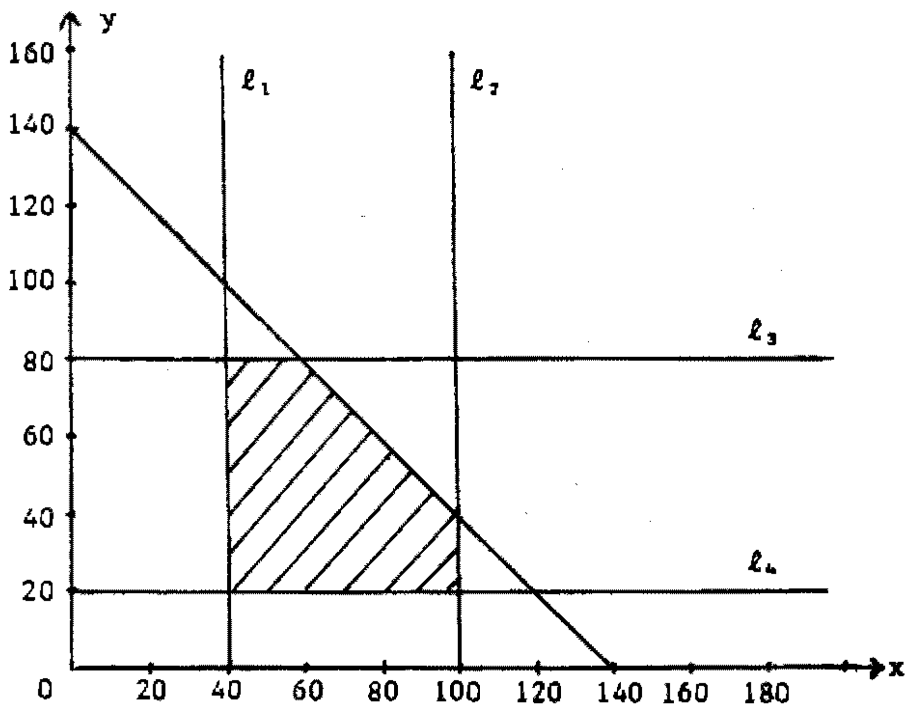
$$\frac{-2}{x} = |x| + t$$

will have ONE root only. (5)

Total Marks: 14

QUESTION SEVEN

The figure shows the graph of a system of inequalities depicting the feasible set for maximizing profit in a particular sales department of a firm selling two types of products, A (x units) and B (y units).



7.1 If ℓ_1 and ℓ_2 are parallel to the y-axis and ℓ_3 and ℓ_4 are parallel to the x-axis, write down the inequalities in terms of x and y that are simultaneously satisfied by the solution set (shaded area) in the figure. (7)

7.2 The objective function $P = x + y$ must be maximized.

7.2.1 Explain how you would maximize P. (2)

7.2.2 Use the graph and a curve of the objective function to determine the value(s) of x and y such that P is a maximum. (5)

7.2.3 Hence, determine the maximum value of P. (2)

Total Marks: 16

P.T.O. .../QUESTION EIGHT

QUESTION EIGHT

8.1 Determine $f'(x)$ in each of the following:

8.1.1 $f(x) = 3x^2(x - 1)$ (3)

8.1.2 $f(x) = \dfrac{(2x - 3)^2}{x - 1{,}5}$ (4)

8.2 If $g : x \to \frac{1}{3}x^3 - x^2 - 3x$, determine

8.2.1 $g'(x)$. (2)

8.2.2 the equation of the tangent line to the curve of f at $x = 3$. (4)

8.3 Find the value of the constant p if

$$f(x) = 2x^2 + \frac{x}{p}$$

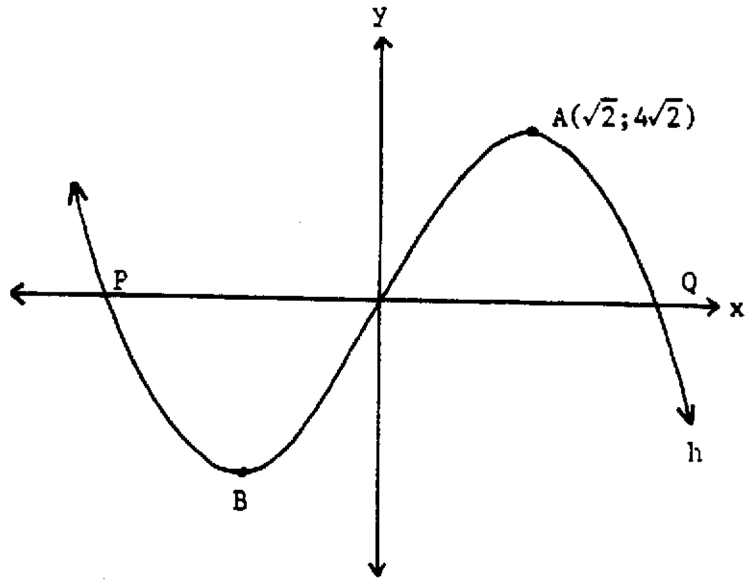
has a local minimum at the point where $x = 2$. (4)

Total Marks: 17

P.T.O. .../QUESTION NINE

QUESTION NINE

The sketch represents the graph of h where $h : x \to -x^3 + cx$.
A and B are the local maximum and minimum turning points respectively of the graph of h .
The point B is symmetrical to the point $A(\sqrt{2} ; 4\sqrt{2})$ with respect to the origin.



9.1 Show that $c = 6$. (5)

9.2 Determine

9.2.1 the length of PQ where P and Q are the x-intercepts of the graph of h . (3)

9.2.2 the value(s) of n such that the equation $-x^3 + cx = n$ will have one root only, this root being negative. (3)

9.3 Prove that the tangent lines to the graph of h which are parallel to the line AP will cut the graph of h at

$$x = \pm\sqrt{\frac{2\sqrt{3} - 8}{-3}} .$$ (6)

Total Marks: 17

P.T.O. .../QUESTION TEN

QUESTION TEN

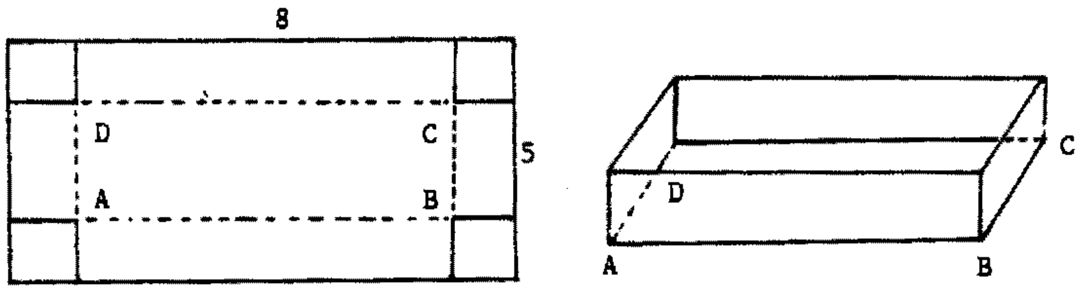
10.1 It is estimated that t years from now the population of a certain species of animal will be $P(t) = 20 - \frac{6}{t}$ thousand.

10.1.1 Determine the rate of change in the population in 3 years time. (3)

10.1.2 By how many will the population increase during the second year? (4)

10.1.3 What will happen to the population in due course? (1)

10.2 A rectangular sheet of cardboard measures 8 units by 5 units. Equal squares are cut out at each of the corners and the remainder is folded so as to form an open box.



If the squares which are cut out have sides of x units each

10.2.1 write down the length, breadth and depth of the box in terms of x . (3)

10.2.2 find the volume in terms of x . (1)

10.2.3 calculate the maximum volume of the box. (7)

Total Marks: 19

TOTAL MARKS : 200

SC	
M A T H E M A T I C S	
HIGHER GRADE	
PAPER TWO	

ADMINISTRATION: HOUSE OF DELEGATES
DEPARTMENT OF EDUCATION AND CULTURE
SENIOR CERTIFICATE EXAMINATION
NOVEMBER/DECEMBER 1991

M A T H E M A T I C S
HIGHER GRADE
PAPER TWO

3 HOURS
200 MARKS

INSTRUCTIONS TO CANDIDATES

1. Answer ALL questions.
2. Number your answers exactly as the questions are numbered.
3. All working details must be clearly shown. Where CALCULATORS/ TABLES are used, answers must be rounded off to TWO DECIMAL PLACES, unless otherwise stated.
4. Marks will be deducted for incorrect statements and slovenly work.
5. The following method of naming angles should be used where possible:

   $\hat{A}_2$, $\hat{C}_1$ etc.
6. Note that diagrams are not necessarily drawn to scale.
7. Where diagrams are supplied on the loose DIAGRAM SHEET, DO NOT REDRAW THESE DIAGRAMS but use the ones supplied and show your working in the answer book.
8. Write your EXAMINATION NUMBER on the DIAGRAM SHEET which must be placed inside the front cover of your answer book when handed in.
9. Unless otherwise stated answers must NOT be obtained by construction and measurement.

P.T.O. .../QUESTION ONE

QUESTION ONE

1.1

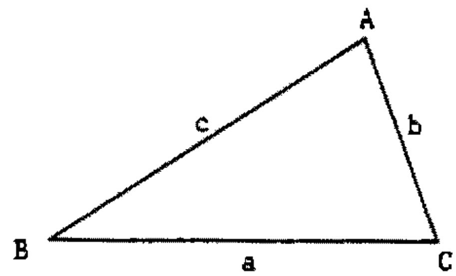
1.1.1 Write down the expansions of $\cos(A + B)$ and $\sin(A + B)$ in terms of the sine and cosine functions of A and B.
Hence derive the expansion for $\tan(A + B)$ in terms of $\tan A$ and $\tan B$. (7)

1.1.2 Without using a calculator, show that
$\tan 165° = \sqrt{3} - 2$. (8)

1.2

1.2.1 In the accompanying figure, $AB = c$, $AC = b$ and $CB = a$.
With reference to ΔABC complete the following expression:

$$\sin C = \frac{c \times \ldots}{\ldots}$$



(2)

1.2.2 In ΔDEF, $\hat{D} = 30°$ and $e = 10$ units.
If $d = 4$ units, show by claculation that ΔDEF does NOT exist and hence write down the minimum value of d for ΔDEF to exist. (5)

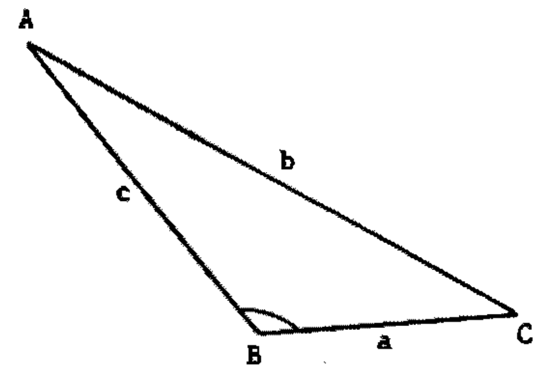
$\boxed{22}$

QUESTION TWO

2.1 In the accompanying figure, $\hat{B}$ is obtuse, $AB = c$, $AC = b$ and $BC = a$. Copy the figure in your answer book and use it to prove that

$$b^2 = a^2 + c^2 - 2ac \cos B \ .$$

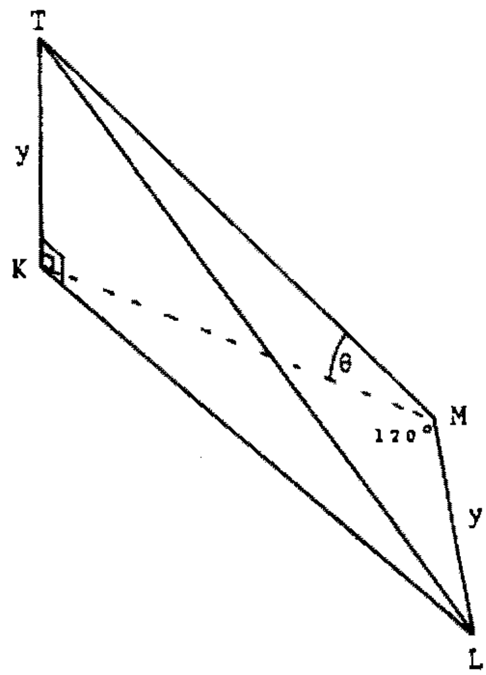
(Show all constructions on the diagram.)



(6)

P.T.O. .../2.2 In the

2.2 In the accompanying figure, K , M and L represent the positions of three points in the horizontal plane such that $K\hat{M}L = 120°$. T represents the position of a point vertically above K so that $TK = LM = y$ and $T\hat{K}M = T\hat{K}L = 90°$. The angle of elevation $T\hat{M}K = \theta$.



2.2.1 Express KM in terms of y and a trigonometric ratio of θ . (2)

2.2.2 Show that $KL = y.\sqrt{\cot^2\theta + \cot\theta + 1}$ (5)

2.2.3 If $y = 15$ metres and $\theta = 22°$, calculate the following:

2.2.3.1 KL , correct to one decimal place. (3)

2.2.3.2 the size of $K\hat{T}L$, correct to one decimal place. (3)

$\boxed{19}$

QUESTION THREE

$f = \{(x;y) \mid y = \cos(x - 30°),\ x \in [-90°,90°]\}$ and

$g = \{(x;y) \mid y = \sin x,\ x \in [-90°;90°]\}$

3.1 Use the set of axes provided in figure 3 on the loose diagram sheet to sketch the graphs of f and g. (7)

3.2 <u>Use your graphs to answer the following questions</u>:

3.2.1 Write down the range of f. (2)

3.2.2 For what values of x is $f(x)$ decreasing as x increases? (2)

3.2.3 If $x \in [-90°;0]$, for what values of x is $f(x).g(x) \geq 0$? (2)

3.3 If $x \in [-90°;90°]$, solve the equation $\cos(x - 30°) = \sin x$ and hence write down the values of x in the interval $[-90°;90°]$ for which $g(x) > f(x)$. (4)

$\boxed{17}$

P.T.O. .../QUESTION FOUR

QUESTION FOUR

4.1 Determine the general solution of the equation:

$$5 \sin x.\cos x - 2 + 5 \sin x - 2 \cos x = 0 .$$

(Write down your answers correct to one decimal place.) (9)

4.2 Prove that $\tan A + \cot A = 2 \operatorname{cosec} 2A$.
Hence determine the minimum value of $\tan A + \cot A$ in the interval $[0°;90°]$, and the value of A for which this minimum occurs. (11)

$\boxed{20}$

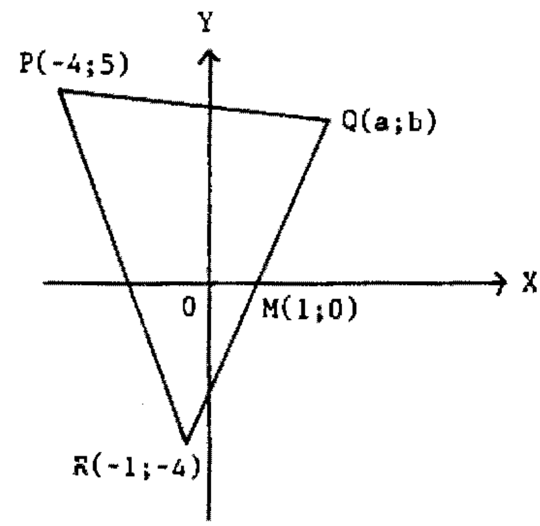
P.T.O. .../QUESTION FIVE

QUESTION FIVE

5.1 Determine the equation of a straight line making an intercept of 3 on the x-axis and -4 on the y-axis.
(Write your answer in the form $ax + by + c = 0$.) (3)

5.2 The straight line with equation $px + qy + 2 = 0$ makes an angle of 135° with the positive x-axis and passes through the point (2;-1) .
Calculate the values of p and q . (5)

5.3 In the accompanying figure, P (-4;5) , Q (a;b) and R (-1;-4) are the vertices of a triangle and M (1;0) is the midpoint of QR .



5.3.1 Calculate the values of a and b . (2)

5.3.2 Determine the equation of the perpendicular bisector of QR . (5)

5.3.3 If the perpendicular bisector of PQ has equation $7x - y + 8 = 0$, and cuts the perpendicular bisector of QR at the point T , calculate the co-ordinates of T . (5)

5.3.4 Determine the equation of the circumcircle of ΔPQR and write your answer in the form $Ax^2 + Bx + Cy^2 + Dy + E = 0$. (6)

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QUESTION SIX

6.1 A circle has equation $x^2 + y^2 + 4x + 6y - 51 = 0$

6.1.1 Determine the co-ordinates of the centre and the radius of the circle. (4)

6.1.2 Show that the straight line with equation $x = 6$ is a tangent to the circle. (3)

6.1.3 Show that the point $P\ (6;3)$ lies outside the circle and calculate the length of a tangent from the point $P\ (6;3)$ to the circle. (6)

6.2 The equation of a circle with centre M is $(x + 1)^2 + (y - 2)^2 = 20$.

6.2.1 Determine the equation of the tangent to the circle at the point $A\ (-3;6)$. (6)

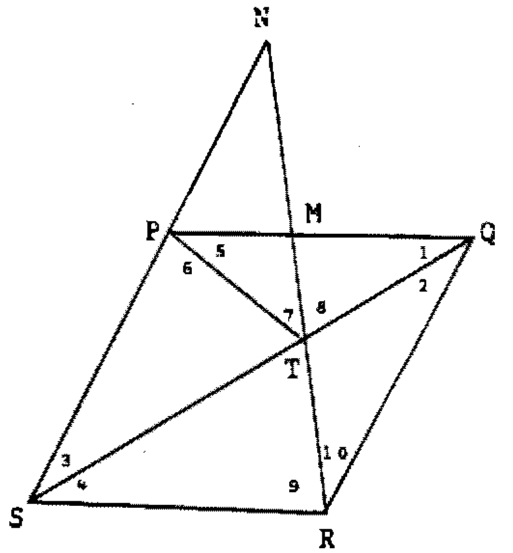
6.2.2 If Q is a point on the circle such that the tangent to the circle at the point Q is parallel to the tangent at A , calculate the co-ordinates of Q . (8)

$\boxed{27}$

P.T.O. .../QUESTION SEVEN

QUESTION SEVEN

In the accompanying figure, T is any point on the diagonal SQ of a rhombus PQRS .
RT produced cuts PQ at M and meets SP produced at N .



7.1 Prove the following:

7.1.1 $\Delta PQT \equiv \Delta RQT$ (5)

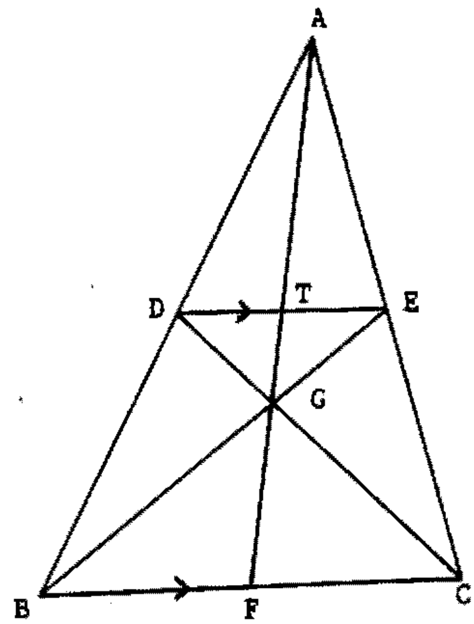
7.1.2 $\Delta PMT \ ||| \ \Delta NPT$ (6)

7.2 If it is further given that $TM = k.TN$ where $k \in \mathbb{R}$,
prove that $\frac{TP^2}{TN^2} = k$. (3)

14

QUESTION EIGHT

In the accompanying figure, CD is a median of triangle ABC and DE is parallel to BC . BE and DC intersect at G , AG cuts DE at T and AG produced meets BC at F .



8.1 Prove that BF = FC . (5)

8.2 Calculate the following, giving reasons for your answers:

8.2.1 $\frac{AG}{GF}$ (2)

8.2.2 $\frac{GT}{TA}$ (5)

12

QUESTION NINE

9.1 Complete the following statements in your answer book:

9.1.1 "The exterior angle of a cyclic quadrilateral ..." (1)

9.1.2 "The straight line drawn from the centre of a circle perpendicular to a chord ..." (1)

9.2 In figure 9, (use diagram supplied on loose sheet, DO NOT REDRAW), O is the centre of the circumscribed circle of triangle ABD. The circle passing through the points O, A and B cuts DB at C, and CO produced meets AD at T. Let $A\hat{O}B = 2x$ and $O\hat{B}A = y$.

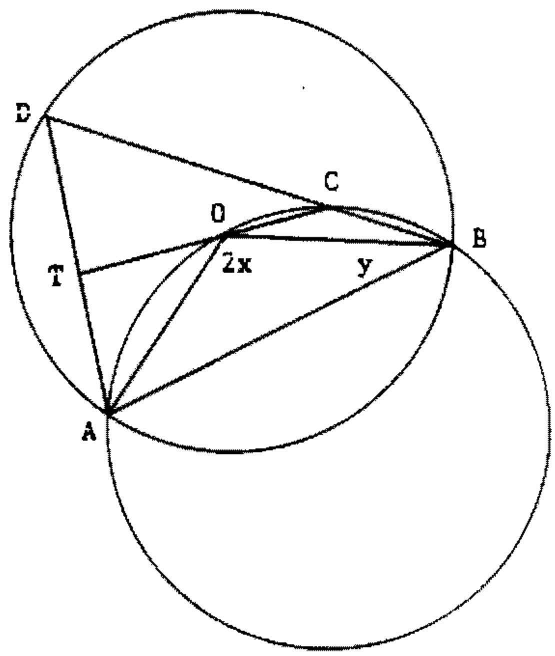


FIG. 9

9.2.1 Express $\hat{D}$ and $O\hat{C}D$ in terms of x and y respectively. Give reasons for your answers. (6)

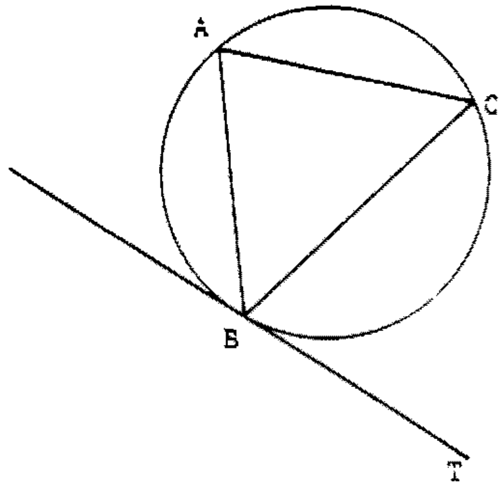
9.2.2 Prove that OT bisects AD. (4)

9.2.3 If $OT = \frac{3}{5}r$, where r is the radius of the circumscribed circle of ΔABD, calculate the length of AD in terms of r. (5)

17

QUESTION TEN

10.1 In the accompanying figure, BC is a chord of the circle and BT is a straight line such that $C\hat{B}T = B\hat{A}C$. Copy the diagram in your answer book and use it to prove the theorem which states that BT is a tangent to the circle at the point B. (Show all construction lines on the diagram.) (7)



10.2 In figure 10, (use diagram supplied on loose sheet, DO NOT REDRAW), AB is a common chord to the two circles, and PT which is the tangent to the smaller circle at A meets the larger circle at T. S is a point on the larger circle such that AT = AS and AC is drawn parallel to TS. The point B is joined to C and S.

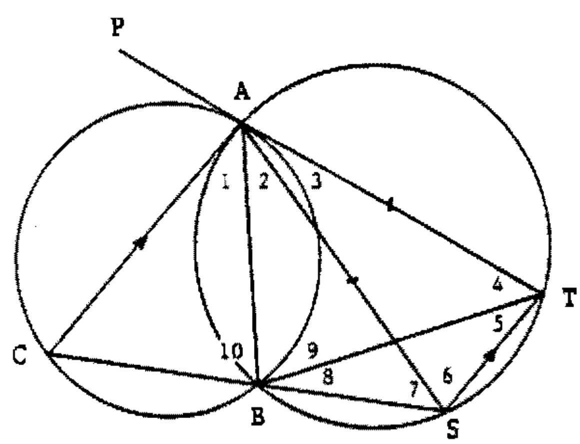


FIG. 10

Prove that

10.2.1 CBS is a straight line. (8)

10.2.2 AB bisects $C\hat{B}T$. (7)

10.2.3 AC is a tangent to the circle ABT at the point A. (4)

26

TOTAL MARKS : 200

SC M A T H E M A T I C S STANDARD GRADE PAPER ONE	17 (P.P. 12) Page 1

ADMINISTRATION: HOUSE OF DELEGATES
DEPARTMENT OF EDUCATION AND CULTURE
SENIOR CERTIFICATE EXAMINATION

NOVEMBER/DECEMBER 1991

M A T H E M A T I C S
STANDARD GRADE
PAPER ONE

3 HOURS
150 MARKS

INSTRUCTIONS TO CANDIDATES

1. Number your answers exactly as the questions are numbered.

2. All details of working must be clearly shown unless otherwise stated.

3. Answer ALL questions.

4. Calculators or mathematical tables may be used, in which case answers must be given correct to two decimal places.

P.T.O. .../QUESTION ONE

QUESTION ONE

1.1 If $g = \{(x;y) : y = 1 - \sqrt{3 - x}\}$,

1.1.1 calculate $g(-1)$. (2)

1.1.2 write down the domain of g. (2)

1.1.3 determine the value of r if $g(r) = -5$. (3)

1.2 Solve for x :

1.2.1 $(x - 3)(2x + 1) = 4$ (4)

1.2.2 $c + \frac{ax}{b} = x$, $b \neq 0$ (3)

1.3 Determine the value(s) of k if k is a root of the equation $x^2 + kx - 8 = 0$. (4)

Total Marks: 18

P.T.O. .../QUESTION TWO

QUESTION TWO

2.1 Simplify without the use of a calculator:

2.1.1 $(15\frac{5}{8})^{-\frac{2}{3}}$ (3)

2.1.2 $\left[(\sqrt{6} - 3)(3 + \sqrt{6})\right]^2$ (3)

2.1.3 $\log_3 \frac{1}{27} + \log_8 32$ (5)

2.1.4 $\dfrac{\log 16 - \log 4}{\log 16 + \log 4}$ (3)

2.2 Prove without using a calculator that

$$\log 8 + 2 \log \frac{1}{5} - 5 \log 2 = -2$$ (4)

Total Marks: 18

QUESTION THREE

3.1 Solve for x :

3.1.1 $2(3^x) = 54$ (3)

3.1.2 $2^x - 2^{x-2} = 12$ (4)

3.1.3 $\log_3 x = -2$ (2)

3.1.4 $\log(x + 3) - \log 2 = 1$ (3)

3.1.5 $5^x = 1000$ (3)

3.2 Solve for m and n in the simultaneous equations

$$m + 2 = 2n$$

and $$2m^2 - n^2 + 1 = 0$$ (6)

Total Marks: 21

QUESTION FOUR

4.1 Given $(x + 3)^2 = m$.
WITHOUT rewriting the equation, write down the value(s) of m for which the roots of the equation are

4.1.1 equal.

4.1.2 unreal.

4.1.3 real and unequal. (3)

4.2 For what value(s) of p are the roots of the quadratic equation $px^2 - 4x + 1 = 0$ real. (4)

Total Marks: 7

P.T.O. .../QUESTION FIVE

QUESTION FIVE

5.1 Given $2x^3 - 9x^2 + 7x + 6 = 0$.

5.1.1 Show that 2 is a root of the equation. (2)

5.1.2 Hence, or otherwise, determine the values of two other roots of the equation. (4)

5.2 When the polynomial $2x^2 - mx - 2kx + 5$ is divided by $2x + 1$ there is a remainder of k . Determine the value of m . (4)

Total Marks: 10

P.T.O. .../QUESTION SIX

QUESTION SIX

6.1 A trust fund of R20 000 earns interest at the rate of 18% per annum, compounded half yearly.

6.1.1 What will the fund amount to in 5 years? (4)

6.1.2 How much interest will it have earned in that time? (1)

6.2 A school for the handicapped bought a minibus for R64 000. Depreciation is calculated at 11% per annum of its diminishing value. After how many completed years will the book value of the minibus for the first time be less than half its original value? (6)

Total Marks: 11

P.T.O. .../QUESTION SEVEN

QUESTION SEVEN

7.1 Evlauate: $\sum_{n=3}^{5} (2^n + 2n - 1)$ (2)

7.2 A geometric sequence has common ratio 5. If the third term is 3, determine

7.2.1 the first term of the sequence. (2)

7.2.2 which term of the sequence is equal to $15(5^{27})$. (4)

7.3 Given the arithmetic series $6 + 20 + 34 + \ldots$.

7.3.1 Show that the sum of the first n terms of the series is $7n^2 - n$. (3)

7.3.2 Hence, find the sum of the first 16 terms of the series. (1)

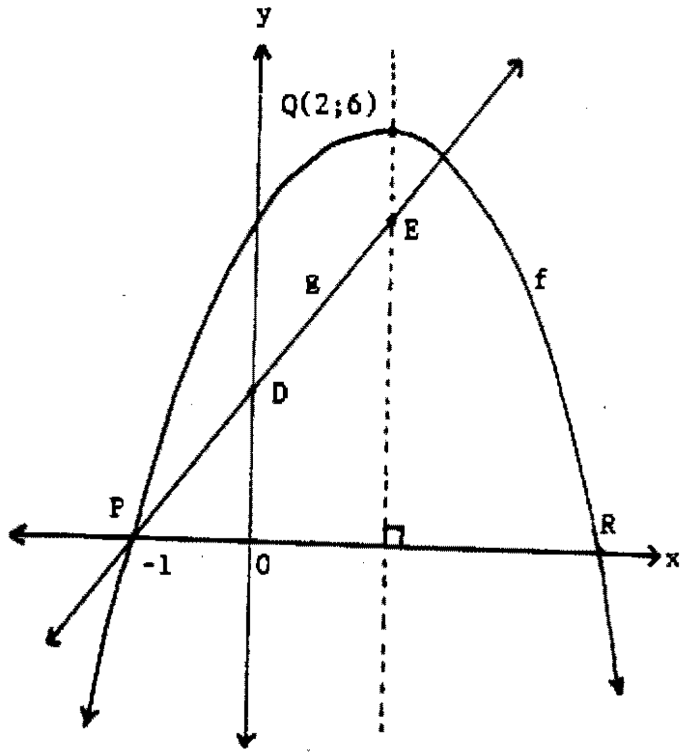
7.3.3 How many terms of the series must be added to give a sum equal to 690? (3)

Total Marks: 15

QUESTION EIGHT

The sketch shows the graphs (not drawn to scale) of $f = \{(x;y) : y = -x^2 + bx + c\}$ and $g = \{(x;y) : y = mx + n\}$

The parabola has turning point $Q(2;6)$ and intersects the x-axis at P and R. D and E are points on the graph of g and $P(-1;0) \in f \cap g$.



Determine the following:

8.1 the co-ordinates of R (1)

8.2 the values of b and c (3)

8.3 the co-ordinates of D if $PD = \frac{\sqrt{13}}{2}$ (3)

8.4 the values of n and m (3)

8.5 the shortest distance from E to the x-axis (2)

Total Marks: 12

P.T.O. .../QUESTION NINE

QUESTION NINE

9.1 Determine $\lim_{h \to 0} \left[\frac{(5 - h)^2 - 25}{h}\right]$ (3)

9.2 Find $\frac{dy}{dx}$ in each of the following:

9.2.1 $y = 2x(2x - \frac{2}{x})$ (3)

9.2.2 $y = \sqrt{x} - \frac{2}{x^2}$ (3)

9.3 Calculate the co-ordinates of the point P where the gradient of the curve $y = 4x - x^2$ is equal to -2. (4)

Total Marks: 13

P.T.O. .../QUESTION TEN

QUESTION TEN

10.1 $$h(x) = -x^3 + 7x^2 - 15x + 9$$
$$= (1 - x)(3 - x)^2$$

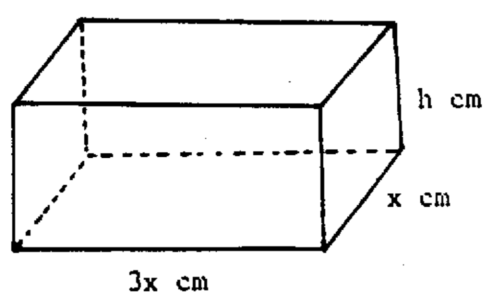
10.1.1 Calculate the intercepts of the graph of h on the x and y axes. (3)

10.1.2 Determine the co-ordinates of the turning points of the curve of h. (5)

10.1.3 Use the above data to sketch the graph of h. (4)

10.1.4 For what value(s) of x is the graph of h increasing? (2)

10.2 A rectangular block of marble is $3x$ cm long, x cm wide and h cm high.



10.2.1 Find an expression for the total length of all the edges of the block in terms of x and h. (2)

10.2.2 Express h in terms of x if the total length of the edges is 96 cm. (1)

10.2.3 Prove that the volume, V cm^3, of the block in terms of x is given by the equation $V = 72x^2 - 12x^3$. (2)

P.T.O. .../10.2.4 Determine the

10.2.4 Determine the value of x for which V is a maximum. (4)

10.2.5 Hence, or otherwise, calculate the maximum volume of the block. (2)

Total Marks: 25

<u>TOTAL MARKS : 150</u>

SC MATHEMATICS STANDARD GRADE PAPER TWO	18 (P.P. 12) Page 1

ADMINISTRATION: HOUSE OF DELEGATES
DEPARTMENT OF EDUCATION AND CULTURE
SENIOR CERTIFICATE EXAMINATION

NOVEMBER/DECEMBER 1991

MATHEMATICS
STANDARD GRADE
PAPER TWO

3 HOURS
150 MARKS

INSTRUCTIONS TO CANDIDATES

1. Answer ALL questions.
2. Number your answers exactly as the questions are numbered.
3. All working details must be clearly shown. Where CALCULATORS/TABLES are used answers must be rounded off to TWO DECIMAL PLACES, unless otherwise stated.
4. Marks will be deducted for incorrect statements and slovenly work.
5. The following method of naming angles should be used where possible: $\hat{A}_2$, $\hat{C}_1$ etc.
6. Note that diagrams are not necessarily drawn to scale.
7. Where diagrams are supplied on the loose DIAGRAM SHEET, DO NOT REDRAW THESE DIAGRAMS but use the ones supplied and show your working in the answer book.
8. Write your EXAMINATION NUMBER on the DIAGRAM SHEET which must be placed inside the front cover of your answer book when handed in.
9. Unless otherwise stated answers must NOT be obtained by construction and measurement.

P.T.O. .../QUESTION ONE

QUESTION ONE

1.1 Use the set of axes provided in figure 1.1 on the loose diagram sheet to sketch the graphs of the functions
$f = \{(x;y) \mid y = \cos 2x\}$ and $g = \{(x;y) \mid y = \sin x\}$
for the domain $[0°;180°]$. (7)

1.1.1 What is the range of f ?
(Write your answer in the space provided below figure 1.1.) (2)

1.1.2 By using capital letters of the alphabet indicate the point(s) on the appropriate axis where you will read off the solution of the equation
$\cos 2x - \sin x = 0$. (2)

1.2 In figure 1.2 (which also appears on the loose diagram sheet), the graph of the function
$h = \{(x;y) \mid y = \tan 2x\}$
is shown for the domain $[0°;180°]$.

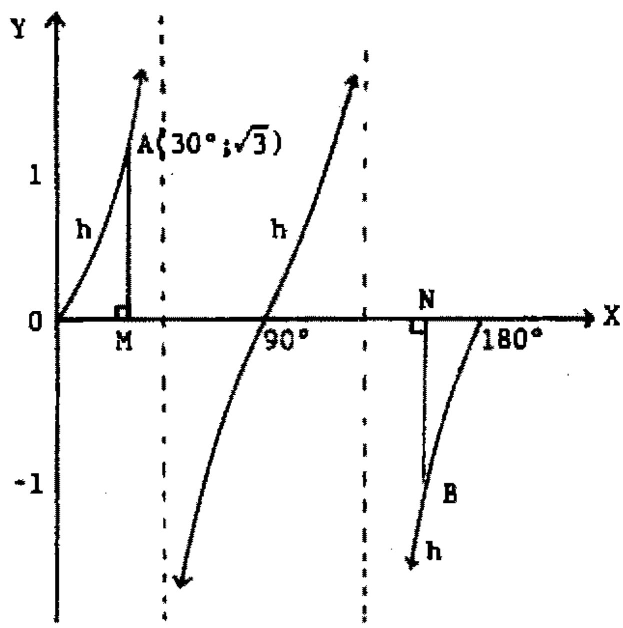


FIG. 1.2

Use your graph to answer the following questions:
(Write your answers to questions 1.2.1 and 1.2.2 in the space provided below figure 1.2 on the loose diagram sheet. Use figure 1.2 on the loose diagram sheet to answer questions 1.2.3 and 1.2.4.)

1.2.1 Which values of x are excluded from the domain of h ? (2)

1.2.2 If $A\ (30°;\sqrt{3}) \in h$ and $AM = BN$, write down the co-ordinates of B . (2)

1.2.3 Show dotted lines on the graph in order to indicate how you would determine the value of $\tan 240°$. (2)

1.2.4 If $2 + \tan 2x = 1$, show clearly on the graph by using the letters P , Q ... etc. how you would read off the values of x . (3)

$\boxed{20}$

P.T.O. .../QUESTION TWO

QUESTION TWO

2.1 If p , $q \in [0°;90°]$, determine the values of p and q in the following equations:

2.1.1 $\tan 55° = \cot p$ (1)

2.1.2 $\sin 200° = -\sin q$ (1)

2.2 If $x \in [0°;180°]$, use your calculator to find the value(s) of x correct to one decimal place in each of the following equations:

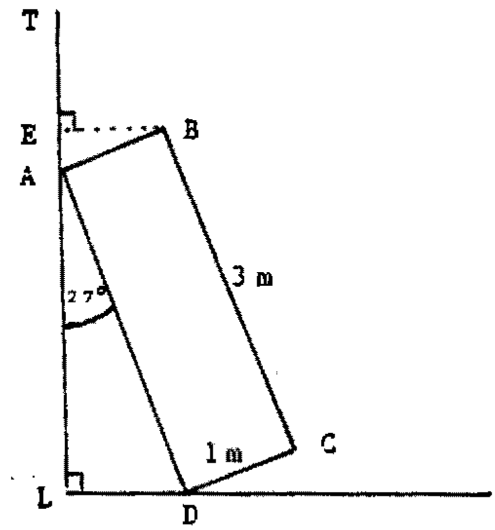
2.2.1 $\tan x = 2 \cos 306{,}2°$ (2)

2.2.2 $4 \cos^2 x = 3$ (3)

2.3 Without using a calculator find the value of:

$$\frac{\operatorname{cosec} 150°}{\sec 330°}$$ (5)

2.4 In the accompanying figure, ABCD represents a rectangular paving stone which rests against a vertical wall, LT such that the points A , L and D lie in the same vertical plane. If BC = 3 metres , DC = 1 metre, $L\hat{A}D = 27°$ and $BE \perp LT$, calculate:



2.4.1 the length of AL correct to <u>three</u> decimal places. (3)

2.4.2 the height (correct to two decimal places) of the highest point of the paving stone above the ground. (5)

$\boxed{20}$

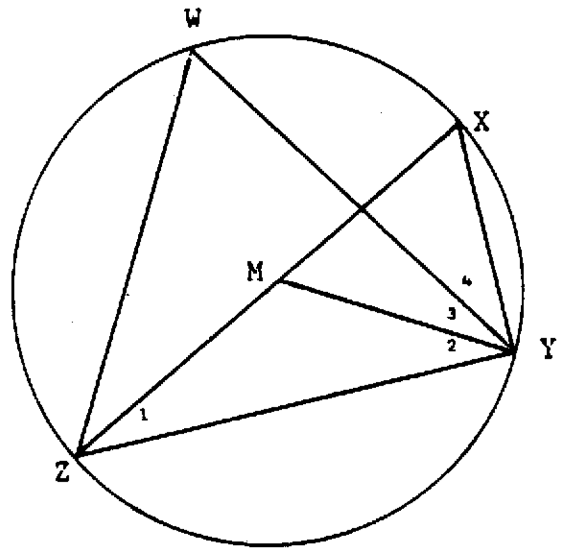
QUESTION THREE

In the accompanying figure, XZ is a diameter of the circle WXYZ whose centre is at the point M .

If $W\hat{Z}X = 26°$ and

$Z\hat{M}Y = 98°$,

calculate, giving reasons, the size of each of the following angles:



3.1 $\hat{W}$ (2)

3.2 $\hat{Z}_1$ (2)

3.3 $\hat{Y}_3$ (5)

$\boxed{9}$

QUESTION FOUR

The points A (1;0) , B (5;-2) and C (3;4) are the vertices of a triangle.

4.1 Determine the gradient of AB . (3)

4.2 Prove that ΔABC is right-angled with the right angle at A . (3)

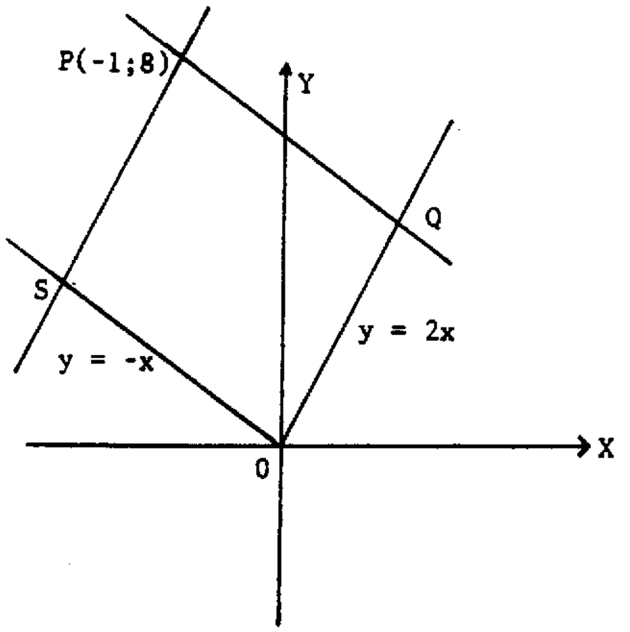
4.3 Determine the equation of the perpendicular bisector of BC in the form $ax + by + c = 0$ and show that it passes through A (1;0) . (9)

$\boxed{15}$

P.T.O. .../QUESTION FIVE

QUESTION FIVE

In the accompanying figure, P , Q , O and S are the vertices of a parallelogram. The equations of the sides OS and OQ are $y = -x$ and $y = 2x$ respectively and P is the point (-1;8).



5.1 Write down the gradient of PS . (2)

5.2 Determine the equation of PS . (3)

5.3 Determine the co-ordinates of S . (4)

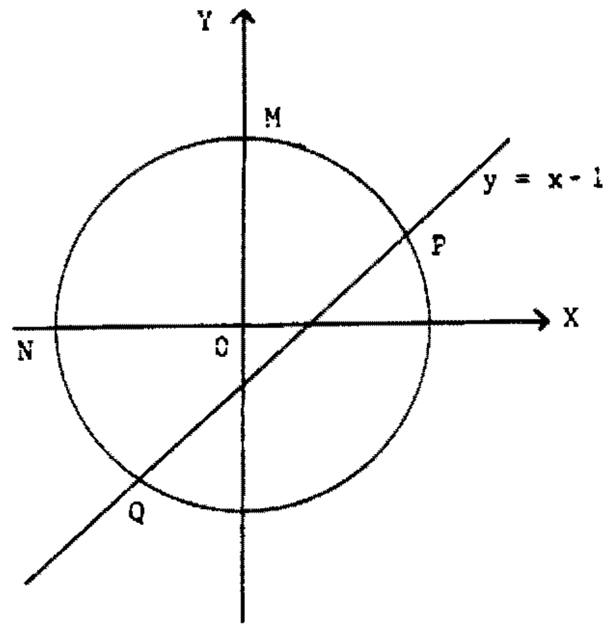
5.4 Calculate the size of $S\hat{O}Q$, correct to the nearest degree. (5)

$\boxed{14}$

P.T.O. .../QUESTION SIX

QUESTION SIX

In the accompanying figure, the circle with equation $x^2 + y^2 = 25$ cuts the y-axis at M and the x-axis at N .
The straight line with equation $y = x - 1$ cuts the circle at the points P and Q .



6.1 Write down the co-ordinates of M and N . (2)

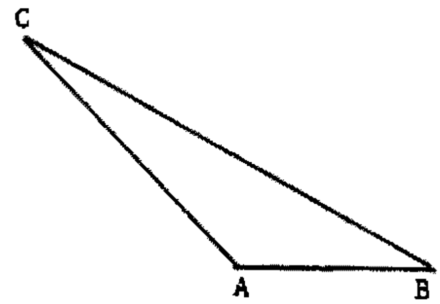
6.2 Calculate the co-ordinates of P and Q . (8)

6.3 Calculate the length of the chord PQ , leaving your answer in surd form. (4)

$\boxed{14}$

QUESTION SEVEN

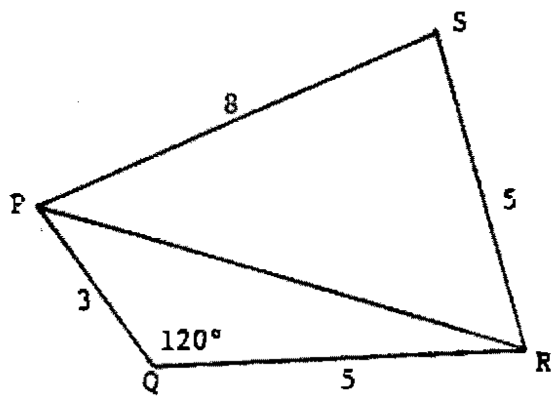
7.1 In the accompanying figure, $\hat{A}$ of triangle ABC is obtuse.



Copy the figure in your answer book and use it to prove that:

$$a^2 = b^2 + c^2 - 2bc \cos A \ . \qquad (6)$$

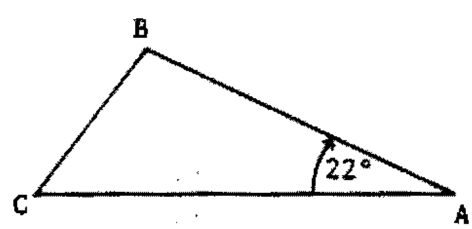
7.2 In the accompanying figure, PQRS is a quadrilateral with PQ = 3 units, PS = 8 units, QR = RS = 5 units and $\hat{PQR} = 120°$.



7.2.1 Calculate the length of PR . (4)

7.2.2 Calculate the size of $\hat{S}$ and hence state, giving a reason, what type of quadrilateral PQRS is. (6)

7.3 In the accompanying figure, AB = 2 BC and $\hat{A} = 22°$.

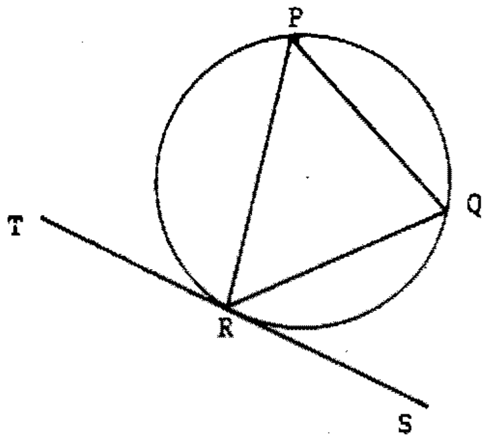


Calculate the size of $\hat{C}$ correct to one decimal place. (5)

21

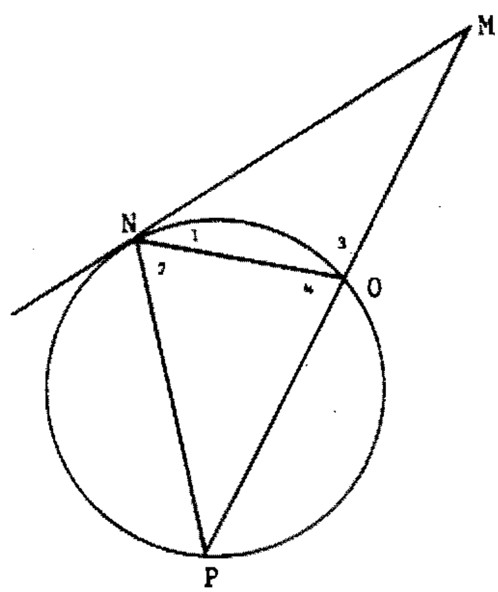
QUESTION EIGHT

8.1 In the accompanying figure, tangent ST touches the circle PRQ at R. Copy the diagram in your answer book and use it to prove the theorem which states that $T\hat{R}P = \hat{Q}$. (Show all necessary construction lines on the diagram.)



(10)

8.2 In the accompanying figure, MN is a tangent to the circle PON at the point N and POM is a straight line.



8.2.1 Prove that $\Delta MON \parallel\!| \Delta MNP$. (4)

8.2.2 If OP = 5 units and MO = 4 units, calculate the length of MN. (4)

8.2.3 If it is further given that NP is a diameter of the circle NOP, show that $r = \frac{1}{2}\sqrt{5}$ units, where r is the radius of the circle. (6)

QUESTION NINE

In figure 9, (use diagram supplied on loose sheet, DO NOT REDRAW), AOBC is a cyclic quadrilateral with BC || OA and AC = OB . A smaller circle with O as centre passes through A and B , and CO produced meets this circle at D .
Let $A\hat{O}C = x$.

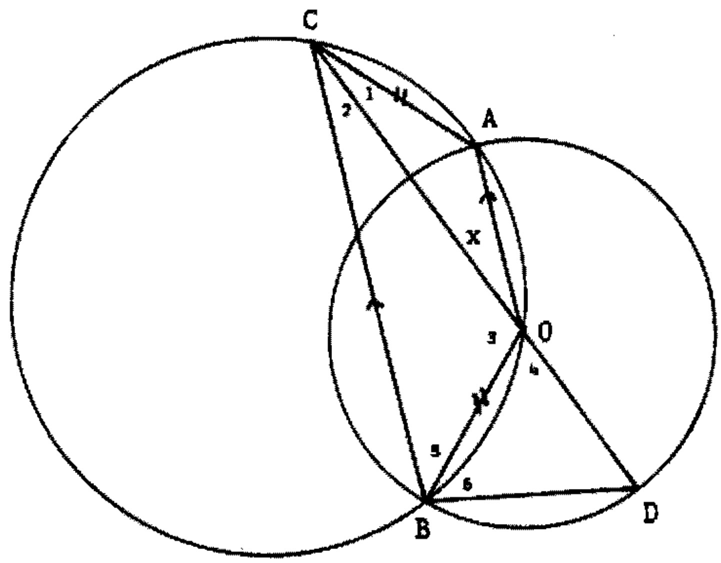


FIG. 9

9.1 Prove that $\hat{C}_1 = \hat{C}_2$. (5)

9.2 Calculate, giving reasons, the size of $\hat{B}_5$ in terms of x . (3)

9.3 If $x = 20°$, prove that ΔBOD is equilateral. (5)

[13]

TOTAL MARKS : 150

APPENDIX 5

TEACHER SCHEDULE: COGNITIVE LEVELS OF QUESTIONS

c/o Private Bag X 54323
Durban
4000

1 August 1992

The Senior Mathematics Teacher

Dear Sir/Madam

CLASSIFICATION OF 1991 S.C. MATHEMATICS QUESTIONS ACCORDING TO LEVELS OF ABILITIES (OBJECTIVES)

I am carrying out an investigation entitled: AN EVALUATION OF THE EFFICACY OF THE AIMS AND OBJECTIVES OF THE SENIOR CERTIFICATE MATHEMATICS CURRICULUM IN THE RSA

As the study involves, inter alia, an analysis of the performance of candidates in the subquestions set in Mathematics in the 1991 S.C. Examination in terms of 3 levels (groups) of objectives, it is necessary to obtain the level of objective each subquestion is testing.

Relevant literature reveals that since the classification of an item to a particular objective is a fairly subjective process, we are consequently obliged to rely on the consensus judgement of the experienced educators in the field; if the majority of the educators agree that a certain question measures say, comprehension, for the majority of the candidates, then we accept that question as one testing comprehension.

In view of the above, you assistance is kindly sought in identifying the level of objective that is being tested by each of the subquestions set in Mathematics HG P1 and P 2 and in Mathematics SG P1 and P2 in the 1991 S.C. Examination of the HOD. Kindly note that only 3 levels (groups) of objectives are to be used, viz., L1 (Knowledge and Skills), L2 (Comprehension/Understanding) and L3 (Application, Analysis, Synthesis, Evaluation and Creative).

A grid sheet (attached) which has been compiled to expedite the exercise, is enclosed. Kindly write down the level of objective being tested by each subquestion next to the appropriate subquestion number in Annexure B. For easy reference, copies of the 4 Mathematics papers are enclosed.

Kindly submit only the grid sheet, duly completed, as soon as possible to me, using the enclosed self addressed envelope.

Thank you for your co-operation and the time set aside for the research.

Yours sincerely

....

H. RAMBEHARI
Principal Education Planner
Department of Education and Culture (HOD)

1991 SENIOR CERTIFICATE EXAMINATION - MATHEMATICS
CLASSIFICATION OF SUBQUESTIONS ACCORDING TO LEVELS OF ABILITIES (L1, L2, L3)

HIGHER GRADE P I		HIGHER GRADE P II		STD. GRADE P I		STD. GRADE PII	
SUB QUEST NO.	LEVEL	SUB QUEST NO.	LEVEL	SUB QUEST NO.	LEVEL	SUB QUEST NO.	LEVEL
1.1.1		1.1.1		1.1.1		1.1	
1.1.2		1.1.2		1.1.2		1.1.1	
1.2.1		1.2.1		1.1.3		1.1.2	
1.2.2		1.2.2		1.2.1		1.2.1	
1.2.3		2.1		1.2.2		1.2.2	
1.2.4		2.2.1		1.3		1.2.3	
1.3		2.2.2		2.1.1		1.2.4	
2.1		2.2.3.1		2.1.2		2.1.1	
2.2.1		2.2.3.2		2.1.3		2.1.2	
2.2.2		3.1		2.1.4		2.2.1	
2.2.3		3.2.1		2.2		2.2.2	
2.3		3.2.2		3.1.1		2.3	
3.1		3.2.3		3.1.2		2.4.1	
3.2		3.3		3.1.3		2.4.2	
3.3.1		4.1		3.1.4		3.1	
3.3.2		4.2		3.1.5		3.2	
4.1		5.1		3.2		3.3	
4.2		5.2		4.1		4.1	
4.3		5.3.1		4.2		4.2	
4.4.1		5.3.2		5.1.1		4.3	
4.4.2		5.3.3		5.1.2		5.1	
5.1		5.3.4		5.2		5.2	
5.2.1		6.1.1		6.1.1		5.3	
5.2.2		6.1.2		6.1.2		5.4	
5.3.1		6.1.3		6.2		6.1	
5.3.2		6.2.1		7.1		6.2	
6.1		6.2.2		7.2.1		6.3	
6.2		7.1.1		7.2.2		7.1	
6.3		7.1.2		7.3.1		7.2.1	
7.1		7.2		7.3.2		7.2.2	
7.2.1		8.1		7.3.3		7.3	
7.2.2		8.2.1		8.1		8.1	
7.2.3		8.2.2		8.2		8.2.1	
8.1.1		9.1.1		8.3		8.2.2	
8.1.2		9.1.2		8.4		8.2.3	
8.2.1		9.2.1		8.5		9.1	
8.2.2		9.2.2		9.1		9.2	
8.3		9.2.3		9.2.1		9.3	
9.1		10.1		9.2.2			
9.2.1		10.2.1		9.3			
9.2.2		10.2.2		10.1.1			
9.3		10.2.3		10.1.2			
10.1.1				10.1.3			
10.1.2				10.1.4			
10.1.3				10.2.1			
10.2.1				10.2.2			
10.2.2				10.2.3			
10.2.3				10.2.4			
				10.2.5			

APPENDIX 6
LETTER OF AUTHORITY FROM THE HOD

HDE 1

Republic of South Africa
Republiek van Suid-Afrika

ADMINISTRATION: HOUSE OF DELEGATES
ADMINISTRASIE: RAAD VAN AFGEVAARDIGDES

Department of Education and Culture
Departement van Onderwys en Kultuur

☎ (031) 3606911 Fax: (031) 374261

Truro House
Trurohuis
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Victoria Embankment 17
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4000

Ref. No. / Verw. No. A10/29/2/22

Enquiries / Navrae S. Pillay

1992 -08- 11

Mr H. Rambehari
43 Bikaner Road
Merebank
DURBAN
4052

Sir

REQUEST FOR PERMISSION TO CONDUCT RESEARCH
Your letter dated 27 July 1992 has reference

1. Permission is hereby granted to you to conduct your research at the schools as indicated in your letter provided that :

1.1 prior arrangements are made with the principals concerned;

1.2 participation in the research is on a voluntary basis;

1.3 completion of questionnaires is done outside normal teaching time;

1.4 the IQ scores are used but pupils are not identified by name.

1.5 all information pertaining to pupils and teachers is treated confidentially and used for academic purposes only.

2. Permission is also granted to you to use information contained in Departmental publications and S.C. Examination scripts, so long as confidentality is maintained.

3. Kindly produce a copy of this letter when approaching schools.

4. The Department wishes you every success in your research and looks forward to receiving a copy of the findings.

Yours faithfully

APPENDIX 7
TABLES

TABLE 7.1 THE COGNITIVE LEVEL (LEVEL), THE FACILITY INDEX (FI) AND SD, AND DISCRIMINATION INDEX (DI) OF QUESTIONS SET IN MATHEMATICS HG PAPER 1 (1991 S.C. EXAMINATION - HOD)

Question	Level	FI	SD	DI
1.1.1	LL	89,9	26,5	0,29
1.1.2	ML	73,2	40,0	0,53
1.2.1	HL	71,5	33,7	0,54
1.2.2	HL	50,4	38,3	0,58
1.2.3	ML	66,2	34,9	0,49
1.2.4	ML	57,8	39,3	0,67
1.3	HL	11,8	29,3	0,55
2.1	ML	56,7	41,5	0,51
2.2.1	ML	68,6	37,3	0,56
2.2.2	ML	53,6	44,8	0,57
2.2.3	HL	43,2	34,4	0,58
2.3	HL	28,4	36,7	0,58
3.1	ML	80,2	33,8	0,48
3.2	ML	22,9	27,8	0,56
3.3.1	ML	33,3	21,7	0,61
3.3.2	HL	7,1	15,5	0,33
4.1	ML	84,2	28,6	0,45
4.2	HL	82,3	26,9	0,48
4.3	HL	16,2	34,8	0,31
4.4.1	HL	63,6	40,6	0,53
4.4.2	HL	29,7	39,1	0,59
5.1	LL	52,3	42,4	0,48
5.2.1	ML	89,5	26,5	0,31
5.2.2.	HL	76,2	38,4	0,51
5.3.1	HL	15,3	31,5	0,42

Question	Level	FI	SD	DI
5.3.2	HL	11,9	27,5	0,44
6,1	ML	67,5	33,8	0,54
6.2	ML	26,9	35,5	0,59
6.3	HL	22,1	24,5	0,28
7.1	ML	48,4	34,0	0,51
7.2.1	ML	30,2	36,2	0,46
7.2.2	HL	24,2	20,7	0,41
7.2.3	ML	69,5	45,9	0,35
8.1.1	ML	94,8	15,6	0,20
8.1.2	ML	33,8	37,5	0,49
8.2.1	LL	92,4	20,9	0,28
8.2.2	ML	58,3	36,8	0,54
8.3	HL	47,1	43,1	0,63
9.1	ML	76,7	34,8	0,49
9.2.1	HL	53,1	44,0	0,52
9.2.2	HL	6,0	21,0	0,38
9.3	HL	18,0	28,0	0,66
10.1.1	ML	34,3	45,5	0,50
10.1.2	ML	42,1	40,5	0,30
10.1.3	ML	4,5	20,7	0,21
10.2.1	ML	76,7	37,3	0,39
10.2.2	ML	80,3	39,8	0,36
10.2.3	HL	43,2	39,1	0,62

TABLE 7.2 THE COGNITIVE LEVEL (LEVEL), THE FACILITY INDEX (FI) AND SD, AND DISCRIMINATION INDEX (DI) OF QUESTIONS SET IN MATHEMATICS HG PAPER 2 (1991 S.C. EXAMINATION - HOD)

Question	Level	FI	SD	DI
1.1.1	LL	82,4	26,7	0,39
1.1.2	ML	45,3	38,5	0,64
1.2.1	LL	83,3	37,1	0,29
1.2.2	ML	69,0	35,2	0,41
2.1	LL	81,8	30,2	0,36
2.2.1	ML	74,2	41,0	0,50
2.2.2	HL	41,0	46,0	0,63
2.2.3.1	ML	59,9	40,3	0,52
2.2.3.2	LL	33,8	41,2	0,50
3.1	LL	87,6	22,0	0,38
3.2.1	ML	59,6	37,6	0,31
3.2.2	ML	55,1	42,8	0,42
3.2.3	ML	36,8	38,4	0,46
3.3	HL	49,0	34,2	0,60
4.1	HL	48,2	41,1	0,63
4.2	HL	46,7	27,3	0,61
5.1	LL	58,7	40,4	0,45
5.2	ML	36,8	34,2	0,55
5.3.1	ML	83,8	33,7	0,41
5.3.2	ML	69,8	37,6	0,62
5.3.3	HL	63,1	40,0	0,61
5.3.4	HL	36,7	39,0	0,67
6.1.1	ML	86,5	28,0	0,39
6.1.2	HL	45,0	46,4	0,50
6.1.3	HL	28,5	31,6	0,57
6.2.1	ML	67,7	38,1	0,51
6.2.2	HL	26,4	38,4	0,50
7.1.1	ML	83,8	26,6	0,29
7.1.2	ML	39,0	38,2	0,54
7.2	HL	36,2	45,3	0,54
8.1	ML	33,6	41,8	0,54
8.2.1	ML	62,0	48,4	0,35
8.2.2	ML	7,8	21,7	0,42
9.1.1	LL	97,6	15,2	0,11
9.1.2	LL	96,2	19,2	0,16
9.2.1	ML	65,7	32,3	0,42
9.2.2	ML	23,0	40,0	0,48
9.2.3	HL	44,1	44,0	0,55
10.1	LL	59,7	47,0	0,42
10.2.1	ML	37,6	40,0	0,53
10.2.2	HL	51,2	48,3	0,59
10.2.3	HL	30,8	41,1	0,53

TABLE 7.3 THE COGNITIVE LEVEL (LEVEL), THE FACILITY INDEX (FI) AND SD, AND DISCRIMINATION INDEX (DI) OF QUESTIONS SET IN MATHEMATICS SG PAPER 1 (1991 S.C. EXAMINATION - HOD)

Question	Level	FI	SD	DI
1.1.1	LL	78,1	36,6	0,31
1.1.2	ML	5,2	18,6	0,16
1.1.3	ML	20,0	35,0	0,45
1.2.1	ML	67,9	41,9	0,55
1.2.2	ML	20,9	34,5	0,56
1.3	HL	13,0	28,8	0,38
2.1.1	ML	32,2	42,1	0,59
2.1.2	ML	47,9	43,2	0,54
2.1.3	ML	25,9	31,6	0,59
2.1.4	ML	36,2	39,3	0,52
2.2	HL	27,0	35,0	0,59
3.1.1	ML	65,9	46,0	0,55
3.1.2	ML	27,9	39,2	0,58
3.1.3	LL	35,8	43,7	0,60
3.1.4	ML	21,4	34,8	0,59
3.1.5	ML	23,4	39,7	0,48
3.2	ML	54,2	37,4	0,61
4.1	ML	26,4	34,4	0,37
4.2	ML	44,5	36,3	0,56
5.1.1	ML	82,6	36,5	0,34
5.1.2	ML	36,9	40,2	0,56
5.2	ML	34,6	35,8	0,61
6.1.1	ML	51,2	31,8	0,56
6.1.2	LL	71,1	45,4	0,25
6.2.	HL	42,9	37,2	0,51
7.1	ML	25,9	36,7	0,40
7.2.1	ML	39,8	46,2	0,57
7.2.2	ML	16,5	24,4	0,56
7.3.1	ML	44,5	43,7	0,60
7.3.2	LL	53,3	49,9	0,49
7.3.3	ML	27,1	33,8	0,45
8.1	LL	31,8	46,6	0,32
8.2	ML	15,6	31,6	0,54
8.3	ML	18,9	33,8	0,53
8.4	ML	17,9	33,7	0,54
8.5	HL	4,1	19,0	0,31
9.1	ML	53,0	42,2	0,49
9.2.1	ML	42,1	39,0	0,62
9.2.2	ML	31,3	39,4	0,65
9.3	HL	17,9	31,5	0,53
10.1.1	ML	55,6	38,6	0,55
10.1.2	HL	30,7	36,6	0,65
10.1.3	ML	29,4	32,9	0,60
10.1.4	HL	6,7	21,8	0,31
10.2.1	ML	11,9	31,1	0,32
10.2.2	ML	14,7	35,4	0,45
10.2.3	HL	10,6	26,0	0,41
10.2.4	HL	14,0	31,3	0,53
10.2.5	ML	11,6	30,7	0,46

TABLE 7.4 THE COGNITIVE LEVEL (LEVEL), THE FACILITY INDEX (FI) AND SD, AND DISCRIMINATION INDEX (DI) OF QUESTIONS SET IN MATHEMATICS SG PAPER 2 (1991 S.C. EXAMINATION - HOD)

Question	Level	FI	SD	DI
1.1	LL	81,9	25,3	0,43
1.1.1	ML	56,2	42,7	0,39
1.1.2	ML	40,7	41,9	0,41
1.2.1	ML	44,2	47,4	0,35
1.2.2	ML	44,5	40,0	0,47
1.2.3	ML	12,8	30,1	0,31
1.2.4	HL	15,7	30,7	0,42
2.1.1	ML	40,5	49,1	0,21
2.1.2	ML	35,8	48,0	0,36
2.2.1	ML	60,0	42,5	0,48
2.2.2	HL	17,6	29,3	0,44
2.3.	HL	63,5	37,3	0,48
2.4.1	ML	47,1	48,4	0,61
2.4.2	HL	13,8	31,7	0,44
3.1	ML	76,9	40,3	0,42
3.2	ML	60,1	48,8	0,50
3.3	ML	49,3	46,4	0,56
4.1	LL	77,4	39,1	0,45
4.2	ML	61,6	43,0	0,62
4.3	HL	24,1	27,3	0,65
5.1	ML	29,9	45,2	0,58
5.2	ML	48,8	43,3	0,56
5.3	HL	11,2	29,1	0,52
5.4	HL	5,8	19,6	0,45
6.1	LL	49,0	45,1	0,50

Question	Level	FI	SD	DI
6.2	ML	29,8	41,9	0,60
6.3	HL	45,6	45,8	0,53
7.1	LL	50,0	43,1	0,56
7.2.1	ML	55,5	42,8	0,64
7.2.2	ML	38,2	43,3	0,66
7.3.	HL	34,2	42,6	0,54
8.1	LL	28,4	37,7	0,64
8.2.1	ML	45,9	39,2	0,68
8.2.2	ML	15,4	33,4	0,51
8.2.3	HL	6,2	21,6	0,44
9.1	ML	43,5	42,0	0,55
9.2	HL	20,9	34,7	0,48
9.3	HL	17,8	34,4	0,52

APPENDIX 8

STATISTICAL METHODS USED IN THIS STUDY

1. Mean, Standard Deviation and Standard Error of Mean

The arithmetic mean is calculated using the formula:

$$\overline{X} = \frac{\Sigma X}{N}$$

Where X = the score; ΣX = sum of scores; and N = the number of cases (Downie & Heath 1974 : 39-40)

The standard deviation is calculated as follows:

$$SD = \sqrt{\frac{\Sigma x^2}{N}} \quad \text{where } x = X - \overline{X}$$

(Downie & Heath 1974 : 54)

The standard error of the mean yields a confidence interval of the mean and is given by the formula:

$$S\bar{x} = \frac{SD}{\sqrt{N - 1}}$$

(Downie & Heath 1974 : 54)

2. Z-scores and T-scores

z-scores and T-scores are also referred to as standard scores. The formula for computing z-scores is:

$$\text{z-score} = \frac{X - \overline{M}}{SD}$$

Where X = any raw score; $\overline{M}$ = arithmetic mean of raw scores; and SD = standard deviation of raw scores.

T-scores can be obtained by multiplying the z-score by 10 and adding the product to 50. Thus,

T-score = 50 + 10 (z-score).

(Gronlund & Linn 1990:352)

3. <u>Discrimination Index</u>

The discrimination index, which is the Pearson Product Moment Correlation coefficient, is computed using the following formula:

$$D = r = \frac{N\,\Sigma XY - (\Sigma X)\ (\Sigma Y)}{\sqrt{[N\Sigma X^2 - (\Sigma X)^2][N\,\Sigma Y^2 - (\Sigma Y)^2]}}$$

where Σ = sum of

N = total number of scores

X = scores on the question

Y = total scores on the paper

r = correlation coefficient between X and Y.

(Behr 1973 : 99)

4. <u>Reliability</u>

A reliability estimate is obtained by using the Kuder-Richardson Formula 20 which is defined by:

$$r_{tt} = k/k -1\ (1 - \Sigma\, pq/\ (SD)^2)$$

where r_{tt} = reliability coefficient of test

K = number of items in the test

$(SD)^2$	=	variance of test
p	=	proportion of pupils getting an item correct (obtained from the difficulty index)
q	=	(1 - p), i.e., proportion of pupils getting an item incorrect.

(Downie & Heath 1974 : 240)

5. <u>Analysis of Variance (ANOVA)</u>

One-way ANOVA (samples of unequal size)

* Sum of squares for the BETWEEN GROUPS is given by

$$(SS)_b = \Sigma n_s \ (M_s - \ M_t \ )^2$$

where n_s = number of cases in a specified group

M_s = mean of this group

M_t = mean of all observations

* Sum of squares for the WITHIN GROUPS is given by

$$(SS)_w \ = \Sigma x_s{}^2 = \ (X_s - \ M_s)^2$$

where x_s is the deviation of a score in a particular group from the mean of that group.

* F- ratio = $\dfrac{(SS)_b}{(SS)_w}$

d.f. for between groups is (no. of groups - 1)

d.f for within groups is $\Sigma(n_s - \ 1)$

(See Guilford & Fruchter 1973 : 230-239)

Significance levels for the F-Distribution were read from tables supplied by Slakter (1972: 437 - 442)

6. Multivariate Analysis of Variance (MANOVA)

In MANOVA, similar calculations are carried out for more than one score (such as levels 1, 2 and 3). From these matrices of between groups (H) and within groups (E), sums of squares and cross products are calculated. Statistics, similar to the F-test for ANOVA, are calculated for MANOVA (e.g., Wilks' lamda which is the determinant of E over the determinant of (E + H).

In particular, repeated measure MANOVA was used to construct tests for differences between scores (e.g., levels, subjects and interaction between them). This was combined with MANOVA tests for differences in the class variables (e.g., location, grade, gender and interactions between them) and any interactions between class variables and repeated score variables.

The rsth elements of the H and E matrices are:

$$h_{rs} = \sum_{j=1}^{k} \frac{1}{N_j} T_{jr} T_{js} - \frac{1}{N} G_r G_s$$

$$e_{rs} = \sum_{j=1}^{k} \sum_{i=1}^{N_j} x_{ijr} x_{ijs} - \sum_{j=1}^{k} \frac{1}{N_j} T_{jr} T_{js}$$

where x_{ijr} = ith observation on response r under treatment j

$T_{jr} = \sum_{i=1}^{N_j} x_{ijr}$ = sum of all observations on rth response in presence of treatment j

$G_r = \sum_{j=1}^{k} T_{jr}$ = grand total of all observations on rth response

$N = N_1 + \cdots + N_k$

(Morrison 1967 :180)

7. Chi-Square (x^2)

The Chi-square statistic is a test of significance which compares observed frequencies (O) with expected frequencies (E) (Downie & Heath 1974 : 189). Observed frequencies are obtained empirically while expected frequencies are generated on the basis of some hypothesis or theoretical speculation (Ferguson 1981 : 191)

The general formula for Chi-square is given by

$$x^2 = \sum \frac{(O - E)^2}{E}$$

where O = observed frequency
E = expected frequency, and
n = number of frequencies

The Chi-square is used to test the null hypothesis that the observed frequencies do not differ from the expected frequencies by chance. The level of significance is read from probability tables for (n-1) degrees of freedom (Downie & Heath 1974: 190-191 & 307).