CREATING AND LEARNING ABSTRACT ALGEBRA: HISTORICAL PHASES AND CONCEPTUAL LEVELS

by

EDITH GLENDA NIXON

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PROMOTER: PROF J HEIDEMA

JOINT PROMOTERS: PROF D C J WESSELS
PROF M D DE VILLIERS

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Summary

Piaget observed that various stages involved in the construction of different forms of knowledge are sequential and that the same sequential order is evident in history. There seem to be three main stages in the development of algebra involving the independent and general solution of equations followed by the evolution of abstract algebra. Piaget referred to these as the intra, inter and transoperational stages but they are termed the levels of percepts, concepts and abstractions here. The perceptual level involves isolated forms, the conceptual level concerns correspondences and transformations amongst forms whilst the abstract level is characterised by the evolution of structures of forms.

Historically the overall perceptual level of abstract algebra lasted from antiquity to the middle of the eighteenth century. The conceptual level followed, lasting for approximately one century and the subsequent abstract level has prevailed from the middle of the nineteenth century onwards. Each of these levels involve numerous sublevels but instead of being continually broken down into more and more sublevels, in this study a spiral of learning is being considered. Each round of the spiral contains a perceptual, conceptual and abstract level. The way in which perceptual levels can arise from previous abstract levels gives an indication of how knowledge is reorganised and expanded in new unexplored directions as the spiral is climbed. The important aspects of proof and axiomatisation are also addressed here.

The historical emergence of abstract algebra reveals a significant pattern concerning the development of mathematics. The levels of thinking involved are important and reveal a general trend of algebraic thought. Hence careful consideration needs to be paid to the revelations arising from historical investigations so that these may help contribute to the encouragement of learning in students of algebra. The idea of levels of learning has been substantiated by many researchers and investigations undertaken in the past. The main characteristics of the three relevant levels and sublevels as well as insights gained from the historical emergence of algebra are being united here to form a comprehensive theory of learning algebra at both the secondary and tertiary levels of study.

Student Number: 398-200-9

I declare that
CREATING AND LEARNING ABSTRACT ALGEBRA: HISTORICAL PHASES AND CONCEPTUAL LEVELS is my own work and that all the sources that I have used or quoted have been indicated and acknowledged by means of complete references.

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(Mrs E.G.Nixon) Date

Acknowledgements

I wish to express my gratitude to:

Jesus Christ the same yesterday,
and today, and forever (Hebrews 13: 8)

I will love thee, O Lord, my strength.
The Lord is my rock, and my fortress,
and my deliverer; my God, my strength,
in whom I will trust; my buckler,
and the horn of my salvation,
and my high tower.
I will call upon the Lord,
who is worthy to be praised. (Psalm 18: 1,2,3)

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Auctus tortuosus algebrae est mirificus et aeternus.

The spiral growth of algebra is wonderful and eternal.

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